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Implementation of Hot-wire Anemometry at DSTO for High-Subsonic and Transonic Mach number applications

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Abstract

A complete hot-wire anemometry system has been designed for implementation at DSTO in high-subsonic and transonic Mach number flows. The implementation includes: the design of apparatus and novel techniques for custom fabrication of robust oxidation-resistant hot-wire sensor filaments; high-performance constant-temperature hot-wire anemometer instrumentation; and a compressed-air-driven nozzle, which forms a jet which is used for stand-alone calibrations. Unsteady velocity measurements and turbulence statistics will provide important information about high-speed flows. An immediate application involves measurements in the high-speed shear layer above a vehicle cavity, which could provide vital information about the resonant interactions in the shear layer and the extraordinary loud SPL (Sound Pressure Level) generated by the flow inside the cavity.

Introduction

The hot-wire anemometer has been used to measure turbulence quantities in subsonic flows for more than 75 years. More recently. the instrument has also been used successfully, to measure supersonic flows. However, hot-wires have not been used extensively in transonic flows because the large dynamic pressures tend to cause wire breakage. Another reason why hot-wires are not often used in transonic flows is that the sensitivity to velocity and density fluctuations are difficult to determine, since they both depend on Mach number, and this makes calibration difficult for transonic flows.

In subsonic incompressible flow the heat transfer from a wire is a function of the mass flow, total temperature and the wire temperature. Since density is effectively constant at low Mach numbers, then all of the mass flow variations can be attributed to velocity fluctuations. The Nusselt number is assumed to be a function of the Reynolds number and the Prandtl number alone. In most applications the Prandtl number can be assumed to be constant, which leads to the familiar King's law calibration for hot-wires operating in incompressible subsonic flows.

The effects of fluid compressibility on the heat transfer from the wire must be taken into account at higher speeds. In supersonic flow it was discovered experimentally that the heated wire is sensitive only to mass flow and total temperature. The reason for this behaviour is that a strong bow shock is located upstream of the wire in supersonic flow. Therefore the wire is operating in the locally subsonic flow downstream of the shock.

The most difficult flow regimes occur at high-subsonic and transonic Mach numbers, since the heat transfer from the wire is a function of velocity, density, total temperature, and the wire temperature, *i.e.* $\mathrm{Nu} = f(Re, M, \theta)$. Because of this complexity, these flow regimes were largely ignored until the 1970's and 1980's when attempts were made to develop methods applicable for these flows.

A proper and complete hot-wire calibration in compressible flow would require a facility in which the density, total temperature, and velocity can all be varied independently of each other. However, evidence is presented to justify various assumptions that allow adequate hot-calibration for compressible flow to be performed in facilities without all of the these capabilities.

Hot-wire characteristics in compressible flow

Under steady conditions the joule heating generated by the current passing through the wire is balanced by the heat transfer to the flow over the wire. Following Morkovin [3] and Rose & McDaid [5], the steady state energy balance is given by

$$\overline{E}^2 = \overline{\mathrm{Nu}}_t\ (\overline{M},\ \overline{Re}_t,\ \overline{\theta})\ \pi\ l\ \overline{k}_t\ \overline{R}_w\ (\overline{T}_w - \ \overline{\eta}\ \overline{T}_t) \qquad (1)$$

where $\overline{E}$ is the voltage at the top of the anemometer bridge, $\overline{\mathrm{Nu}}$ is the Nusselt number, $\overline{M}$ is the Mach number, $\overline{Re}_t$ is the Reynolds number based on wire diameter, $\overline{\theta}$ is the overheat ratio ($=T_w/T_t$), l is the length of the hot-wire filament, $\overline{R}_w$ is the hot resistance of the wire, $\overline{k}_t$ is the thermal conductivity of the fluid, $\overline{\eta}$ is the recovery factor, and the subscript t denotes quantities evaluated at the total temperature of the fluid, $\overline{T_t}$. The Nusselt number is defined in terms of a convective heat transfer coefficient with reference to $\overline{T_w} - \overline{T_e}$, where $\overline{T}_w$ is the average wire temperature. The dimensionless variables in equation (1) have the following functional dependence:

$$\begin{aligned}
\overline{M} &= \overline{M}\ (\overline{U},\ \overline{T_t}) \\
\overline{Re_t} &= \overline{Re}_t\ (\overline{\rho},\ \overline{U},\ \overline{T_t}) \\
\overline{\theta} &= \overline{\theta}\ (\overline{T_t}) \\
\overline{\eta} &= \overline{\eta}\ (\overline{\rho},\ \overline{U},\ \overline{T_t})
\end{aligned}$$

