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Direct Numerical Simulation of 3D Steady Streaming Induced by Honji Instability

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Abstract

Direct numerical simulation of sinusoidal oscillatory flow around a circular cylinder is carried out to study the Honji Instability. Three-dimensional Navier-Stokes equations are solved by finite element method. Numerical study is performed at *KC* number of 2 and β number ranging from 100 to 600 with an interval of 50. The steady streaming is calculated by averaging the velocity component over one oscillatory flow period. It is found that the flow is two-dimensional at β = 100 and the steady streaming structure is identical to that observed at $KC \ll 1$. For β=150 and 200, regularly distributed Honji vortices are observed around the cylinder. Three-dimensional steady streaming for these two β numbers is characterized by evenly distributed streamwise vortex-pairs. For $250 \leq \beta \leq 550$, the streamwise vortices in steady streaming become uneven along the cylinder. For β = 600, the flow transition to turbulent happens while the streamwise vortices still exist in the steady streaming. For case with unstable Honji vortices or turbulent flow, the steady streaming is time dependent because the flow is aperiodic. The intensity of streamwise vortices in the steady streaming decreases with the increase of the number of flow periods over which the steady streaming is calculated.

Introduction

Oscillatory flow around a stationary circular cylinder and flow induced by a circular cylinder oscillating in an otherwise quiescent viscous fluid has been of significant academic and practical interest. This flow is specified as $u = U_m \sin(2\pi t/T)$, where U_m, the maximum speed of the flow, and T, the period of oscillation. Key dimensionless parameters are then the Keulegan -Carpenter number *KC* and the Reynolds number *Re*

$$KC = U_m T/D \text{ and } Re = U_m D/\nu \quad (1)$$

where D is the diameter of the cylinder and ν is the kinematic viscosity of the fluid. A frequency number given by the ratio of *Re* and *KC* is also used frequently, which is written as

$$\beta = Re/KC = D^2/\nu T \quad (2)$$

When a periodic oscillatory flow passes around a circular cylinder, the interaction between the flow and the cylinder leads to a non-zero period-averaged flow field. This phenomenon is referred to as steady streaming ([1] and [2]), which has been of great interest in the area of acoustics and hydrodynamics. Most of the work on steady streaming since the early stage of the investigation has been summarized in the review paper of Riley [3].

Most of the previous research work was carried out under the condition of small amplitude oscillation ($KC \ll 1$), where the flow remains two-dimensional and analytic solution of steady streaming is achievable ([4] and [5]). In addition steady streaming can also be observed directly in experiments ([4], [6] and [7]).

Steady streaming has been demonstrated to exist at high *KC* numbers, where separated flow and vortex shedding happens ([8] and [9]). An et al. [10] carried out two-dimensional numerical simulations and captured six steady streaming patterns corresponding to the six vortex shedding regimes observed by Williamson [11].

It has been known that oscillatory flow around a circular cylinder remains two-dimensional only when *KC* and β are lower than certain values, beyond which Honji instability [12] occurs. Honji instability is a three-dimensional instability, which is characterised by mushroom shaped vortex structures. The flow structure of Honji Instability has been known well based on the previous work [12 – 17].

The motivation of the present study is to identify the structure of steady streaming around the cylinder when the Honji instability is present. This question will be addressed by conducting a series of direct numerical simulations.

Governing Equations and Numerical method

In this study, a sinusoidal oscillatory flow around a fixed circular cylinder is simulated numerically. The governing equations are the non-dimensional Navier-Stokes equations and the continuity equation. The governing equations in Cartesian coordinate system *xyz* (shown in Figure 1) read:

$$\frac{1}{KC}\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial x_j^2} = 0 \quad (3)$$

$$u_{i,i} = 0 \quad (4)$$

where u_i represents velocity component in the x_i-direction, $(x_1, x_2, x_3) = (x, y, z)$, $(u_1, u_2, u_3) = (u, v, w)$, and p is the pressure.

The initial conditions for the problem are

$$u_i(x, y, z) = 0 \qquad i = 1,2,3 \quad (5)$$

Non-slip boundary condition is given on surface of the cylinder. On the inlet boundary, velocity is specified as:

$$(u, v, w) = (U(t), 0, 0) \quad (6)$$

where the free stream velocity is given as:

$$U(t) = U_m \sin(2\pi t / T) \quad (7)$$

Periodic boundary condition is used on the two boundaries in the axial direction of the cylinder. Free slip boundary condition is implemented on two side boundaries. At the outflow boundary, the pressure is set to be zero and velocity gradients in the flow direction are set to zero.

