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Numerical simulation of a bifurcating jet within a radially confined domain

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Abstract

Direct Numerical Simulation is used to model the discharge of a periodically perturbed jet of fluid into a pipe that is five times the diameter of the jet. A planar flapping perturbation at the jet inlet, when used on its own or when combined with an axisymmetric perturbation, leads to a bifurcation of the jet some distance downstream. The present observations help resolve a previous discrepancy between experimental and numerical results concerning the presence of this bifurcation phenomenon. Across all periods of perturbation, the bifurcation is strongest, and occurs closer to the jet inlet, when the flapping perturbation is used exclusively. For both single mode and bimodal perturbation, bifurcation is observed across Strouhal numbers (the ratio of the product of the inlet diameter and axial perturbation frequency to the inlet bulk speed) in the range 0.33–1. This is an expansion of the previously reported range for which this phenomenon has been observed.

Introduction

The discharge of a jet from a wall into a body of quiescent fluid arises in many applications, ranging from the exhausts of jet engines to many manufacturing processes involving the mixing or reaction of fluids. In these flows, the nature of the mixing between the injected and quiescent fluid is important.

Past studies (detailed in [8]) have revealed that the addition of periodic perturbations to the initial jet velocity profile can lead to interesting downstream flow patterns and enhanced fluid mixing. When an axisymmetric periodic perturbation is applied to the jet efflux velocity, vortex rings form in the jet shear layer. With the addition of a planar "flapping" perturbation, these vortex rings can be made to tilt away from the plane of the jet nozzle. Tilted vortex rings will naturally propagate along their axis, and thus away from the central axis.

If the flapping perturbation causes successive vortex rings to tilt in different directions, then their mutual interaction can amplify the tilting. By controlling the frequencies of the axial and flapping perturbations, it has been observed experimentally [8] that jets can be made to branch into two, three or four streams, or bloom out in all directions. It has been observed both experimentally [8] and via direct numerical simulation [5] that such flow structures can greatly enhance mixing between the jet efflux and surrounding fluid.

This investigation considers a perturbed inlet jet profile of the form

$$U_P = U(r)\left[1 + A_0 \sin(2\pi f_0 t) + A_1 \sin(2\pi f_1 t + \phi)\cos(\theta)\frac{2r}{D}\right] \qquad (1)$$

where, in a cylindrical coordinate system (z,r,θ) with origin at the centre of the jet, $U(r)$ is a steady parabolic axial velocity profile through an inlet of diameter D. Here A and f give the amplitude and frequency of the perturbations, and the subscripts 0 and 1 denote the axial and flapping perturbations respectively. ϕ is a constant that specifies the phase difference between the two perturbations. This jet inflow is of the same form used in [4] and [5].

For a spatially and temporally averaged inlet bulk speed U_m, we may characterise the flow using at least five dimensionless parameters: the Reynolds number $Re = U_m D/\nu$, the Strouhal number of the axial disturbance $St_a = f_0 D/U_m$, the ratio of axisymmetric and flapping perturbation frequencies $\beta = f_0/f_1$, and the ratios A_0 and A_1 of the amplitudes of the axial and azimuthal disturbances to that of the base flow. For bimodal perturbations, we only consider the case where $\beta = 2$, meaning that successive axial pulses correspond to flapping pulses in opposing directions. This leads to successive vortex rings being tilted in opposite directions, potentially leading to the bifurcation of the jet stream. Bifurcation can be defined to have occurred when the streamwise velocity splits into two separate peaks as one moves downstream from the jet inlet.

The present study focuses on low Reynolds number (200) and high amplitude perturbations ($A_1 = 0.5$, $A_0 = 0$ or 0.5 depending on whether or not axisymmetric perturbations are used), with no phase difference between the two perturbation modes (i.e. $\phi = 0$).

Domain and Numerical Method

The simulations use spectral element discretisation in the meridional semiplane and Fourier discretisation in the azimuthal direction to solve the incompressible Navier–Stokes equations. A cylindrical domain of diameter $5D$ and axial length $25D$ is used. Each of eight meridional semiplanes consists of 650 spectral elements, whose geometry is shown in figure 1. Tensor-products of eighth-order Gauss-Lobatto-Legendre Lagrange shape functions are used within each element. Temporal integration uses a second order backwards time differencing velocity correction scheme. More details of the numerical method is given in [1].