Hence, for steady flow past a finite-length normal hot-wire,

$$\overline{E} = \overline{E}(\overline{\rho},\ \overline{U},\ \overline{T_t}) \qquad (2)$$

Assuming that equation (1) applies at each instant of time in an unsteady flow and using the following notation for mean and fluctuating components, then for small fluctuations, the fluctuating output voltage of the anemometer can be expressed in terms of perturbations in velocity, density and total temperature:

$$\frac{e'}{\overline{E}} = S_u \frac{u'}{\overline{U}} + S_\rho \frac{\rho'}{\overline{\rho}} + S_{T_t} \frac{T_t{}'}{\overline{T_t}} \qquad (3)$$

where the sensitivity coefficients are given by:

$$\begin{aligned}
S_u &= \left.\frac{\partial \ln \overline{E}}{\partial \ln \overline{U}}\right|_{\overline{\rho},\overline{T_t},\overline{T_w}} \\
S_\rho &= \left.\frac{\partial \ln \overline{E}}{\partial \ln \overline{\rho}}\right|_{\overline{U},\overline{T_t},\overline{T_w}} \\
S_{T_t} &= \left.\frac{\partial \ln \overline{E}}{\partial \ln \overline{T_t}}\right|_{\overline{\rho},\overline{U},\overline{T_w}}
\end{aligned}$$

Noting that

$$\frac{\partial \ln \overline{E}}{\partial \ln \overline{U}} = \frac{\overline{U}}{\overline{E}} \frac{\partial \overline{E}}{\partial \overline{U}} \tag{4}$$

then equation (3) can be written as:

$$e' = \frac{\partial \overline{E}}{\partial \overline{\rho}}\, \rho' + \frac{\partial \overline{E}}{\partial \overline{U}}\, u' + \frac{\partial \overline{E}}{\partial \overline{T_t}}\, T_t{}' \tag{5}$$

Morkovin [3] proposed the following relations for the sensitivities to density and velocity fluctuations:

$$S_\rho = \frac{1}{2}\left(\frac{\partial \ln \mathrm{Nu}_t}{\partial \ln Re_t} - \frac{1}{\tau_{w_r}} \frac{\partial \ln \eta}{\partial \ln Re_t}\right) \tag{6}$$

$$S_u = S_\rho + \frac{1}{2\alpha}\left(\frac{\partial \ln \mathrm{Nu}_t}{\partial \ln M} - \frac{1}{\tau_{w_r}} \frac{\partial \ln \eta}{\partial \ln M}\right) \tag{7}$$

Morkovin argued that the derivatives with respect to Mach number are very nearly zero in equation (7). Hence the sensitivities to velocity and density fluctuations are very nearly equal for Mach numbers, $M > 1.2$. Using this approximation, equation (3) can be written as

$$\frac{e'}{\overline{E}} = S_{\rho u} \frac{(\rho u)'}{\overline{\rho U}} + S_{T_t} \frac{T_t{}'}{\overline{T_t}} \tag{8}$$

where $S_{\rho u} = S_\rho = S_u$.

The formulation proposed by Morkovin is the key for implementation of hot-wires in supersonic flow, since it allows the modal analysis techniques used by Kovasznay [2] to determine the different fluctuating quantities by operating the hot-wire at different overheat ratios.

Calibration at high-subsonic & transonic Mach numbers

The problem is that the formulation proposed by Morkovin does not appear to be applicable in transonic flow since the variation of the derivatives of Nusselt number and the recovery factor are not independent of Mach number, i.e. $S_u \neq S_\rho$. However, Rose & McDaid [6] and Horstman & Rose [1] conducted investigations which demonstrate that the ratio of S_u/S_ρ is insensitive to overheat ratio for large overheat ratios. They found that for $Re_t > 20$ and high overheat ratios, that S_u is independent of M and equal to S_ρ.

Rong, Tan & Smits [4] investigated this result for operating conditions experienced in the facilities of the Gas Dynamics Laboratory at Princeton University. The Princeton group use a calibration technique which assumes the semi-empirical relation for the heat transfer:

$$\mathrm{Nu} = \frac{H}{\pi l k(\overline{T_w} - \eta \overline{T_t})} = X + Y \overline{Re_t}^n \tag{9}$$

where H is the power dissipated by the filament. See Smits, Hayakawa, & Muck [8] for further details.

