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Statistical Modeling to Determine Resonant Frequency Information Present in Combustion Chamber Pressure Signals

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Abstract

A statistical modeling method to accurately determine combustion chamber resonance is proposed and demonstrated. This method utilises Markov-chain Monte Carlo (MCMC) through the use of the Metropolis-Hastings (MH) algorithm to yield a probability density function for the combustion chamber frequency and find the best estimate of the resonant frequency, along with uncertainty. The accurate determination of combustion chamber resonance is then used to investigate various engine phenomena, with appropriate uncertainty, for a range of engine cycles. It is shown that, when operating on various ethanol/diesel fuel combinations, a 20% substitution yields the least amount of inter-cycle variability, in relation to combustion chamber resonance.

Introduction

Fluid phenomena frequently involve pressure fluctuations which are analysed using a variety of techniques. In-cylinder pressure has been a focal point of engine research since its introduction[8, 13]. Routinely during engine research in-cylinder pressure is used to investigate the indicated power, indicated mean effective pressure, peak pressure, maximum rate of change of pressure, heat release, and thermal efficiency of engines[10, 16]. However, as the greatest rate of change of pressure occurs during combustion near top dead centre (TDC) of the piston motion, where the velocity of the piston is low, it is necessary to examine the Pressure vs Crank-angle (p-θ), or a Pressure vs Time, plot to obtain the detailed characteristics of combustion[10]. It is this region that is of interest.

Some interesting studies have been performed by other researchers in relation to resonant frequencies in combustion chambers from in-cylinder pressure[11, 15, 17, 18]. Isolation of the resonant frequency is important as it is related to the speed of sound and hence temperature[11, 15, 17, 19]. Since, many assumptions are made when analysing data, it is of great importance that the implications of these assumptions are understood[3]. Typically, researchers interested in combustion resonance have used Fast Fourier Transforms (FFT) to find the desired frequency information[11, 17, 19].

Fast Fourier transforms are common practice in basic spectral analysis; mostly because of their ease of use and computational efficiency[4, 6, 14]. However, assumptions which underpin them, for example the assumption of periodic stationary frequencies, and their limited resolution can make them an undesirable tool for use in spectral analysis[3, 12]. Further, FFTs, and many other traditional techniques, also have problems when there is noise or incomplete data[12]. Limited data is also a major draw back with the FFT approach to spectral analysis, to produce any useful results multiple periods of data are required[9]—in some applications, such as the one proposed, analysing multiple periods simultaneously is counter to the aim.

Bretthorst explains that if there are complex phenomena or evidence of more than a single stationary harmonic frequency the Fourier method may yield incorrect or misleading results[3]. Such results obtained with FFTs are not incorrect because the FFT is wrong; they are simply not correct because the FFT is attempting to answer a different question[5]. Hence, other methods of spectral analysis need to be investigated to ensure that the information found is accurate and therefore usable for making decisions from or determining frequency information present in the data. Emphasis should be given more to scientific interest and less to mathematical convenience[2].

For the purpose of this investigation spectral analysis was performed using Markov-chain Monte Carlo (MCMC) in a Bayesian framework; specifically, using an implementation of the Metropolis-Hastings Algorithm[7]. In situations where the user can approach the problem from an informed perspective, using this methodology has the advantage that it requires the user to explicitly state any assumptions being made in the calculation[3]. Consequently, the user always knows exactly what problem is being solved. A further advantage is that this methodology can allow for noise in the data—which is always present for in-cylinder pressure data.

Experimental Configuration

Experiments were conducted on a modern turbo-charged 6-cylinder Cummins diesel engine (ISBe220 31) at the QUT Biofuel Engine Research Facility (BERF). The engine has a capacity of 5.9 l, a bore of 102 mm, a stroke length of 120 mm, a compression ratio of 17.3:1 and maximum power of 162 kW at 2500 rpm. The engine was coupled to an electronically controlled hydraulic dynamometer with load applied by increasing the flow rate of water inside the dynamometer housing. In-cylinder pressure was measured by a Kistler piezoelectric transducer with a Data Translation simultaneous analogue-to-digital converter connected to a desktop computer running National Instruments LabView. Data was collected at a sample rate of 200,000 samples per second. During testing the engine was run on neat diesel fuel and diesel/ethanol at 2000 rpm on full load.

Experimental Data

Data was collected with the engine running on neat diesel fuel, and also with ethanol fumigation substituting 10%, 20%, 30%, 40% and 50% of the diesel on an energy basis. Specific data that was collected included: in-cylinder pressure, band-pass filtered in-cylinder pressure (1–80 kHz), crank-angle information (updated every degree) and injection information. In-cylinder pressure was collected as a differential voltage signal and for this application is not converted to absolute pressure.

