# Geothermal Fluid Dynamics

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## Abstract

This article reviews the development of geothermal fluid dynamics from the early studies of convection in a porous medium in the 1940s up to the present complex numerical models of geothermal fields. The early studies, and many others since, consider a simple problem with constant fluid properties except in the buoyancy term where linear dependence of density on temperature is assumed (the Boussinesq approximation). Even with this simple model complex behaviour is observed with a transition from conduction to convection at a Rayleigh number (*Ra*) of $4\pi^2$. At higher values of *Ra* transitions from steady to oscillatory and then to irregular convection occur.

In the 1970s numerical methods for studying general geothermal flows, including boiling water and steam, were developed, and have been applied to models of many geothermal systems over the last 40 years. Because of the highly nonlinear behaviour of boiling zones in hot geothermal systems geothermal simulators such as TOUGH2 [79] use implicit time stepping and upstream weighting of mobilities.

Geothermal fluid dynamics is still a challenging research area with many topics requiring further work. Some of those discussed here are: automatic model calibration or inverse modelling, double porosity models, chemical reactions, fluid-rock interaction and engineered geothermal systems.

## Introduction

The study of geothermal fluid dynamics began with the work by Horton and Rogers [40] and Lapwood [47] on the onset of convection in a porous medium. They proved the classical result that for a closed square box, convection occurs for a Rayleigh number (*Ra*) greater than $4\pi^2$. Many other studies on the onset of convection in a porous medium for various configurations and boundary conditions have been carried out over the last 65 years since the work by Horton and Rogers. The book by Nield and Bejan [57] provides a good summary of this research.

Post-onset behaviour was investigated by several researchers using perturbation analysis [66],[59],[85], but in order to investigate flows for *Ra* well above the critical value of $4\pi^2$, numerical methods had to be used. These included spectral methods [16],[42],[43],[67],[89],[91], finite difference methods [8],[37],[38],[87] and finite element methods [17]. The results are discussed further below.

The research on the onset of convection and post-onset behaviour all involved simple permeability structures and approximate fluid properties and therefore had limited application to modelling the flow of heat and mass in real geothermal fields which have very complex geological structures and a wide range of temperatures.

Numerical methods suitable for modelling non-isothermal, multi-phase, multi-component flow in heterogeneous, anisotropic porous media developed during the 1970s. A number of geothermal simulators were developed, most of which used fully implicit time differencing and upstream weighting of fluid properties at block interfaces.

Over the last 30 years computer models of many geothermal fields have been set up and modelling has become an almost standard tool in the planning and management of geothermal projects. The author recently reviewed the state-of-the-art of geothermal modelling [62] and gave a historical perspective on the development of geothermal modelling [63]. A brief discussion of the numerical techniques used in geothermal simulators, and their application to the study of reservoir physics and modelling real geothermal systems, is given below.

One of the most difficult tasks faced by a geothermal modeller is the problem of model calibration. This involves choosing model parameters such as permeability and porosity, and model boundary conditions such as the location and strength of deep inflows of heat and mass, so that the model results match the data. Solving this inverse problem is a topic of great current interest and is discussed further in the penultimate section. In the final section other important present and future research topics in geothermal reservoir modelling are discussed.

## Governing equations

Mass and heat flow in a geothermal system can be represented by the following generic conservation equations:

$$\frac{\partial A_m}{\partial t} + \nabla.\mathbf{F}_m = q_m \tag{1}$$

$$\frac{\partial A_e}{\partial t} + \nabla.\mathbf{F}_e = q_e \tag{2}$$

Here $A_m$ and $A_e$ are the amount of mass and heat per unit volume, $\mathbf{F}_m$ and $\mathbf{F}_e$ are the mass flux and heat flux and $q_m$ and $q_e$ are the amount of mass and heat produced or injected by wells.

For single-phase hot water $A_m$ and $A_e$ are defined by

$$A_m = \phi\rho_l \tag{3}$$

$$A_e = (1-\phi)\rho_r c_r T + \phi\rho_l u_l \tag{4}$$

Here $\phi$ is the porosity, $\rho_l$ is the density of water, $u_l$ is the internal energy of water, $\rho_r$, $c_r$ are the rock density and rock specific heat respectively and $T$ is temperature. One of the important properties of geothermal systems is that most of the heat energy is contained in the rock matrix rather than the fluid and typically the first term in equation (4) accounts for 70 to 80% of the total energy content. In equations (3) and (4) and below the subscript *l* is used to denote the properties of liquid water. Similarly the subscript *v* is used below to denote the steam or vapour phase.