In transonic flows the heat transfer is also dependent on the Mach number. Therefore, for a given wire, X and Y are dependent on the overheat ratio $\tau \equiv (\overline{T_w} - \overline{T_r})/\overline{T_r}$ and the Mach number. The output voltage of the anemometer can be calculated, assuming the bridge is perfectly balanced, as:

$$\frac{E^2 R_w}{\pi k l (R_a + R_w)^2 (T_w - \eta T_t)} = A f(\tau) + B g(\tau) Re^n \tag{10}$$

where A and B are constant for a given wire and Mach number. For calibration purposes, equation (10) can be expressed in the more convenient form:

$$E^2 = L + M(\rho u)^n \tag{11}$$

where (ρu) represents the instantaneous mass flow normal to the wire and L and M are constants for a given wire at a given overheat ratio, stagnation temperature and Mach number.

Rong *et al.* [4] performed experiments at a constant Mach number by keeping the position of the hot-wire fixed in a convergent–divergent nozzle, while varying the stagnation pressure. They then applied a curve fit to the data to determined the values of L and M and assuming a value of $n = 0.55$. Variations in stagnation temperature were accounted for by correcting the voltage according to the equation below:

$$E_{\mathrm{corr}} = E + \left.\frac{\partial E}{\partial T_0}\right|_{\rho u} (T_c - T_0) \tag{12}$$

where T_c is some constant reference temperature. The quantity $\partial E/\partial T_0$ is evaluated according to the method suggested by Smits *et al.* [8]. Alternatively, Rong *et al.* [4] used the raw output voltage and made corrections to L and M according to the expressions below:

$$L_{\mathrm{corr}} = L_c + \frac{\partial L}{\partial T_0}(T_0 - T_c) \tag{13}$$

$$M_{\mathrm{corr}} = M_c + \frac{\partial M}{\partial T_0}(T_0 - T_c) \tag{14}$$

where $\partial L/\partial T_0$ and $\partial M/\partial T_0$ are found from equations (10) and (11). The mass flow sensitivity of interest is then given by

$$S_{\rho u} = \frac{\partial \ln E}{\partial \ln \rho u} = \frac{n M \rho u^n}{2E^2{}_{\mathrm{corr}}} \tag{15}$$

In the experiments by Rong *et al.* the velocity is constant and it follows from equation (11) that E_{corr} will have a functional relationship with fluid density of the form:

$$E^2{}_{\mathrm{corr}} = L_u + M_u \rho^n \tag{16}$$

where L_u and M_u are different from the L and M defined in equation (11). The density sensitivity can then be found from

$$S_\rho = \frac{\partial \ln E}{\partial \ln \rho} = \frac{n M \rho^n}{2E^2{}_{\mathrm{corr}}} \tag{17}$$

Rong *et al.* found strong experimental evidence to support these relations using a value of n = 0.55. Furthermore, they found that S_ρ and $S_\rho/S_{\rho u}$ appeared to be independent of Mach number over their experimental range of, $0.5 < M < 1.4$, and for $100 < \overline{Re_t} < 300$, and for overheat ratios of 0.8 and 1.0.

The methods proposed by Rong *et al.* [4], Rose & McDaid [6] and Horstman & Rose [1] demonstrate that at sufficiently large wire Reynolds numbers and over heat ratios that the sensitivity to mass flow variations and the sensitivity to density variations are essentially independent of Mach number and Reynolds number and that they are equal. This makes it possible to distinguish between density and velocity fluctuations in transonic flow, in the mean flow (*rms*) sense, without requiring additional information.

Model of the Constant Temperature Hot-wire Anemometer (CTHWA)

Watmuff [9] conducted an investigation into hot-wire behaviour which was motivated by observations of hot-wire behavior in supersonic flow at the Gas Dynamics Laboratory at Princeton University. A frequency response requirement of 500 kHz is not uncommon in high speed flows. Yet with extremely careful tuning of the system controls, by the most highly-skilled operators and under the most favorable of circumstances, a barely adequate frequency response of around 250kHz can be obtained with commercially available equipment. However the so-called phenomena of strain-gauging (*i.e.* small amplitude high frequency oscillations) was responsible for contamination of about 3 out of every 4 experimental runs [7]. Sometimes the oscillations could be made to disappear by adjusting the anemometer controls. This observation led Watmuff to propose that strain-gauging could be a purely electronic, rather than an electro-mechanical phenomenon and that a more sophisticated model of the system behaviour could lead to further understanding and possible control of this frustrating problem.