As this investigation has been focused on combustion resonance the injection information was used to define the region of interest and the band-pass filtered in-cylinder pressure data was used for the calculation. The frequencies of interest ranged from approximately 5 kHz to 10 kHz, the band window was set to be very wide to ensure that the frequencies of interest were unaffected by the filtering—the methodology employed allows for ignoring or modeling any additional phenomena present in the signal making filter selection less important. In this scenario it was only important to remove the low frequencies dealing with the speed of the engine. An example of this data is shown in Figure 1.

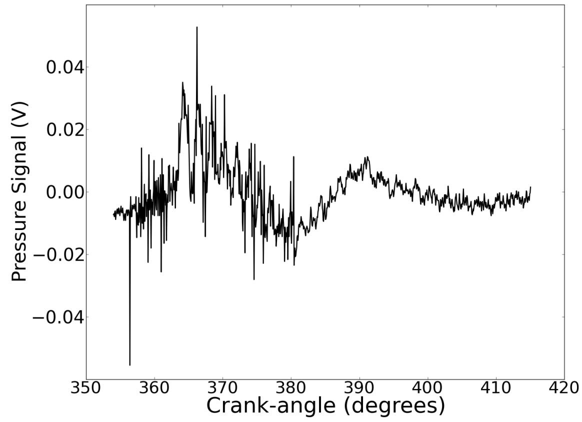

Figure 1: Combustion chamber pressure fluctuation after application of a band-pass filter (1–80 kHz): 2000 rpm, full load, neat diesel fuel

Mathematical Overview

The Metropolis-Hastings algorithm is an adaption of a random walk that uses an acceptance/rejection rule to converge to the specified target distribution[7]. For this implementation the random walk is performed by sampling from a Normal Distribution with the current value as the mean and some adaptable standard deviation. Each parameter has its own adaptable standard deviation that is updated every 300 trials, with the aim of controlling the acceptance to rejection ratio at 40%, using the following formula:

$$\sigma^* = \sigma e^{Acc-0.4}, \tag{1}$$

where σ^* is the updated standard deviation, σ the current standard deviation and Acc is the ratio of acceptance/rejection for the previous 300 trials. This ensures that the results obtained are useful; a 100% acceptance rate would indicate that we are not exploring the space properly and a 0% acceptance rate would indicate that we are not exploring the space at all.

Acceptance or rejection of each candidate value is driven by the data and by prior information. If the signal is defined as being normally distributed with a time varying mean:

$$y(n) \sim N(\mu(n), \tau), \tag{2}$$

where $y(n)$ is the signal, $\mu(n)$ is the mathematical model that we define and τ is the standard deviation—modeling this way accounts for noise present in the data—then the probability of acceptance, R, can be defined as:

$$R = min\left(1, \frac{\prod p(y_i|\mu_i^*, \tau)}{\prod p(y_i|\mu_i, \tau)} \cdot \frac{p(\theta^*)}{p(\theta)}\right), \tag{3}$$

where θ is the parameter in question and * denotes the use of the candidate parameter. The more probable that the candidate value is the true value, compared to the current value, the closer to 1 R becomes. For the case that $R < 1$ a random number is sampled between 0 and 1 from a uniform distribution and if R is greater than this value the candidate value is accepted.

In this implementation each parameter is updated 5 times per iteration. Further, before any values are saved there is a "burn-in" period of 10,000 iterations to ensure that each parameter has converged to a value. At the end of this period 1000 iterations are then performed where each parameter of interest is saved in an array at the end of each iteration. These saved values allow a probability density function (pdf) of each parameter to be produced where the mode is taken to be the best estimate of the parameter and the spread gives an indication of the uncertainty.

The Model

Examination of Figure 1 reveals four important features. Firstly, a period of very high frequency fluctuation associated with diesel injection; secondly, a peak associated with ignition; thirdly, a rapidly decaying low frequency shape; finally, the resonant frequency information which can be seen superimposed on the basic signal. Ignoring all but the resonant frequency and the low frequency shape, a model can be created that fits this data and finds the required information. More complex models can be employed if further information is desired, for example a term could be included to model the injection frequency or other frequency modes (radial, circumferential, axial); however, in this investigation, of interest is the first circumferential mode resonant frequency only. In this instance, to eliminate the need to model the injection, or model anything before the initial peak, a step function is used. Modeling just these features fits the data sufficiently to produce usable results whilst still being general enough to work across a range of cycles and engine operating modes, allowing cyclic behaviour to be examined as well as a comparison between engine operating conditions. The model is, therefore, as follows:

$$\begin{aligned} \mu(n) &= H(n-\delta)\left(A_1 e^{-\lambda_1 n}\sin\left(\frac{2\pi\omega_1 e^{-a_1 n}}{samplerate}n + \phi_2\right)\right. \\ &\quad \left. + \; A_2 e^{-\lambda_2 n}\sin\left(\frac{2\pi\omega_2 e^{a_2 n}}{samplerate}n + \phi_2\right)\right), \end{aligned} \tag{4}$$

where $H(n-\delta)$ is a step function that is equal to 0 when $n < \delta$ and 1 when $n \geq \delta$, and $\omega_2 e^{a_2 n}$ represents the resonant frequency.