No-slip boundary conditions are implemented at the wall from which the jet enters, and across the circumferential boundary of the domain. This represents a different approach to that taken in [4] and [5], where fluid is allowed to advect across the sidewalls of the domain. This distinction is potentially important, as a jet efflux entering into ambient fluid naturally entrains flow from the far field (for example [9]). The domain described could more closely represent the finite diameter domains present in the applications of flows of this nature. It can be noted that axisymmetric pipe expansions have been studied in numerous other contexts, such as [3]. Convective boundary conditions [7] are used at the far end boundary, with a bulk outflow convection speed of $0.6U_m$. This allows (potentially) recirculating fluid to advect across the domain boundary.

Figure 1: Geometry of the 650-element mesh in each meridional semiplane. Units are relative to D.

Axisymmetric Pulsatile Flow

When A_1 = 0, the inlet velocity profile is axisymmetric. For a non-zero A_0, a vortex ring is produced for each velocity maximum of the perturbed inlet velocity. This is shown in figure 2 for A_0 = 0.5 and St_a = 0.5, with the behaviour similar for other perturbation frequencies within the range 0.33 ≤ St_a ≤ 1. Due to the relatively low Reynolds number, the vorticity dissipates well before the far end of the domain. This scenario can be used as a basis for comparison for the nonaxisymmetric simulations. The effects of the presence of the far end boundary have been included to give scale to the domain, and to demonstrate that they do not affect the region of interest.

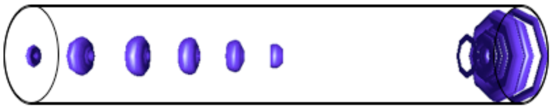

Figure 2: Isosurfaces of the vortex core measure of Jeong and Hussain [5] for an axisymmetric pulsatile just with St_a = 0.5 and A_0 = 0.5.

Bimodal Perturbation

If a flapping perturbation is added to the axisymmetric perturbation to the inlet jet, vortex rings continue to form for each maximum of the axisymmetric perturbation (figure 3). The initial tilt of each vortex ring is not particularly noticeable, however as each vortex ring travels downstream, its interaction with neighbouring vortex rings amplifies this tilt and ultimately leads to its breakdown. For all dimensionless pulse periods, the remnants of the individual vortex rings become indiscernible approximately 13 jet diameters downstream of the inlet.

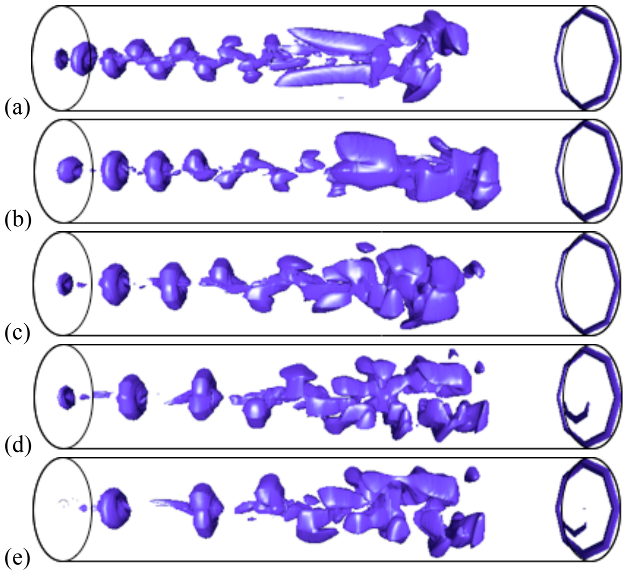

Figure 3: Isosurfaces of the vortex core measure for bimodal perturbation (A_0=A_1=0.5) and St_a = (a) 1, (b) 0.67, (c) 0.5, (d) 0.4 and (e) 0.33. View is of the plane of flapping.

The bifurcation of the jet can be quantified by considering time-averaged contours of axial velocity, as shown in figure 4. We observe that the jet typically breaks off into two distinct streams within the plane of "flapping", with the streams separating at approximately the same axial location as where the vortices break down. This bifurcation is strongest for smaller Strouhal numbers, in the sense that, within cross-sections normal to the inflow, there is typically a larger difference between the peak streamwise velocity and that at the central axis. This could be due to larger spatial separation between vortex rings allowing for greater uninhibited vortex propagation away from the central axis. It is noted here, but will be discussed later, that bifurcation is not apparent for a dimensionless perturbation frequency of 0.67. These observations of bifurcation are in general agreement with experimental observations documented in [8], but run counter to the lack of bifurcation for bimodal perturbation observed in the numerical studies of [4]. There are at least four factors (aside from the different side boundary conditions) that could feasibly account for this difference. To begin with, the present study uses larger perturbation amplitudes (0.5 compared to 0.15) and a different phase difference between the modes of perturbation (ϕ = 0 rather than $\pi/4$) to [4]. Additionally, the simulations in [4] were at St_a = 0.55, which our results indicate could be within a region in parameter space of weak bifurcation. Finally, [4] used a domain of length 15D, a length after which bifurcation was only beginning to become apparent in our simulations.