Petrov-Galerkin finite element method is employed for discretizing the Navier-Stokes equations. In the Petrov-Galerkin formulation the standard Galerkin weighting functions are modified by adding a streamline upwind perturbation, which acts only in the flow direction [18]. Details about the finite element discretization can be found in Zhao *et al* [19]. The parallel computing program code based on message passing interface (MPI) is developed for the calculations. The numerical simulations were conducted on iVEC (Western Australian Computational Facility).

The detail of the mesh and numerical results

A definition sketch of the coordinate system and the detail of mesh around the cylinder are shown in Figure 1. The flow oscillates along the x-axis direction. Eight-node hexahedral elements are employed to discretize the three-dimensional domain. The domain dimensions are 40D, 20D and 4D in *x*-, *y*- and *z*-axis direction, respectively. The cylinder perimeter is discretized by 96 nodes. The minimum element size in the radial direction next to the cylinder surface is 0.001D. The total nodes number of the mesh is 448,950.

Non-slip boundary condition is given on surface of the cylinder. On the inlet boundary, a sinusoidal velocity in x-axis direction is specified. Periodic boundary condition is used on the two boundaries in the axial direction of the cylinder. Free slip boundary condition is implemented on two side boundaries. At the outflow boundary, the pressure is set to be zero and velocity gradients in the flow direction are set to zero.

A series of DNS simulations with a constant *KC* of 2 and β ranging from 100 to 600 (with an interval of 50) are carried out. This numerical model has been validated under steady current condition [19] and oscillatory flow condition [17]. No further validation is given in this study due to the page limit. Three flow regimes are identified from the numerical results. Numerical results show that, for β = 100, the flow is two-dimensional. Honji vortices are observed for β = 150 ~ 550. For β = 600, the flow is in turbulent flow regime.

In this study, the steady streaming is defined as the averaged flow field over one flow period. The steady streaming velocity ($\bar{u}_i, i = 1,2,3$) is calculated as:

$$\bar{u}_i = \int_{t_0}^{t_0+NT} u_i \, dt \qquad i = 1, 2, 3 \qquad (8)$$

where t_0 is an arbitrary instant after the flow has been fully developed and *N* is the number of flow period.

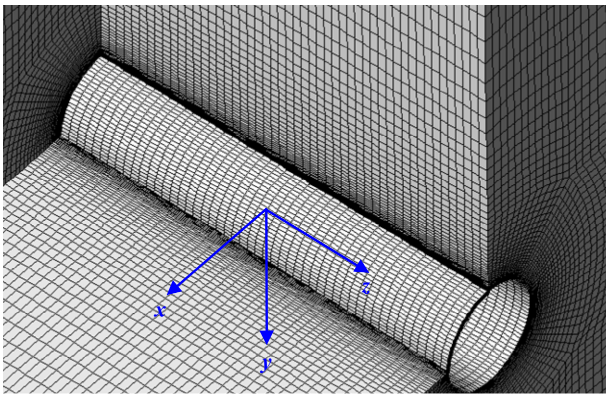

Figure 1. The sketch of co-ordinate system and the detail of mesh around the cylinder.

Steady streaming of two-dimensional flow

Present numerical results show that the flow remains two-dimensional at β = 100. Honji [12] observed that β = 120 is a critical frequency number between two-dimensional flow regime and Honji Instability regime.

Figure 2 show the steady streaming of case of *KC* = 2 and β = 100, in the cross-section of *z* = 0. The steady streaming structure comprises four small recirculating cells in the inner zone near the cylinder surface and four larger cells in the outer zone. The structure of the steady streaming is symmetric with respect to both *x*-axis and *y*-axis. The simulated streaming structure is identical to the experimental results [20] and analytical solution [5]. It should be noted that these experimental and analytical results mentioned above were obtained under low *KC* number and low *Re* number conditions. The flow conditions corresponding to the steady streaming structure observed by Masakazu Tatsuno [20] were *KC* = 0.54 and *Re* = 12.7. It is believed that flow structure shown in Figure 2 is a typical steady streaming pattern for two-dimensional oscillatory flow.

