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A Direct Numerical Simulation investigation of rheology parameter in Non-Newtonian suspension flow in open channels

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Abstract

When self-formed open channels flow at a sufficient gradient or slope, it can generate a certain level of turbulence. The turbulent behaviour of the transportation material can keep the particles in suspension. From our previous study it has been observed that if the slope reduces and the flow rate keeps constant, the intensity of turbulence will decline as well. The mechanism governing particle transportation in turbulent flow has been studied in the past although it not well understood. Limitations on measurements with an opaque fluid result in a less detailed mapping of the fluid velocity in the near wall regions. Therefore a direct numerical simulation approach is required.

Direct numerical simulation (DNS) of the turbulent flow of non-Newtonian fluids in an open channel is modelled using a spectral element-Fourier method. The simulation of a yield–pseudoplastic fluid using the Herschel-Bulkley model agrees qualitatively with experimental results from field measurements of mineral tailing slurries. The simulation results over–predict the bulk flow velocity for the cases considered, however the source of the discrepancy is difficult to ascertain. The effect of variation in yield stress is fully investigated and used to assess the sensitivity of the flow to these physical parameters. This methodology is seen to be useful in designing and optimising the transport of slurries in open channels.

Introduction

The flow of non-Newtonian fluids in open channels is of great importance to the mining industry. Fixed-shape open channels are used to ore slurries and tailings streams as a more economic alternative to pumping when conditions allow. At the present the design of these flumes is often done using crude estimates based on the conditions established for water with a limited set of field observations (Wilson, 1991).

Self-formed channels flow at a sufficient gradient to generate a level of turbulence that is able to maintain all the tailings particles in suspension. However with a shallower gradient, the turbulence intensity reduces, and more solids settle in the channel. It is found that the turbulent coherent structures are likely to contribute to the mechanics of suspending particles away from the channel bed (Verbanck, 2000). Yet, there is a lack of fundamental understanding about many of the mechanisms involved in how a turbulent flow of a non-Newtonian carrier fluid maintains particles in suspension.

Experimental measurement of velocity profiles and turbulence statistics can be used to determine the state of channel flow. If the rheology of the suspension is known, then computational simulation can be used to simulate the flow patterns. Computational modelling of non-Newtonian fluids using direct numerical simulation (DNS) shows significant reliability when compared with reality. The main benefit of using a DNS technique is that once it is validated, it can be used to model flow behaviour and provide a detailed description of the turbulent structure which can then be applied to particle transportation in channels.

In earlier years of research, Kim *et al* (1987) have simulated turbulent channel flow of Newtonian fluids with Reynolds number up to 3300. There have been some DNS of the turbulent flow of polymer solutions as well (Sureskumar *et al*, 1997). Furthermore, Rudman and Blackburn (2003, 2006) have used spectral element method to simulate non-Newtonian flow in pipes. The velocity distribution resulting from the present approach of adopting the Herschel-Bulkley model showed good agreement in terms of shape and magnitude when compared with the experimental data. The objective of this paper is to present some interesting findings from a study undertaken for time independent shear-thinning viscoplastic fluids in an open channel flow, whose rheology can be described using the Herschel-Bulkley model.

Rheological Models

In current work, the viscosity η of the fluid can be described using the Herschel – Bulkley rheological model

$$\eta = \frac{\tau_Y}{\dot{\gamma}} + K\dot{\gamma}^{n-1} \tag{1}$$

Where K is the consistency index, n is the flow behaviour index, and τ_y is the fluid yield stress.