For a two-phase mixture of boiling water and steam, equations (3) and (4) become

$$A_m = \phi(\rho_l S_l + \rho_v S_v) \tag{5}$$

$$A_e = (1-\phi)\rho_r c_r T + \phi(\rho_l u_l S_l + \rho_v u_v S_v) \tag{6}$$

Here $S_l$ and $S_v$ are the liquid and vapour saturations (volume fractions), $\rho_v$ is the density of steam and $u_v$ is the internal energy of steam.

For single phase flow of hot water the mass flux $\mathbf{F}_m$ is defined by Darcy's Law

$$\mathbf{F}_m = \frac{k}{\nu_l}(\nabla p - \rho_l \mathbf{g}) \tag{7}$$

Here $k$ is permeability, $\nu_l$ is the kinematic viscosity, $p$ is pressure and $\mathbf{g}$ is the acceleration due to gravity.

Then the energy flux is given by the sum of an advective and conductive term:

$$\mathbf{F}_e = h_l \mathbf{F}_m - K \nabla T \tag{8}$$

Here $h_l$ is the enthalpy of hot water and $K$ is the effective thermal conductivity of the saturated porous medium.

To represent two-phase flow, relative permeabilities $k_{rl}$ and $k_{rv}$ are introduced and the mass flux of water and steam, $\mathbf{F}_{ml}$ and $\mathbf{F}_{mv}$, are calculated separately:

$$\mathbf{F}_{ml} = \frac{kk_{rl}}{\nu_l}(\nabla p - \rho_l \mathbf{g}) \tag{9}$$

$$\mathbf{F}_{mv} = \frac{kk_{rv}}{\nu_v}(\nabla p - \rho_v \mathbf{g}) \tag{10}$$

The difficulty with using equations (9) and (10) is that the relative permeability functions are not known and cannot be measured in the field. Most experimental studies of $k_{rl}$ and $k_{rv}$ have involved mixtures of liquid and gaseous hydrocarbons rather than steam and water. It is known that at low saturations water is trapped in the fractures and pores and becomes immobile, but the precise value of the residual or immobile water saturation is not known and cannot be measured *in situ*. Conversely at very high water saturations the steam bubbles become trapped and the steam is immobile. One of the few experimental studies that considered boiling flows [98] suggested that straight-line relative permeabilities may be appropriate for geothermal flows.

Various versions of the relative permeability functions have been used in the geothermal context and are available in geothermal simulators such as TOUGH2 [78].

One of the important issues in the mathematical modelling of geothermal flows is how well equations (9) and (10), the two-phase version of Darcy's Law, represent the flow of water and steam through a fractured rock matrix. Some modellers have adopted a dual-porosity approach [4],[44], and in order to match pressures, enthalpies and tracer test results dual-porosity, dual-permeability models have been used [68].

The total mass and energy flows are given by

$$\mathbf{F}_m = \mathbf{F}_{ml} + \mathbf{F}_{mv} \tag{11}$$

$$\mathbf{F}_e = h_l \mathbf{F}_{ml} + h_v \mathbf{F}_{mv} - K \nabla T \tag{12}$$

Here $h_l$ and $h_v$ are the enthalpies of water and steam respectively.

One of the interesting consequences of equations (11) and (12) is the possibility of counter-flow with the flux of rising steam matching the flux of water trickling downwards, giving zero nett mass transfer but a significant upward energy flow. Counter-flow is the main mechanism for heat transfer in geothermal systems with large boiling zones and a small through-flow of mass. This is the case for vapour-dominated systems such as The Geysers in California [100] or Kamojang [92] and Darajat [3],[35] in Indonesia.