The use of prior information for each parameter ensures that there are no issues with label switching and that we fit the data in the intended way. For instance, a_1 is given a uniform prior to contain it between 0.001 and 0.01. This ensures that it is positive, large enough to decay at a rate sufficient to model the lower frequency signal and that it does not cause complications with the modeling of the higher frequency signal. Similarly, a_2 is also given a uniform prior; however, it is contained between 0.0001 and 0.001 to ensure that it models the specific frequency of interest. In this framework it is acceptable to give a parameter an uninformative prior which has little effect on whether a candidate value is accepted or rejected; for example A_1 and A_2 were given normally distributed priors with relatively large standard deviations making them effectively entirely data driven. An example of the model fit, from equation (4), is shown in Figure 2. All results were obtained using NetBeans as the development environment utilising the standard gcc c++ compiler.

Results

In order to determine the value, and uncertainty, of any model parameter we typically plot its multiple values as determined by the Monte Carlo as a pdf. Figure 3 shows the pdf of parameter ω_2 and Figure 4 shows the pdf of a_2. Results obtained from figures 3 and 4 allow characterisation of the resonant frequency as a function of time. However, for the purpose of this investigation the instantaneous frequency located at the top of the first peak—seen in Figure 1—will be termed the reference resonant frequency and henceforth denoted as ω_r. An example of the results running the engine at 2000 rpm at full load for 15 s is shown in Figure 5. This figure depicts the significant inter-cycle variability present in a diesel engine running under normal conditions. ω_r ranges from 5200 to 6400 Hz with the modal reference resonant frequency being 5450 Hz with cycle-to-cycle deviations as great as 1000 Hz.

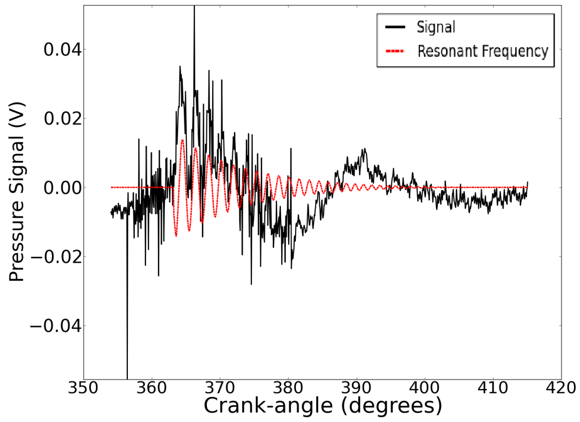

Figure 2: Comparison of expected resonant frequency signal from the second term of the model, using equation (4) on the signal in Figure 1.

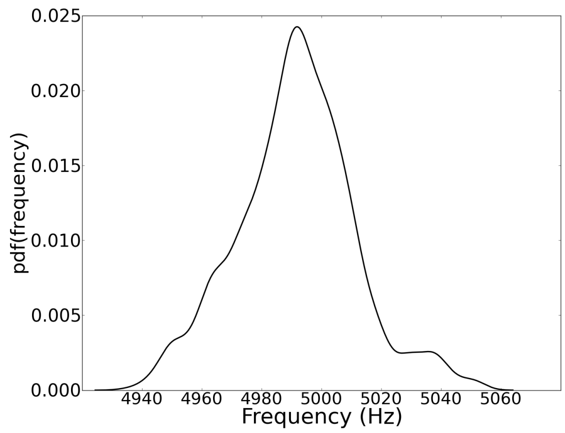

Figure 3: Pdf of ω_2 from equation (4) and the signal in Figure 1.

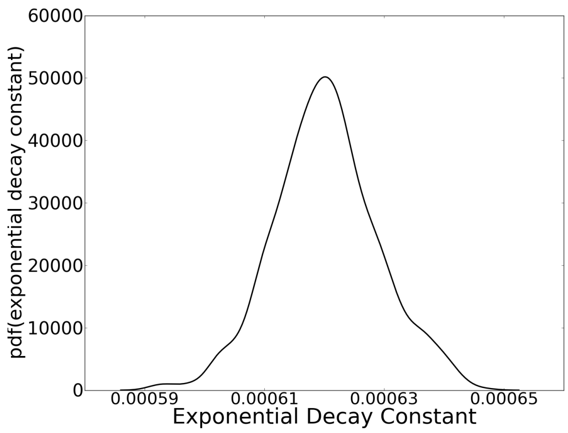

Figure 4: Pdf of a_2 from equation (4) and the signal in Figure 1.