Wall Viscosity

When the viscosity varies in space and time, the appropriate viscosity scale to use in order to define a Reynolds number is obvious. Therefore in this paper, the Reynolds number is defined by mean wall viscosity. It is calculated from the mean wall shear stress, τ_w. It is determined directly from the applied axial pressure gradient.

$$\tau_w = \frac{\delta p}{\delta z} \rho \frac{A}{C} \tag{2}$$

Where

$$C = R\theta \tag{3}$$

Assuming a Herschel-Bulkley rheology,

$$\eta_w = K^{1/n} \frac{\tau_w}{(\tau_w - \tau_y)^{1/n}} \tag{4}$$

Wall Units

Wall units are introduced with the wall viscosity replacing of the non-Newtonian viscosity. Therefore the friction velocity is defined as $U^* = \sqrt{\tau_w / \rho}$, the non-dimensional velocity is $U^+ = U / U^*$ and the non-dimensional distance from the wall is written $y^+ = (\rho U^* / \eta_w) y$.

Experimental Method

The experiment was conducted at the Sunrise Dam Gold Mine, in Western Australia. The flume channel had a 340 mm internal diameter with semi-circular cross section. A plunge box was located at the upstream end of the flume. The tailings slurry was supplied to the plunge box through a 150 mm High Density Polyethylene (HDPE) pipe with an outlet 20 cm above the plunge box floor. A diaphragm valve was installed in the pipe to allow adjustment of the flow rate of tailings slurry.

At the downstream end of the flume, a flow measuring box was placed to measure the flow rate of the flume. A level indicator and a stopwatch were also used to record the time taken for the flume discharge to fill a specific volume. Local velocity at specific locations within the channel was measured by Delft E-30 velocity probe, which generated an electromagnetic field from 5 electrodes mounted in the bottom surface of an ellipsoid head with 33 mm in diameter and 11 mm thick. Due to the requirement for all five electrodes to be immersed in the fluid without any air interaction at the surface, it was found that the E-30 probe could not measure fluid velocities at depths less than 5 mm. It was discovered that the width of the ellipsoid head of the probe prevented velocity measurements from being taken within about 17 mm of the boundaries of the half pipe. Further details of this experiment and associated instrumentation are given in Jewell *et al* (2006).

Numerical Method

The numerical method is based on spectral element/Fourier formulation. The three-dimensional spatial discretisation uses isoparametrically mapped quadrilateral spectral elements in the cross section (x,y) that allows arbitrary geometry in the (x,y) plane, and z (out-of-plane) direction. The z-axis is aligned with the flow direction and uses a Fourier expansion that results in a periodic geometry in this direction. Details of numerical method may be found in Rudman and Blackburn (2006).

In order to drive the flow in the axial (z) direction, a body force per unit mass equivalent to the pressure gradient measured in the experiments is applied to the z-momentum equation. This approach allows the pressure to be periodic in the axial direction

The computational domain consists of fifty one 15th order elements in the channel cross section and 384 Fourier modes in the axial direction, with domain lengths of 0.5πD. The reason for choosing a small domain length is because with a longer domain length and current number of planes (384) the simulation time would be too great. The coherent structure presented in this domain length is enough to justify the current domain length = 0.5πD (Guang *et al*, 2010). A larger number of z planes can increase the resolution of the contour thus the observation of the result become a lot easier. Therefore the domain length has been shortened to meet the demand of number of z planes. This is the summary of simulation parameter.

Simulation run	n	K	τy	Reynolds number	Wall viscosity
Control- Simulation result based on experiment	0.81	0.0506	2.249	8294	0.01998
Simulation A Yield stress +20%	0.81	0.0506	2.698	7295	0.02149
Simulation B Yield stress +30%	0.81	0.0506	2.923	7022	0.02232
Simulation C Yield stress +50%	0.81	0.0506	3.374	6481	0.02419
Simulation D Yield stress -20%	0.81	0.0506	1.799	8405	0.01865
Simulation E Yield stress -30%	0.81	0.0506	1.574	8688	0.01805
Simulation F Yield stress -50%	0.81	0.0506	1.124	9258	0.01693

Tab. 1 Parameters for simulation

Boundary Conditions

In this simulation, boundary conditions can be only of Dirichlet or Neumann type. Therefore the wall region has no-slip condition, and at free surface it is forced to have a stress free surface condition.

Experimental results

The transport characteristics of the slurry were measured. The generalised Reynolds number is based on a wall viscosity that is calculated by wall shear stress.