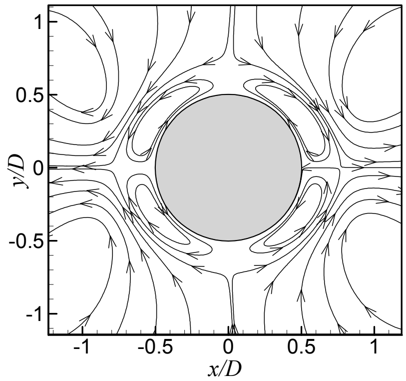

Figure 2. Stream lines and velocity distribution of steady streaming in the *x-y* plane with *z* = 0.0 (*KC* = 2 and β = 100).

Steady streaming of stable Honji vortices

For cases of β = 150 and 200, Honji Instability happens. Honji vortices are evenly distributed along the spanwise direction of the cylinder. A typical example of stable vortices is plotted in Figure 3(a) (β = 200) in which 'streamlines' are plotted using the velocity vectors (*v*, *w*) to visualize the flow structure in the *y-z* plane. Distinctive mushroom-like shapes appear – each one corresponds to a Honji vortex. Five equally-sized vortices are seen evenly distributed on the top side of the cylinder. Another five vortices are found on the lower side of the cylinder. The vortices on the two sides of the cylinder are distributed in staggered manner. The position of each Honji vortex does not change with time. These features are in good agreement with the observations [12] which are reproduced in Figure 3 (b).

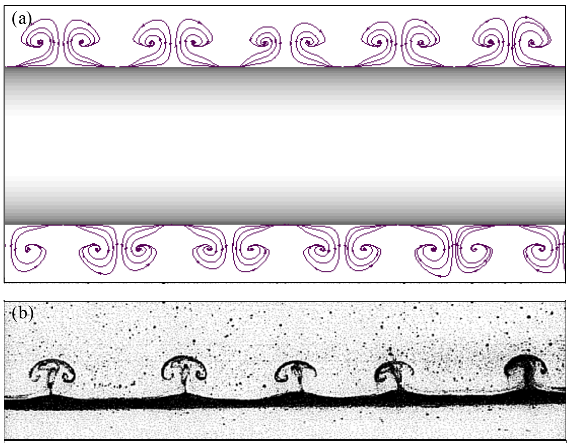

Figure 3. Honji vortices in the plane *x* = 0. In a) is shown the numerical result at time *t* = 75*T* for *KC* = 2 and β = 200; the equivalent experimental results of Honji (1981) are reproduced in b).

The steady streaming velocity is calculated using Eq. (8) with t_0 = 99*T* and *N* = 1. The numerical results of this case (β = 200) show strong periodicity. The steady streaming does not change with t_0 and *N* values. In Figure 4(a) the three-dimensional steady streaming structures are given by means of the iso-surface of λ_2 = -0.5, where λ_2 is the second eigenvalue of the symmetric tensor

$D^2 + \Omega^2$, with $\boldsymbol{D}$ and $\boldsymbol{\Omega}$ being the symmetric and the anti-symmetric parts of the velocity gradient tensor. Jeong and Hussain [21] demonstrated that λ_2 represents the vortex core locations of three-dimensional vortices. In Figure 4 (a), six pairs of streamwise vortices exist on the top side of the cylinder ($y > 0$), corresponding to six Honji vortices. The distance between two adjacent Honji vortices is 0.67D, which is in good agreement with the measurement of Honji [12]. These streamwise vortices are often named as rib structures. Each pair of ribs wraps nearly half of the circumference of the cylinder. Spanwise vortices also exist in between the ribs and the cylinder surface, which are often called as rollers. Figure 4 (b) shows the three-dimensional streamlines around the Honji vortices. The streamlines are started to be drawn along a straight line that is parallel to the z-axis and very close to the cylinder surface. All the streamlines erupt from six focuses on the crown line of the cylinder ($x = 0$ and $y = 0.5D$). A zoom-in of the streamlines in one Honji vortex is given in Figure 4 (c). The zoom-in figure shows that all the streamlines creep on the cylinder surface at the streamline start locations, and then accumulate to one point on the crown of the cylinder, from where the streamlines eject outwards and form the stem of the mushroom. All the streamlines curl towards the cylinder and form the crown of the mushroom when they reach a height of about 0.3D. After that, the streamlines are separated into four parts and flow out in the oscillatory direction. Such a steady streaming structure can transport mass, energy and heat away from the local area of the cylinder surface continuously. This explains how the dye streaks formed in the experiments of Honji [12] and Sarpkaya [13].

