17th Australasian Fluid Mechanics Conference
Auckland, New Zealand
5-9 December 2010

Rheological controls on the dynamics of channeled lava flows

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Abstract

We report the results of a series of analogue laboratory experiments which investigate the cooling, convection and solidification of a channelized viscoplastic lava flow. In the experiments, slurries of kaolin and molten polyethylene glycol flowed with a constant flux down an inclined channel under cold water. Experiments with cooling but no solidification showed that thermal convection occurs in organised rolls aligned with the downstream flow. Three crust cover regimes were identified in solidifying flows: (a) a 'tube' regime, in which the surface crust covered the entire channel and jammed against the channel wall and (b) two 'mobile crust' regimes, in which a mobile raft of crust formed in the centre of the channel, separated from the channel walls by open shear zones. In the first mobile crust regime the crust width is determined by a balance between shear and solidification, while in the second it is determined by the width of the unyielded region in the flow centre. The degree of surface crust coverage and the transition between these regimes is quantified in terms of the Bingham number and a dimensionless parameter ϑ, which parameterises the effects of shear and convection on crust growth.

Introduction

The dynamic evolution and final state of lava flows are strongly influenced by the nature and timing of cooling and solidification [6]. This thermal history is predominantly determined by complex interactions between the flow and its environment at the flow surface. Here the flow loses heat most rapidly to the surrounding environment, leading to the formation of a surface crust.

The presence of a surface crust inhibits radiative and convective heat transfer from the flow to the environment (see [9] for example), allowing the flow to travel much further than if the surface crust was continuously disrupted [1]. An effective model of lava flow dynamics must therefore predict the formation and distribution of any surficial crust. Unfortunately, it is not feasible to explicitly include surface solidification in numerical models, as instabilities within the flow lead to complex crustal behaviour (such as surface folding, rifting and fracturing) which can be difficult to capture adequately. However, progress in understanding this complex problem has been made using analogue laboratory experiments using polyethylene glycol (PEG), a clear wax with a freezing point close to room temperature (e.g. [4, 5, 7, 10]).

Griffiths and Fink [4] carried out quantitative experiments studying the horizontal spreading of point- and line-sourced flows. They showed that crust morphology regimes could be parameterised using a dimensionless group representing the ratio of an advection timescale to the time taken for the flow surface to reach the solidification temperature:

$$\psi = \frac{U t_s}{H},$$

where U and H are characteristic velocity and height scales for the flow, i.e. U/H is a characteristic shear strain rate scale and t_s is the time taken for the surface to solidify, calculated from a one-dimensional thermal model of the flow surface.

Motivated by the fact that many lava flows form localised channels which can transport lava many kilometres from an erupting vent, Griffiths, Kerr and Cashman [7] carried out an investigation of the processes controlling solidification in PEG channel flows. They showed that the crust cover reached a quasi-stable distribution, and classified the crust cover on the flows into two regimes: a 'mobile crust' regime in which a central raft of crust was carried down the centre of the channel, separated from the side walls by open shear zones, and a 'tube' regime in which solidification of the flow surface was rapid enough to form a stationary roof beneath which molten material continued to flow. By examining the interactions between surface shear, internal convection and crust growth, they developed the following dimensionless grouping:

$$\vartheta = \psi \left(\frac{\mathrm{Ra}}{R_0} \right)^{1/3}, \tag{1}$$

where $(\mathrm{Ra}/R_0)^{1/3}$ is an approximation describing heat transfer to the surface of the flow due to internal convection at a Rayleigh number Ra (R_0 is an empirical constant, see [2, 7]). Tube regimes have low values of ϑ, while mobile crust flows have high values of ϑ.

What is not clear from these channel experiments is the role that the internal rheology of the lava plays in determining the distribution of crust on the surface of the flow. Partial crystallisation driven by cooling and degassing can generate a touching network of crystals which can bear a stress in addition to the viscous response of the melt fraction of the lava; the lava develops a viscoplastic rheology [11]. In this study we use a series of analogue laboratory experiments to extend the study of Griffiths et al. [7] to flows with non-Newtonian, viscoplastic rheologies and examine the effects that rheology has on the development of surficial lava crusts.