Clapp (1961) reported the results of experimental measurements of the turbulent pipe flow of power law fluids with flow behaviour indices in the range of 0.698-0.813. Clapp determined that the logarithmic velocity profile for the turbulent flow of power law fluids is a function of the flow index, *n*, and satisfies

$$\hat{U} = \frac{A}{n} + \frac{B}{n} \ln \hat{y} \tag{5}$$

Where

$$\hat{y} = \left[\frac{\left(\rho^n \tau_w^{\,2-n} \right)^{1/2}}{K} \right] y^n \tag{6}$$

The value of these coefficients for well-developed turbulent flow of Newtonian fluids (where *n* equals to 0) are now generally accepted to be A=5.5, B=2.5. However in this case, the logarithmic profile used for all plots is

$$U^+ = 5.5 + 2.5 \ln y^+ \tag{7}$$

The mean axial velocity data at the centreline for the slurry is presented in Figure 1, in wall units, together with the logarithmic velocity profile. The experimentally measured velocity profile has a linear relationship between U^+ and y^+ in the near wall region. In the logarithmic region, the actual velocity profile for the slurry is slightly higher than the logarithmic velocity profile. At the free surface region, the measured velocity profile declined after a maximum. According to Joung *et al* (2007), Nezu (2005) this could be concluded as a secondary flow effect where the lower velocity/momentum material is advected into the otherwise high velocity/momentum region. Therefore there is a dip in the axial velocity profile after maximum velocity has been reached. This happens regularly in non circular channel flow. It is also suggested by Nezu and Nakagawa (1993) that this phenomenon is called the "velocity-dip", and it is peculiar to open channel flows. The bulk velocity of the experimental result is 1.06 m/s.

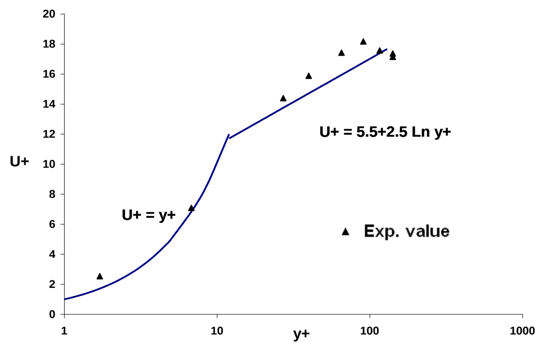

Figure. 1. Experimentally measured velocity profile for slurry

Discussion of Results

The computationally predicted profile for the control Simulation at Reynolds number = 8294 is presented in Figure 2. DNS turbulent pipe flow at Reynolds number = 7500 flow data from Rudman and Blackburn (2006) is also presented in the same plot. The velocities have been non-dimensionalised. The non-dimensionalisation is undertaken using the wall viscosity give in Equation 2. The channel profile is also in good agreement with accepted profile for turbulent pipe flow. All profiles have a linear relationship between U^+ and y^+ in the near wall region. In the logarithmic region, the simulation profile and Rudman and Blackburn (2006) profile is above logarithmic velocity profile. This is consistent with experimental results. However, Rudman and Blackburn (2006)'s profile used a power law fluid rather than a Herschel-Bulkley fluid.

Furthermore, experimental data from Eckelmann (1974) is plotted on Figure 3; this represents data for an oil channel flow in a square channel. In Figure 3, the mean axial velocity from Eckelmann's work is compared to DNS result. The normalised velocity profiles are almost identical for both experimental and current DNS results up to approximately $y^+ = 20$, where they begin to show some discrepancies. The bulk velocity of the simulation is 1.48 m/s.

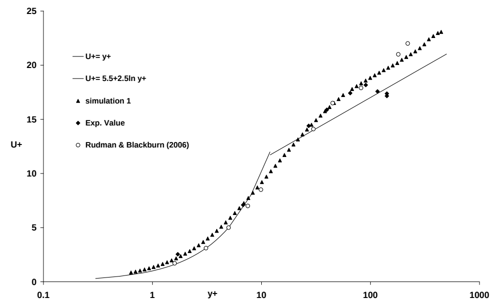

Figure.2. Experimentally measured velocity profile in conventional wall units for slurry in comparison of Simulation results.