Watmuff [11] completely generalized his original analysis and developed an algorithm for deriving the transfer functions of the constant temperature hot-wire anemometer of arbitrary complexity. He demonstrated that a minimum of two equivalent amplifiers are required to model the feedback amplifier properly. He showed that in general, the poles of the transfer functions for electronic and velocity perturbations will always be identical, regardless of complexity of the system, the frequency response characteristics of the feedback amplifier and the nature and quantity of components used to model the bridge impedances. The feedback amplifier must have the potential for both a high gain and a high frequency response in order for the system as a whole to achieve stable high frequency operation.

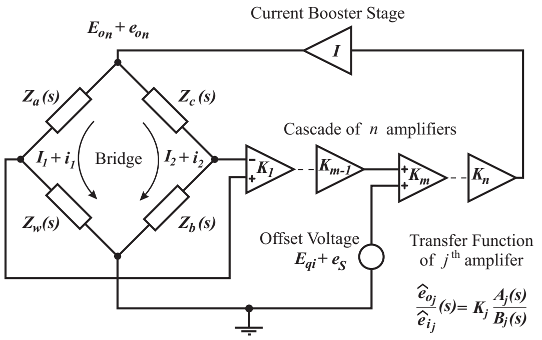

Figure 1: Model of constant-temperature hot-wire anemometer. From Watmuff [11].

High–Performance Hot–Wire Anemometer Design

Practical constant temperature hot-wire anemometer designs should use a cascade of amplifiers rather than a single feedback amplifier because of the gain-bandwidth product limitations of physical devices. A cascade of n amplifiers is used in the model shown in figure 1. These and other theoretical considerations led to a practical design which is specified in detail in the NASA Contractor Report by Watmuff [10]. The report provides details, such as fabrication drawings and a parts list, to enable the instrument to be constructed by others. The design allows the highest possible frequency response while maintaining system stability. Further enhancements include the use of two modular power supplies to achieve a high signal-to-noise ratio. A total of 40 anemometers were built at NASA Ames Research Center based on this design.

A total of four of these hot-wire anemometers have been built at DSTO using the same chassis and PCB design as specified by Watmuff. A complete and updated list of components, revised part numbers and other modifications will be documented in a DSTO Technical Report.

Fabrication of robust oxidation-resistant hot-wire filaments

Platinum is attractive for hot-wire filaments since it is highly resistant to oxidation at elevated temperatures. However, platinum has a relatively low tensile strength of around 200 MPa. On the other hand, tungsten is nearly an order-of-magnitude stronger than platinum with a tensile strength of around 1,500 MPa. For this reason, tungsten hot-wire filaments have been used in the more demanding situations involving high–speed subsonic and transonic flows.

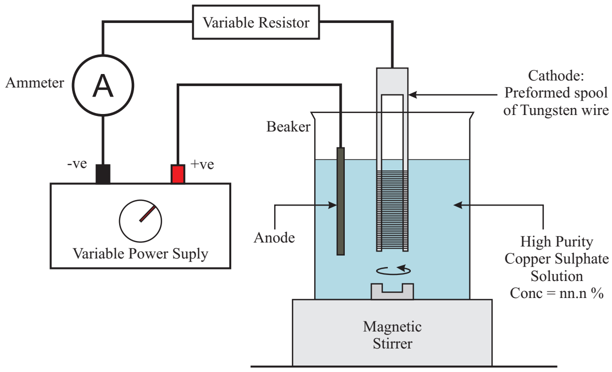

Figure 2: Schematic of apparatus for copper plating of spool of tungsten wire.

However, one of the shortcomings of tungsten is that it readily oxidizes at elevated temperatures. During research conducted at NASA Ames Research Center the first author became aware of a form of tungsten wire that was flash coated with an extremely thin coating of platinum. This material offers the benefits of both materials, i.e. the high strength of tungsten and the oxidation resistance of platinum. This wire material is no longer manufactured, but we have managed to obtain sufficient quantities of this material with a diameter, d = 2.5 μm.

We are refining a technique for manufacturing robust oxidation-resistant hot-wire sensor filaments that was developed by the first author in conjunction with technical staff at NASA Ames Research Center. The raw flash–coated tungsten wire is wound onto a large spool which is then supported in a beaker containing copper sulphate solution, as shown in the schematic in figure 2. A copper coating is electrolytically deposited onto the tungsten wire that will end up forming the active portion hot-wire filament.

The copper coating has a thickness of about 40μm and it makes it easier to handle the extremely fine wire filament for mounting onto the prongs of the hot-wire probe. The central region of the copper coating is then etched away with a fine stream of dilute sulphuric acid, leaving the active section of the hot-wire filament. The remnant of the copper coating attached to each of the prongs form stubs, which support the active filament out away from any flow disturbances that may be introduced by the prongs. The procedure for etching the copper is very similar to the procedure that is widely used to etch away the silver from a hot-wire filament constructed from Wollaston wire.