Fluid rheology and flow scales

A viscoplastic material behaves as a solid up to a critical magnitude of an applied shear stress, τ_0, called the yield strength of the material. The material deforms once the yield strength is exceeded. The simplest viscoplastic rheology is the Bingham fluid, which has the following constitutive equation:

$$\left.\begin{array}{lll} \tau = \left(2\mu + \sqrt{2}\tau_0/|\dot{\gamma}|\right)\dot{\gamma} & \text{if} & |\tau| > \tau_0 \\ \dot{\gamma} = 0 & \text{if} & |\tau| \le \tau_0 \end{array}\right\} \tag{2}$$

where μ is the dynamic viscosity of the yielded fluid, and

$$|\dot{\gamma}| = (\dot{\gamma}\cdot\dot{\gamma})^{1/2} \text{ and } |\tau| = (\tau\cdot\tau)^{1/2}$$

are the second tensor invariants of the shear strain rate ($\dot{\gamma}$) and deviatoric stress tensors (τ) respectively.

The Bingham rheology divides a viscoplastic flow into two sets of domains; one set, the 'yielded' regions, are described by the

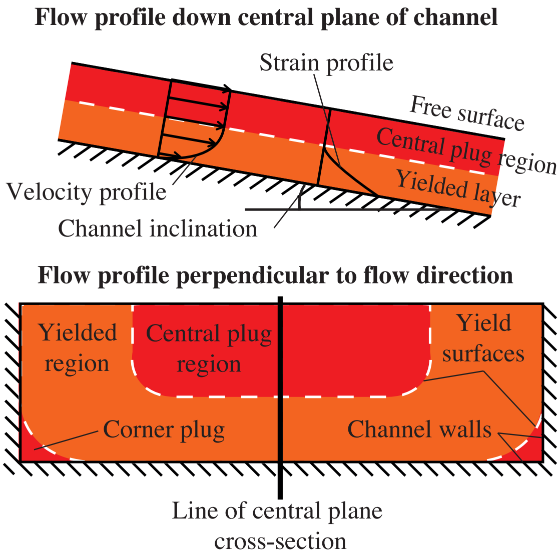

Figure 1: A fully-developed isothermal viscoplastic channel flow, showing the three plug regions which develop in a flow with a rectangular crosssection.

first branch of (2) in which the yield strength of the fluid is exceeded and the fluid deforms. The other set, termed 'plug' regions, are described by the second branch of (2). In these regions the shear stress is low enough that the yield strength of the fluid prevents deformation. The plug boundary (where $|\tau| = \tau_0$) is called the 'yield surface' (see Figure 1).

The low Reynolds number, isothermal flow of a purely viscous fluid is used to provide a scale for the analysis of our flows. The fluid has a density ρ and flows with a constant volume flux Q under an ambient fluid of density ρ_a. The channel's width is w, and its base is inclined at an angle θ to the horizontal. Then the effective down-channel buoyancy force acting on the flow is $g' = (1 - \rho_a/\rho) g \sin\theta$.

The dimensionless parameters of interest in this study are the dimensionless grouping ϑ, defined in (1), an aspect ratio:

$$\beta = H/w$$

and the Bingham number, which is the ratio of the yield strength to a characteristic viscous stress:

$$\mathrm{B} = \frac{\tau_0 H}{2\rho\nu U}, \qquad (3)$$

where $\nu \equiv \mu/\rho$ is the kinematic viscosity of the yielded fluid, and the following depth and velocity scales are characteristic of this flow at low β:

$$H = \left(\frac{3Q\nu}{wg'}\right)^{1/3} \quad \text{and} \quad U = \left(\frac{9Q^2 g'}{8w^2\nu}\right)^{1/3}.$$

Apparatus and measurements

Our experiments involved the continuous release of slurries down an inclined channel under cold water. The slurries comprised of 75% polyethylene glycol wax (PEG) and 25% kaolin by weight. The apparatus used consisted of a 1600 mm long acrylic channel placed in a larger tank (400×200×2000 mm, see Figure 2). Slurries were fed through a tube from an overhead reservoir into a lock at the top of the sloping acrylic channel. A sluice gate prevented water in the channel mixing with