**Onset of convection**

In the simple analysis described here the Bousinesq approximation is made, i.e. the density of water is assumed constant except in the buoyancy term where it is defined by:

$$\rho_l = \rho_0[1 - \alpha(T - T_0)] \tag{13}$$

Then, in the absence of external sinks or sources (i.e. wells), equations (1) and (2) can be simplified and written in terms of the volume flux $\mathbf{Q}$. For 2D flows with $\mathbf{Q}=(u,v,0)$ they become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{14}$$

$$\lambda \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \tag{15}$$

Here $\lambda$ is the ratio of the thermal mass of the saturated rock matrix to that of water:

$$\lambda = [(1-\phi)\rho_r c_r + \phi \rho_0 c_0]/\rho_0 c_0 \tag{16}$$

The specific heat of water $c_0$ is assumed to be constant. The diffusivity $\kappa$ is given by:

$$\kappa = K/\rho_0 c_0 \tag{17}$$

In deriving equations (14) and (15) the assumption is made that the dependence of $u_l$ and $h_l$ on pressure is negligible.

Using equation (13) Darcy's law can be written as:

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial x} \tag{18}$$

$$v = -\frac{k}{\mu}\left( \frac{\partial p}{\partial y} + \rho_0 g[1 - \alpha(T - T_0)] \right) \tag{19}$$

The classical onset problem considers a closed box with a temperature $T_1$ on the base and $T_0$ on the top. In order to use analytic methods to solve equations (14), (15), (18) and (19) for this problem it is necessary to assume that all parameters are constant with respect to pressure and temperature. A few previous studies [41],[90] have included the more physically realistic temperature dependence of the viscosity $\mu_l$ but this modification requires the use of numerical methods.

It is convenient for solving the onset problem to introduce the following nondimensional variables:

$$x^* = x/a, \quad y^* = y/a, \quad t^* = \kappa t/a^2$$

$$u^* = au/\kappa, \quad v^* = av/\kappa$$

$$p^* = (p - [p_0 - \rho_0 g(y - a)])k/\kappa\mu$$

$$T^* = [T - T_0]/[T_1 - T_0]$$

Here $a$ is the thickness of the porous layer.

The introduction of a stream function also helps:

$$u^* = \frac{\partial \psi^*}{\partial y^*}, \quad v^* = -\frac{\partial \psi^*}{\partial x^*}$$

Now the governing equations can be reduced to two coupled equations for the nondimensional temperature and stream function, $T$ and $\psi$ (dropping the * for convenience):

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -Ra \frac{\partial T}{\partial x} \qquad (20)$$

$$\lambda \frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \qquad (21)$$

The Rayleigh number $Ra$ is defined by:

$$Ra = \frac{\alpha(T_1 - T_0)\rho_0 gka}{\kappa\mu_l} \qquad (22)$$

The nondimensional boundary conditions for the closed box problem are shown in Fig. 1.
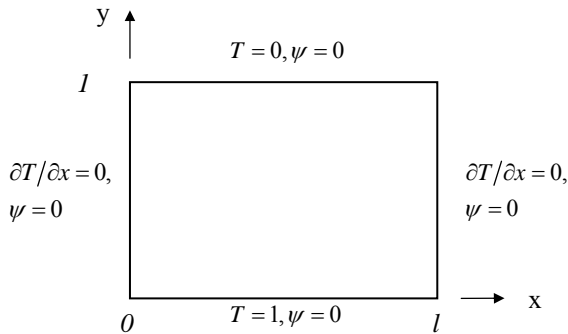


Figure 1. Boundary conditions for the nondimensional closed box problem

The onset problem is then easily solved by assuming steady flow and using linear stability analysis [57]. The conduction solution is perturbed using

$$\psi = \varepsilon\psi^{(1)}, \quad T = 1 - y + \varepsilon T^{(1)} \qquad (23)$$

Equations (20) and (21) are easily solved using (23) giving solutions:

$$\psi^{(1)} = A \sin\frac{m\pi x}{l}\sin n\pi y$$

$$T^{(1)} = B \cos\frac{m\pi x}{l}\sin n\pi y$$

With

$$Ra = \frac{\pi^2 [m^2/l^2 + n^2]^2}{m^2/l^2}$$

The smallest (critical) value $Ra_{cr} = 4\pi^2$ is given by a square single cell with $m=1$, $n=1$ and $l=1$.

A similar analysis was carried out for other sets of boundary conditions by Nield [56].