The resonant frequency is directly related to the temperature[11]:

$$T \propto f^2, \tag{5}$$

therefore we can use the resonant frequency as a way of commenting on the repeatability of combustion. Figure 5 clearly shows that even whilst running on neat diesel fuel there is significant inter-cycle variability. Further, an observable feature of this plot is that typically prior to a high frequency peak is a cycle with a slightly lower frequency—perhaps an indication of unburned fuel present from one cycle having an effect on the next. Figure 6 shows the pdfs of the ω_r for 250 consecutive cycles with the engine running at 2000 rpm on full load on neat diesel fuel (D100E00) and with 10% (D090E010), 20% (D080E020), 30% (D070E030), 40% (D060E040) and 50% (D050E050) ethanol substitutions.

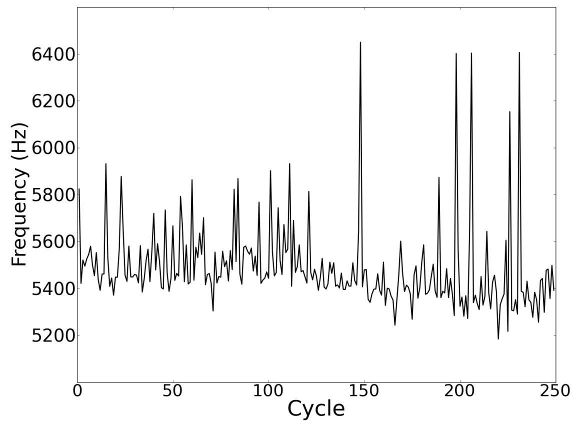

Figure 5: Reference resonant frequency, ω_r, for 250 sequential cycles with the engine at 2000 rpm and full load at selected ethanol substitutions.

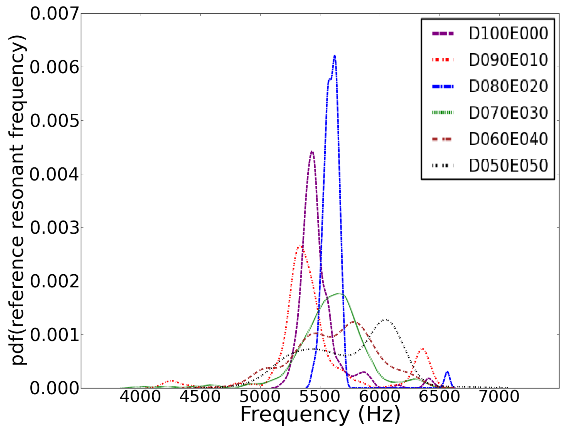

Figure 6: Pdfs of ω_r of various engine operating conditions.

It can be seen in Figure 6 that small ethanol substitutions lead to a slight decrease in ω_r. A 10% ethanol substitution gave a decrease of approximately 2%. Also, from a combustion frequency perspective Figure 6 indicates that at 20% ethanol substitution there is the least variation from cycle-to-cycle and when compared to neat diesel fuel a slight increase in frequency. A 20% ethanol subsitution leads to improved thermal efficiency[1]. This result validates the improved efficiency, indicated by the slightly higher combustion temperature and the improved inter-cycle consistency.

At high ethanol substitutions, 30%, 40% and 50%, the repeatability of each combustion becomes increasingly low, compared to neat diesel and lower ethanol substitutions. It can be seen in Figure 6 that the pdf of ω_r becomes significantly flatter, with a marked increase in standard deviation, as the ethanol substitution increases, indicating the increasing inter-cycle variability as the ethanol substitution is increased.

We conclude that up to 20% ethanol substitution, from a combustion chamber resonance perspective, does not significantly alter inter-cycle variability in a negative way, with some evidence to suggest improved inter-cycle variabilty at the 20% ethanol substitution. However, at ethanol substitutions of 30% and higher, inter-cycle variability is severely influenced in a negative manner.

Conclusions

This paper demonstrated a statistical modeling method for determining resonant frequency information present in combustion chamber pressure signals. As such, accurate determination of instantaneous frequency information was also demonstrated. Results of this were shown for a large number of consecutive cycles with the engine operating under various ethanol substitution rates to demonstrate a novel utility of this methodology. Using this cycle-to-cycle evidence evinces support for improved inter-cycle variability with 20% ethanol substitution. Higher ethanol substitutions were shown to significantly increase inter-cycle variability.

A further implication of the results is that evidence was found to support not using ad hoc methodology, such as cycle averaging, when the desired output is frequency information. This is particularly true when using frequency information to make arguments during potentially unstable operating conditions, such as the example used for this paper of alternative fuel substitutions, with a variation upwards of 2000 Hz. Only through the use of more sophisticated techniques can information such as this be accurately obtained, particularly in light of the small number of data points and noise present.

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