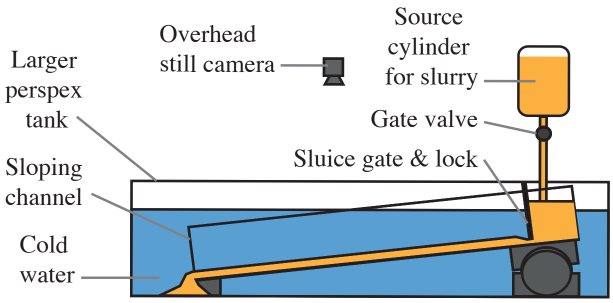

Figure 2: Schematic diagram of experimental apparatus.

the slurry in the lock, and ensured that the generated flow had a uniform depth across the entire channel width. A gate valve and stopcock were used to control the flow rate.

A range of dynamic regimes were obtained by varying the source flux, the channel inclinations (3.5°, 6.5° and 7.3°) and widths (80 mm and 150 mm), and the ambient water temperatures (5–15°C). The water, slurry and solidification temperatures were measured for each experiment. The height of the flow in the channel was also measured for some experiments. Source fluxes were measured by timing the rate of height loss in the overhead reservoir. Crust cover ratios were measured from the overhead photographs of the experiments.

Cooling & convecting flows

In cooling flows of both viscous and viscoplastic rheologies the shear at the free surface generates a laterally-varying thermal boundary layer which is thinnest in the centreline of the channel and thickens towards the walls. This thermal structure is unstable, and drives overturning cells in each half of the flow cross section which are dominated by two laminar sinking plumes at either wall (see Figure 3). The convective overturning flow in both cases is smaller than the downstream flow by several orders of magnitude.

Viscous and viscoplastic experiments with surface cooling but no solidification (with ambient water temperatures around 24°C) were carried out to qualitatively determine the main features of the internal flow dynamics. The Rayleigh numbers were chosen to match those of the solidifying experiments (described in the next section), and those observed in real flows. Neutrally buoyant dyed streams of PEG were used to highlight the overturning (see Figure 4). Dye streams placed in the central portion

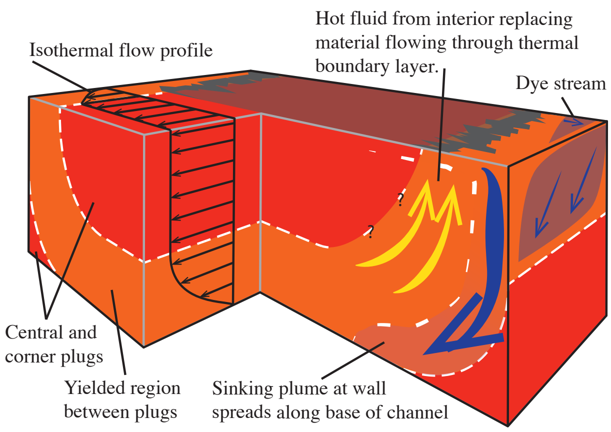

Figure 3: Summary of qualitative observations of convection in viscoplastic flows.

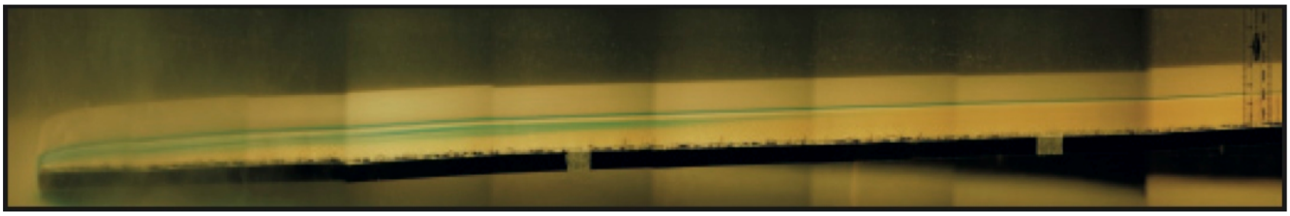

Figure 4: Side view of a cooling viscoplastic experiment with a dye stream highlighting the sinking side wall plume. The effect of the corner plug is shown as dyed fluid fails to reach the bottom of the wall before moving into the flow interior.