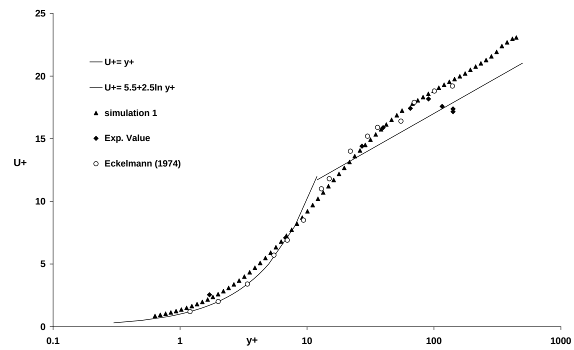

Figure. 3. Eckelmann (1974) experimental profile in comparison of Simulation results.

It is seen that a significant disagreement between simulation and measurement exists. Not only is the predicted velocity is approximately 39% higher than experimental, but the velocity profile has somewhat different shape to the experimental profiles. The simulation profile and experimental profile have the same magnitude in the near wall region. In the free surface region, the simulation profile does not show any secondary flow effect.

Numerical Results

The results from six simulations are presented here. For these simulations, six different yield stresses (see Table 1). Simulation A, B, C have yield stresses increased to 20%, 30% and 50% respectively. Simulation D, E, F have yield stresses decreased to 20%, 30% and 50% respectively. The six simulations are simulated at generalised Reynolds number of 6500-9300. Although simulations are undertaken in a Cartesian coordinate system, all results are presented in a cylindrical coordinate system in which the axial velocity is denoted by U, the radial velocity by V and the azimuthal velocity by W.

Increased Yield Stress

Mean flow profile for Herschel-Bulkley fluids

The mean axial velocity for these three simulations for τ_y = 2.698, 2.923, 3.374 are shown in Figure 4. They are plotted with conventional 'law of the wall' non-dimensionalisation and are compared to control yield stress profile using the same DNS code. As yield stress τ_y increases, the profiles for the Herchel-Bulkley fluids are moving away from the logarithmic profile obtained by theoretical analysis. All three simulations plus the control simulation show indications of a log-layer profile with a greater slope than the theoretical logarithmic profile. The results for τ_y = 3.374 fall sufficiently above the theoretical profile.

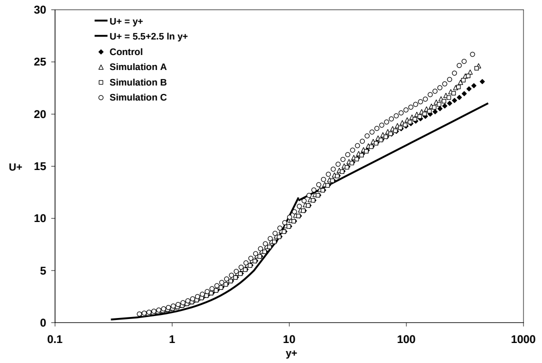

Figure. 4. Mean axial velocity profiles for the turbulent flow of three different Herchel-Bulkley fluids. The profiles have been non-dimensionalised using the conventional non-dimensionalisation with the mean wall viscosity taking the place of the Newtonian viscosity.

Decreased Yield Stress

Mean flow profile for Herschel-Bulkley fluids

The mean axial velocity for these three simulations for τ_y = 1.799, 1.574, 1.125 are shown in Figure 5. They are plotted with conventional 'law of the wall' non-dimensionalisation and are compared to control yield stress profile using the same DNS code. As yield stress τ_y decreases, the profiles for the Herchel-Bulkley fluids are moving closer to the theoretical logarithmic profile, as expected. From Figure 5, it is quite obvious that Simulation F (which represents -50% yield stress) is closer to the logarithmic profile than the other two simulations (Simulation D and Simulation E).