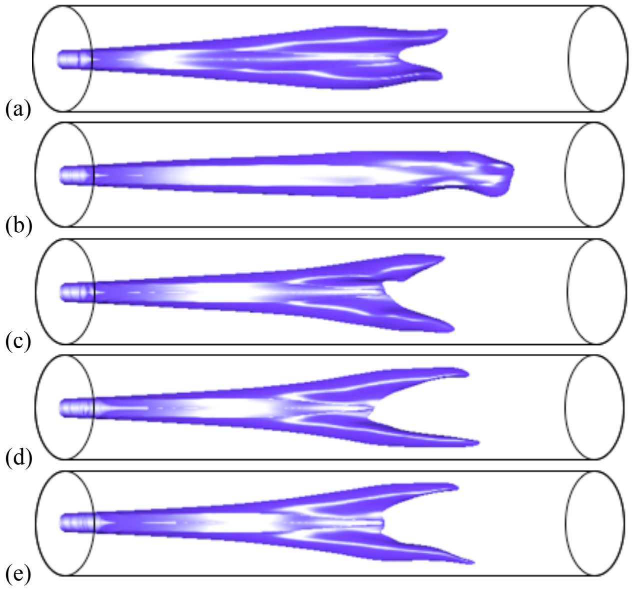

Figure 4: Time-averaged isosurfaces of axial velocity 0.4U_m for bimodal perturbation (A_0=A_1=0.5) and St_a = (a) 1, (b) 0.67, (c) 0.5, (d) 0.4 and (e) 0.33. View is of the plane of flapping.

Flapping-only Perturbation

The characteristics of jet flow with bimodal perturbation can be compared to those where only the flapping perturbation is applied (i.e. A_0 = 0). Although the axisymmetric perturbation mode is not present, for consistency we will continue to parameterise the frequency of perturbation by St_a (with β =2). For flapping-only perturbation, the vortex core measure isosurfaces in figure 5 do not show clear evidence of distinct vortex rings being formed, but rather alternating trails of vorticity corresponding to each half-period of flapping. This periodicity becomes indiscernible closer to the jet inlet than in the bimodal case, suggesting that mixing occurs more effectively with only this single perturbation mode. A further point of difference with the bimodal case is that there appears to be a distinguishable dependence between the perturbation period and the axial location where this breakdown occurs (increasing from about 6D for St_a = 1 to 8.5D for St_a = 0.33). This is potentially a result of smaller pulse periods inherently allowing for more comprehensive interaction between fluid from successive pulses.

Figure 6 shows that the bifurcation is more pronounced, and occurs closer to the inlet, for flapping-only perturbation across all Strouhal numbers. The increase in the level of bifurcation is also evidenced by figure 5, where the vortex core measure isosurfaces appear to split and change shape at approximately the location of bifurcation. Unlike figure 5, however, figure 6 does not display any clear correlation between the pulse period and the nature of bifurcation.

It appears that entraining fluid in distinct vortex rings hinders bifurcation, which, in this set of simulations, does not occur until the vortex rings break down. The presence of bifurcation using flapping-only perturbation is in agreement with [4].

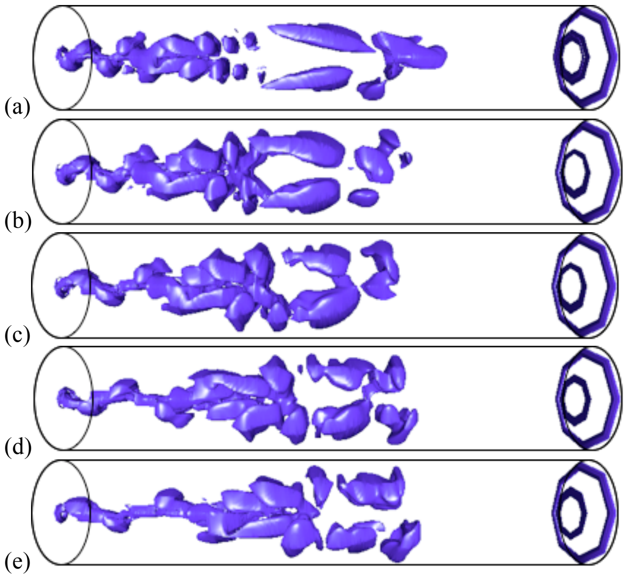

Figure 5: As for figure 3, but with for flapping-only perturbation (A_0=0).