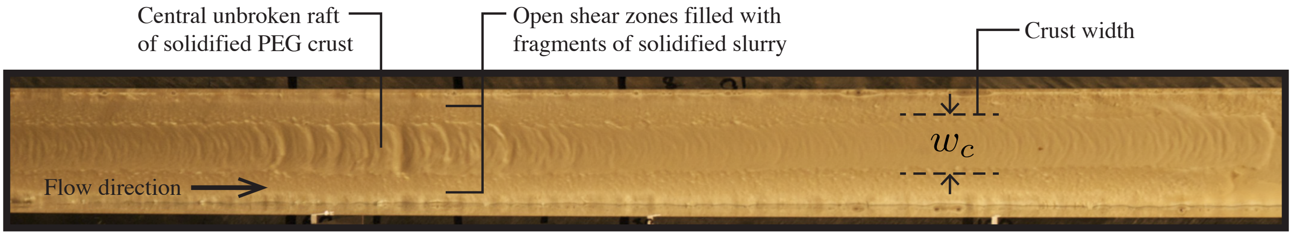

Figure 5: Overhead view of an experimental flow in the mobile crust regime. Parameters values for this flow are $\mathrm{B} = 0.053$, $\vartheta = 6.31$ and $\beta = 0.127$. The channel width is 150 mm.

of the channel failed to sink at all, while streams in the sidewall shear zones did not reach full depth before moving into the flow interior.

The dyed experiments showed that (a) the plug regions are not significantly altered by convection in the channel, and (b) the plugs cause significant qualitative differences in the overturning of viscoplastic flows (compared to the viscous case), although the overturning rates were similar in both cases. A summary of the cooling experiment observations for viscoplastic flows is shown in Figure 3.

Cooling and Solidifying flows

108 experiments were carried out with solidification at the free surface. Figure 5 shows the central crust typical of mobile crust regime flows, with a quasi-stable width for much of the channel. The folding of the crust is associated with increased surface drag as the crust forms. Following Griffiths et al. [7], the flows have been classified into mobile crust and tube regimes. Some flows fell into a reproducible transitional regime where the crust covered the entire channel width but continued to slip along the channel walls until they jammed, either near the end of the channel or on the base of the larger tank.

The dimensionless group ϑ was varied in two ways in this study: firstly by changing the flow rate Q (which controls the flow depth scale H and velocity scale U), and secondly by varying the solidification time t_s (by varying the ambient water temperature). The first group of experiments (with varying Q) vary ϑ and B simultaneously (since (1) and (3) show $\vartheta \sim U \sim Q^{2/3}$ and $\mathrm{B} \sim H/U \sim Q^{-1/3}$), while the second group (with varying t_s) vary ϑ while holding B constant.

Figure 6 shows the ratio of crust to channel widths as a function of the parameter ϑ for a range of experiments. The black and grey symbols represent recalculated viscous flow data from Griffiths et al. [7], included for comparison; all other data are from the current study. The orange data represent flows with constant solidification time t_s with varying flow rate Q, while the blue and green data represent varying t_s at two values of Q. These two datasets allow a separation of the effects of viscoplasticity from those of varying shear rate.

The second plot in Figure 6 shows that viscoplastic flows in the mobile crust regime with varying t_s fall into one of two subregimes: shear-controlled or plug-controlled. Shear-controlled regimes have a crust width ratio which is determined predominantly by the strain rate in the shear layer, unlike plug-controlled regimes where crust width is determined predominantly by the extent of the plug. In contrast, the plug-controlled regime was not seen in flows where ϑ is being controlled by varying Q while holding t_s constant (left plot in Figure 6). The plug still has a minor effect here however: high-B flows generally have wider crusts than the corresponding viscous or low-B flows at the same value of ϑ.