Steady streaming of unstable Honji vortices

For cases of $\beta = 250 \sim 550$, Honji vortices are unstable. With the developing of flow, the amalgamation of two Honji vortices and generation of new Honji vortex exist [17]. These features indicate the steady streaming depends on t_0 and N value. It is found that the intensity of streamwise vortices decrease with the increase of N value. For steady streaming calculated with $N > 100$, the streamwise vortices disappear and the steady streaming becomes two-dimensional. All the three-dimensional features are smoothed out during the averaging process. Therefore, one period averaged flow field is employed to represent steady streaming. Figure 5 shows the steady streaming calculated with $t_0 = 99T$ and $N = 1$. The rib pairs can still be observed in Figure 5(a). However, the rib pairs are not evenly distributed and the strengths of the two vortices of each rib-pair are obviously different from each other. The streamlines around the Honji vortices are shown in Figure 5 (b) and (c). Although the streamlines are not regularly distributed, the mushroom-shaped vortex structures still can be seen clearly.

Steady streaming of turbulent flow

For cases of $\beta = 600$, Honji vortices disappear and the oscillatory flow transition to turbulent happens. Honji [12] observed that the critical β value between Honji Instability regime and turbulent regime is 580 for $KC = 2$. Once again, the steady streaming depends on the value of t_0 and N. The steady streaming calculated with $t_0 = 99T$ and $N = 1$ are plotted in Figure 6. The iso-surface of $\lambda_2 = -0.5$ (Figure 6(a)) shows the rib shaped streamwise vortices still exist in this case, together with some irregular, small-scale vortices. The three-dimensional streamlines are shown in Figure 6(b) and (c), which are distributed irregularly along the cylinder. The streamlines also creep along the cylinder surface at the beginning and then eject upwards around the crown line. The shapes of the streamlines distinctively differ from that of the Honji vortices. One zoom-in three-dimensional structure is shown in Figure 6 (c). All the streamlines roll in one direction after ejecting out from the cylinder surface. Some other vortex structures are even more complicated. Here we conclude that this case belongs to turbulent flow and Honji vortices do not exist here.

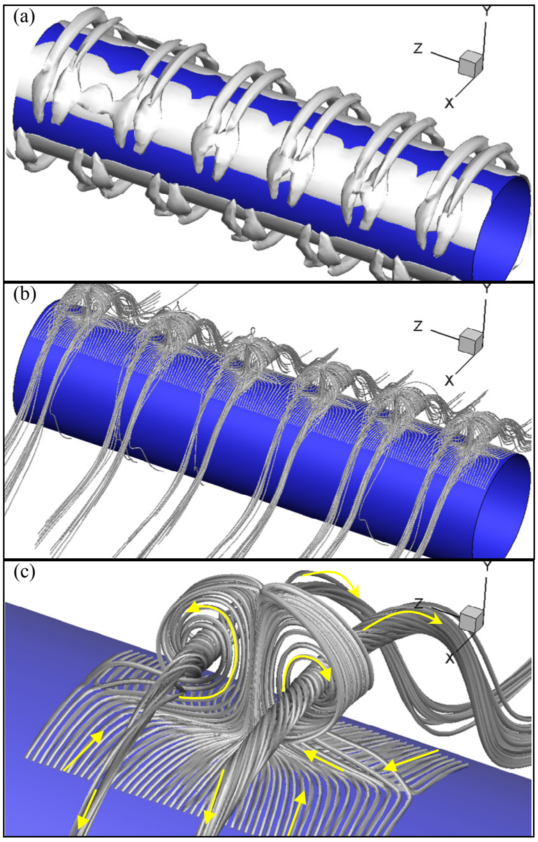

Figure 4. The steady streaming structures represented by the iso-surface of $\lambda_2 = -0.5$ ($KC = 2$ and $\beta = 200$).