The final spool of tungsten wire will contain enough plated material to make sufficient hot-wires for several years of continuous testing. Furthermore, the spacing on the spool makes it relatively easy to attach new wires to the prongs during the repair process. The installation procedure is the same as with Wollaston wire, except that it is the copper (instead of silver) that is etched away to expose the active filament.

One difficulty which remains with the plating process concerns the formation of nodules, *i.e.* the formation of small and uneven random lumps of copper on the wire material. This problem was also experienced during development of the plating process at

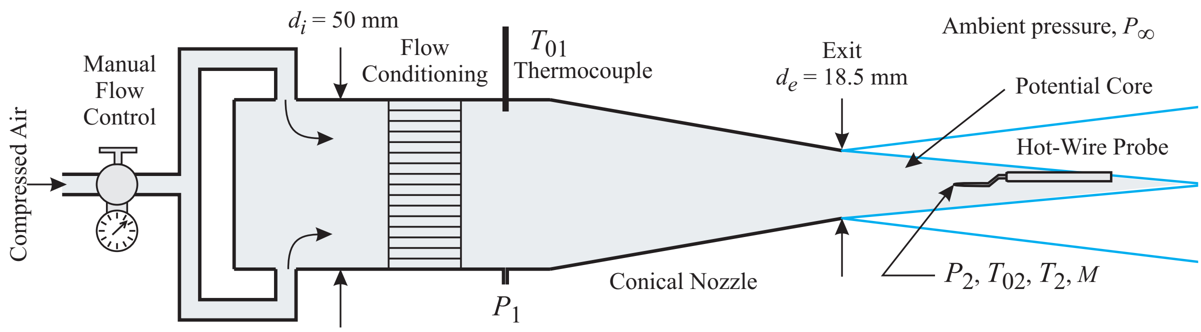

Figure 3: Schematic of apparatus constructed for calibration of hot-wire probes at high-speeds.

NASA. Unfortunately, the procedures used during the plating process were not documented. The magnetic stirrer shown in figure 2 helps to prevent the formation of bubbles and regions of concentrated copper deposition associated with imperfections located on the surface of the wire. Also, it is documented in the literature that very low currents are required to prevent the formation of nodules. We are confident that a solution to this frustrating problem will found very quickly.

High-speed stand-alone hot-wire calibration facility

The open jet facility sketched in figure 3 has been constructed to provide a stand-alone means to calibrate hot-wires at high-subsonic and transonic Mach numbers. The flow is provided by compressed air at a typical volume flow rate, $q = 0.085$ m^3 s^{-1} (180 CFM) and a typical mass flow of approximately, $\dot{m}$ = 0.1 kg s^{-1}. The flow is controlled by a manually operated valve with a pressure gauge and the flow enters around the periphery upstream of the nozzle. Flow conditioning is installed in the form of filter material to remove dust and a honeycomb to straighten the flow before it reaches the nozzle. The contraction ratio of the nozzle is given by $(d_i/d_e)^2 = 7.30$, where d_i and d_e are defined in figure 3.

The static temperature and static pressure are measured upstream of the nozzle and the velocity is small enough in this region to assume that these measurements also represent the total temperature and total pressure. Independent measurements have also been made in the potential core of the jet downstream of the nozzle exit. The stagnation temperature was measured using a thermistor and the velocity was measured using a miniature Pitot-static tube. The measurements have confirmed that the mass flow calculated in the potential core is very accurate when based only on the upstream static pressure and temperature measurements and assuming that the flow is isentropic.

The hot-wire is supported so that the filament is aligned with the exit of the facility. The exit diameter, $d_e = 18.5$ mm, is sufficient to minimize flow blockage by the hot-wire prongs and filament. Profiles have been measured across the jet at various downstream positions which demonstrate that the side-wall boundary layer of the nozzle is extremely thin leading to ”top-hat” shaped velocity profiles in the potential core. This means that position and alignment of the hot-wire probe in the jet does not have to be performed in an extremely careful manner.

By appropriate adjustment of the manually operated valve, the facility will provide the means to impose a range of accurately known mass flows on the hot-wire filament at accurately known temperatures, thereby providing the data required for calibration.

Conclusions

A complete hot-wire anemometry system has been designed for implementation at DSTO in high-subsonic and transonic Mach number flows.

Acknowledgements

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