Therefore viscoplastic flows can potentially have two critical values of ϑ which are of interest: the transition from tube to mobile crust, and (where it occurs) the transition from shear- to plug-controlled regimes. Both of these critical values show a dependence on B. With increasing B, the tube-mobile crust transition occurs at higher ϑ, while the plug-shear transition occurs at lower ϑ. Thus there exists a critical value of B where the two transitional values of ϑ coincide and surface crust forms across the whole surface of the flow for all values of $\vartheta < \infty$. We expect this value to be close to the critical Bingham number for isothermal flow B_c (see e.g. [8]). In contrast, the plug-shear transition moves to $\vartheta \to \infty$ in the limit $\mathrm{B} = 0$, and the viscous case is recovered.

Volcanological implications

These experiments have shown that crust growth on the surface of viscoplastic channel flows can be significantly influenced by the presence of plug regions in the flow interior. We note that the location and extent of plugs in a viscoplastic lava flow are likely to be highly sensitive to a range of additional complexities present in real lava channels such as variations in the side walls and floor, bends, expansions and contractions. Analytic studies of viscoplastic Poiseuille flow in an irregular channel [3] has shown that the central plugs remain intact to first order for irregularities up to the scale of the channel half-width; we therefore expect that the plugs will still play an important role in determining crust formation in real flows under these conditions.

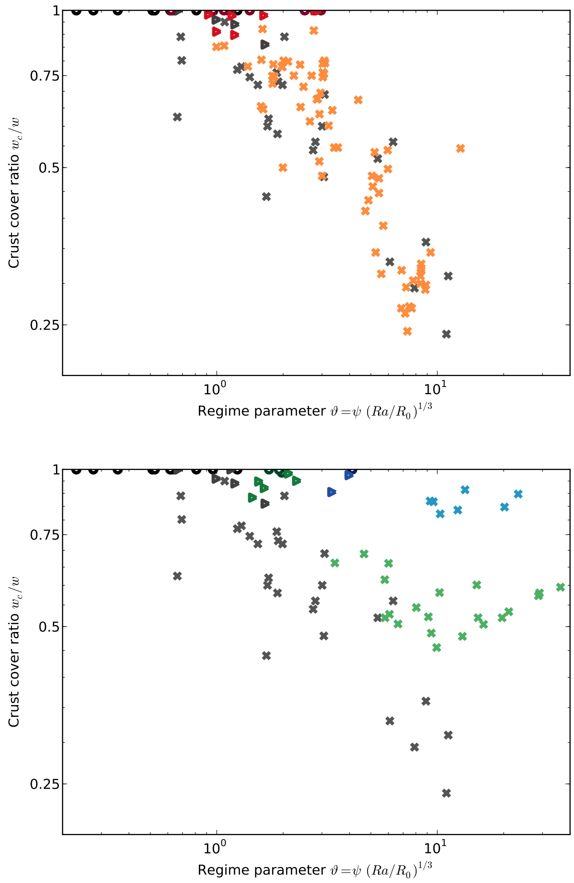

Figure 6: Crust width ratio (w_c/w) as a function of ϑ for all experiments. Symbols denote the crust regime of the flows: tube flows are denoted by circles, mobile crust flows by crosses and transitional flows with triangles. Viscous flows (recalculated data from [7]) are shown in black and grey in both plots, viscoplastic flows with constant t_s are shown in orange in the top plot, while the blue and green data points are viscoplastic flows with constant B of 0.053 and 0.084 respectively, and are shown in the bottom plot. Only low aspect ratio viscous flows ($\beta < 0.5$) from Griffiths et al. [7] are shown, as these best match the viscoplastic flows aspect ratios and low-aspect ratio scaling used in this study.

The two types of crust ratio trends observed in the experimental flows are likely to affect different aspects of spatial and temporal patterns of solidification in real lava flows. Constant t_s trends (like the orange data in Figure 6) are likely to be more important for crust response to temporal flow rate changes at a given location in a flow. In contrast, the constant-Q trend (shown in the blue and green data) should be more important in describing variation in crust cover with variation in lava rheology due to different melt compositions and crystallisation histories.

Acknowledgements

We thank Tony Beasley, Daniel Hunt and Ben Tranter for their enthusiastic assistance with the experimental work, and Graham Hughes and Ross Griffiths for their constructive comments on a draft of this paper. This study was funded by an Australian Research Council Grant (DP077270) and an Australian Postgraduate Award from the Australian Government.

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