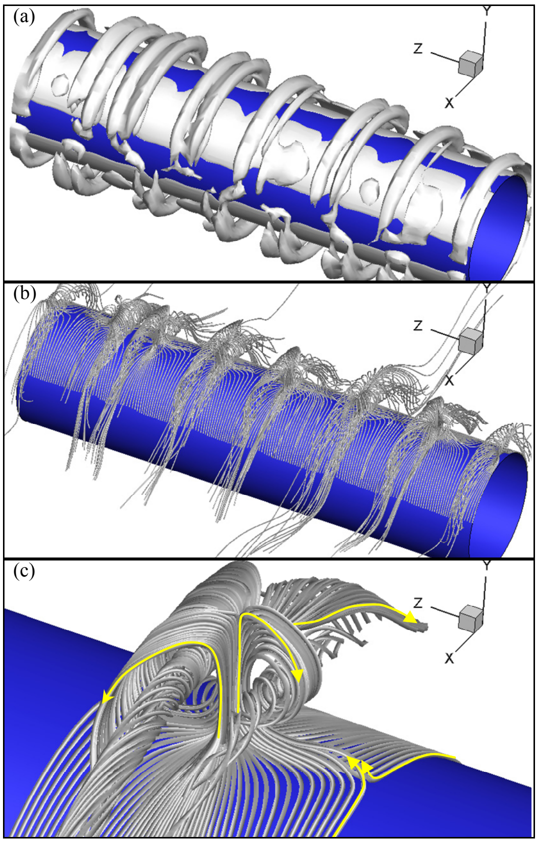

Figure 5. The steady streaming structures represented by the iso-surface of $\lambda_2 = 0.2$ ($KC = 2$ and $\beta = 400$).

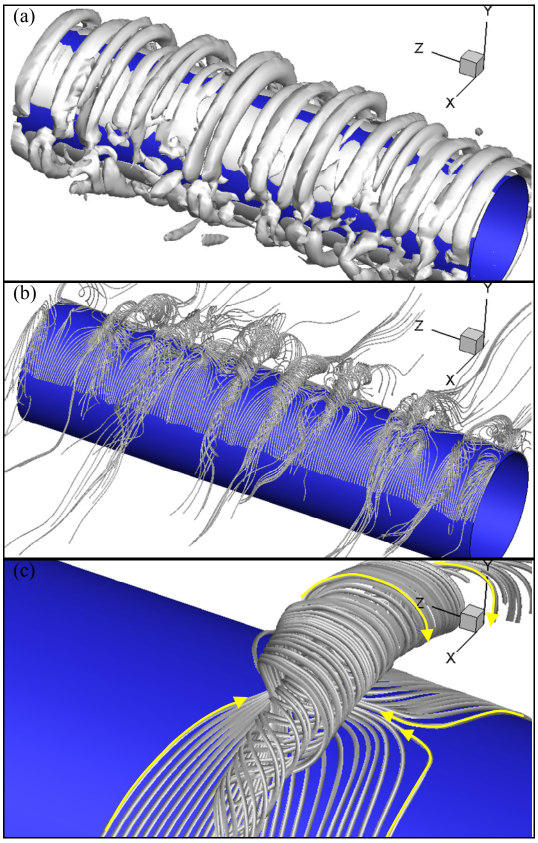

Figure 6. The steady streaming structures represented by the iso-surface of $\lambda_2 = 0.2$ ($KC = 2$ and $\beta = 600$).

Conclusions

Oscillatory flow around a circular cylinder at *KC* number of 2 and β number in the range of 100 to 600 is investigated numerically by solving the three-dimensional Navier-Stokes equations. In this numerical model, finite element method is used to solve the governing equations. The results are summarized as following:

- For β = 100, two-dimensional steady streaming is found. The basic structure is identical to that of cases with $KC << 1$.
- For β = 150 and 200, Honji vortices are observed. three-dimensional steady streaming structure was identified, which is characterised by stable, evenly distributed streamwise vortex-pairs.
- For $300 \le \beta \le 550$, the positions of Honji vortices are unstable. The streamwise vortices are irregularly distributed in the steady streaming.
- For β = 600, the flow is in turbulent regime. Streamwise vortices still exist while the Honji vortices disappear.
- For the cases with unstable three-dimensional vortices, the steady streaming depends on the value of t_0 and *N*. The degree of three-dimensionality of steady streaming decreases with the *N* value.

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