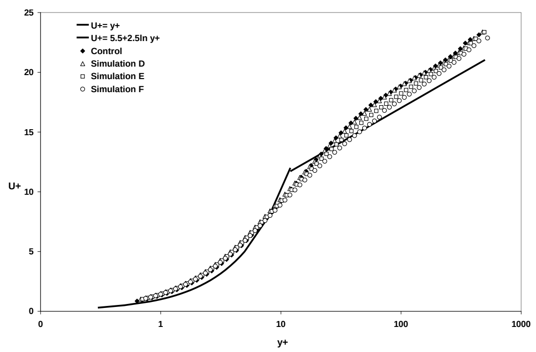

Figure. 5. Mean axial velocity profiles for the turbulent flow of three different Herchel-Bulkley fluids. The profiles have been non-dimensionalised using the conventional non-dimensionalisation with the mean wall viscosity taking the place of the Newtonian viscosity.

Conclusion

The simulation results for a Herschel-Bulkley fluid show some agreement as well as some significant differences with the experimental results. Validation has been done by Rudman and Blackburn (2003) on numerical method in turbulent Newtonian flow and laminar flows of power-law fluids in pipes, and in both cases no errors were found.

A comparison between the experimentally measured velocity profile in an open channel and the simulation data indicates some discrepancy between the results obtained. The study of yield stress is being undertaken to address this issue. It appears that with an increasing yield stress, the turbulent intensities are weaker compared to a Newtonian fluid. Additionally, the mean flow profile deviates further from the Newtonian profile as a yield stress increases. However the yield stress effect could not fully explain the discrepancy between experimental data and simulation data, future work is needed to investigate the other two rheological parameter: K and n. Hopefully, the application of DNS to flows of non-Newtonian fluids can provide us an improved understanding of the effect of rheological parameters on turbulent flow.

REFERENCES

Clapp, R,M., 1961, Turbulent heat transfer in pseudoplastic non-Newtonian fluids, *International Dev of Heat Transfer,* Part III, New York.

Eckelmann, H, 1974, The structure of the viscous sublayer and the adjacent wall region in a turbulent channel flow, *Journal of Fluid Mechanics,* 65(3): 439-459.

Guang, R., Murray, R., Chryss, A., Slatter, P., Bhattacharya, S., 2010, A DNS investigation of the effect of yield stress for turbulent non-Newtonian suspension flow in open channels, *Particulate Science and Technology,* accepted.

Jewell, R.J., Lawson, S., Newman, P.(eds), 2006, *Paste 2006*,

Joung, Y., Choi, S, U., Choi, J, I., 2007 Direct numerical simulation of turbulent flow in a square duct: analysis of secondary flows, *Journal of Engineering Mechanics,* 133(2): 213-221

Kim, J., Moin, P., Moser, R., 1987, Turbulence statistics in fully developed channel flow at low Reynolds number, *Journal of Fluid Mechanics,* 177:133.

Nezu, I., 2005, Open-Channel flow turbulence and its research prospect in the 21st century, *Journal of Hydraulic Engineering,* 31(4): 229-246.

Rudman, M., Blackburn, H.M., 2003, *The effect of shear thinning behaviour on turbulent pipe flow*, Third International Conference on CFD in the Minerals and Process Industries, CSIRO, Melbourne, Australia.

Rudman, M., Blackburn, H.M., 2006, Direct numerical simulation of turbulent non-Newtonian flow using a spectral element method, *Applied Mathematical Modelling*, 130: 1229-1248.

Sureskumar, R., Beris, A., Handler, A., 1997, Direct numerical simulation of the turbulent channel flow of a polymer solution, *Physics of Fluids,* 9:743-755.

Verbanck, M., 2000, Computing near-bed solids transport in sewers and similar sedminet-carrying open-channel flows, *Urban Water,* 2: 277-284

Wilson, K.C., 1991, Slurry transport in flumes, *Slurry handling design of solid liquid system,* Brown, N.P, Heywood, N.I., (eds) Elsevier, London.