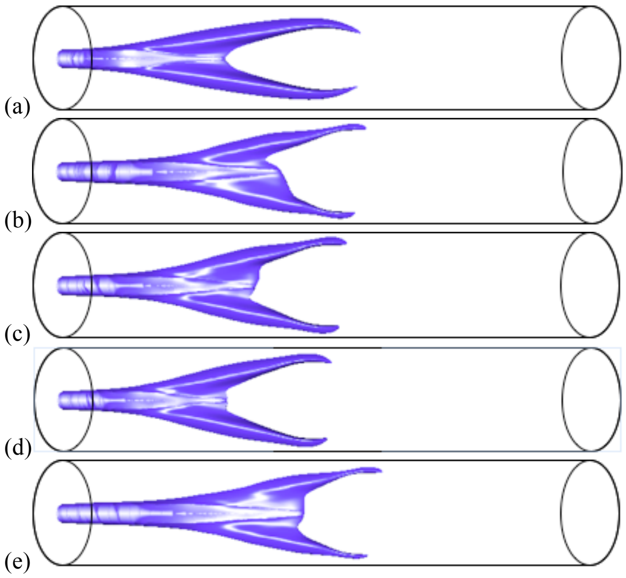

Figure 6: As for figure 4, but with for flapping-only perturbation (A_0=0).

Discussion

We now return to discuss in more detail the apparent abnormality seen in figure 3(b), where no bifurcation is observed for dual-mode perturbation at $St_a = 0.67$. It is possible that here the vortex rings are too close to each other to allow for them to propagate radially outwards after tilting. This is approximately consistent with the experimental findings, which document a maximum St_a (at *Re* between 2800 and 10 000) for bifurcation of 0.7. The fact that we observe bifurcation at $St_a = 1$ now needs to be rationalised. It could be possible that here successive vortex rings are so close to each other that they are never allowed to propagate independently, and instead almost immediately decay into alternating trails of vorticity. In this sense, the bifurcating mechanism here might be closer to what is observed for the flapping-only perturbations. Indeed, looking at the vortex core isosurfaces at the location of bifurcation, it seems as though the characteristics of figure 3(a) more closely resemble those in figure 5 than in 3(b)-(e). Irrespective of the mechanism, it has been found that bifurcation can exist at higher pulse frequencies than previously observed. We can also note that the bifurcation at the lowest Strouhal number tested is approximately at the limit of the lowest pulse frequency for which bifurcation has been documented. Alternating vortex tilting, as seen in figure 3, has been observed as an instability mode in the context of pulsatile stenotic flows [2], with a least stable Strouhal number of 0.31. This perhaps indicates that bifurcation could occur at even lower frequencies than those tested.

Conclusions

Jet bifurcation was observed for both dual-mode (axisymmetric and flapping) and flapping only inlet perturbations for Strouhal numbers between 0.33 and 1 (relating to a real or imagined axisymmetric perturbation frequency that is always twice the flapping frequency). The only conditions where bifurcation was not clearly evident was for bimodal perturbation at a Strouhal number of 0.67. This pulsation period is close to the experimentally determined lower limit for this phenomenon. The presence of bifurcation at the even higher Strouhal number of 1 has not been previously documented, and is possibly a result of a bifurcation mechanism different to the vortex tilting observed experimentally.

Flapping only perturbation was found to both accelerate the onset and increase the extent of bifurcation. This is likely to be due to a differing bifurcation mechanism, which bypasses the formation of and interaction between distinct vortices. Apart from limiting the extent to which spreading could occur, the presence of side walls does not seem to have qualitatively affected the bifurcation phenomenon, which is relevant to potential applications of such flows. Further work could test the effects of altering the radial size of the domain, over an extended range of the governing dimensionless parameters.

Acknowledgments

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