**Post-onset behaviour**

For a Rayleigh number not much greater than the critical value perturbation analysis can be applied [66],[85],[59]. A small parameter $\varepsilon$ defined by:

$$\varepsilon^2 = (Ra - Ra_{cr})/Ra$$

Then expansions for $\psi$ and $T$ are assumed in the form:

$$\psi = \varepsilon\psi^{(1)} + \varepsilon^2\psi^{(2)} + \varepsilon^3\psi^{(3)} + \ldots$$

$$T = 1 - y + \varepsilon T^{(1)} + \varepsilon^2 T^{(2)} + \varepsilon^3 T^{(3)} + \ldots$$

O'Sullivan and McKibbin [59] used expansions truncated at $O(\varepsilon^6)$ and obtained results for Nusselt number vs cell-width that agreed well with numerical results for $Ra$ up to 200.

The closed box convection problem discussed here is one of the simplest examples of hydrodynamic instability but even so it exhibits interesting and complex post-onset behaviour. An early investigation by Straus [89] used stability analysis based on Fourier series representations of the unknowns, to prove two-dimensional convection is unstable for $Ra > 380$. Caltigirone [8] proved a similar result that fluctuating convection occurs for Ra $> 384 +/- 5$, and he also showed that the transition from steady convection to fluctuating convection is strongly dependent on cell-width (the length $l$ in Fig. 1). Fig. 2 shows some of Caltigirone's results, for Nusselt number ($Nu$) vs cell-width for $Ra = 800$. The plot shows that stable convection ceases for a cell-width $> 0.5$.

Aidun and Steen [2] determined that for unicellular two-dimensional flow a Hopf bifurcation occurs at $Ra = 390.7$ that destabilizes the flow. This result was confirmed by Riley and Winter [84] who also investigated the effect of cell-width. They found that the flow becomes less stable as the cell becomes wider, thus confirming the numerical results of Caltigirone [8].
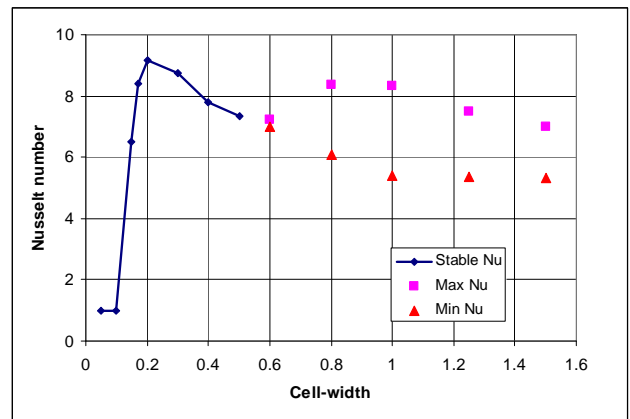


Figure 2. Nusselt number vs cell-width for $Ra$=800 (see Caltigirone [8]).

Kimura *et al.* [42] used a pseudo-spectral numerical scheme to study two-dimensional, unicellular, time-dependent convection in a square box. They found that with increasing $Ra$ the flow evolves from steady S to chaotic (non-periodic) NP through a sequence of bifurcations S $\rightarrow$ P$^{(1)}$ $\rightarrow$ QP$_2$ $\rightarrow$ P$^{(2)}$ $\rightarrow$ NP. Here P$^{(1)}$ and P$^{(2)}$ are periodic regimes and QP$_2$ is a quasi-periodic state with two basic frequencies. The transitions given by Kimura *et al.* [42] are given in Table 1.

A study by Holzbecher [37] investigating the stability of higher modes also confirmed the instability of the unicellular mode at $Ra = 390$. Further it showed that transition to the oscillatory regime occurs at $Ra \sim 510$ for the second mode and $Ra \sim 970$ for the third mode.

| Transition | Ra |
|---|---|
| Conduction to Convection (S) | $4\pi^2$ |
| S to P$^{(1)}$ | 380 - 400 |
| P$^{(1)}$ to QP$_2$ | 500 - 520 |
| QP$_2$ to P$^{(2)}$ | 560 - 570 |
| P$^{(2)}$ to NP | 850 - 1000 |

Table 1. Rayleigh number for various transitions (see Kimura *et al.* [42])

Holzbecher found that for supercritical flows unicellular first mode behaviour is only relevant for low values of $Ra$ and then second or third mode steady convection is preferred to oscillating first mode convection.

The situation becomes even more complex if three-dimensional flow is considered [36],[102],[39],[91],[87]. For example Straus and Schubert [91] investigated convection in a cubic box for $Ra <$ 150 and found that both 2D and 3D steady convection can occur depending on the initial conditions, even though the Nusselt number is greater for 2D flows for $Ra < 97$ but greater for 3D flows for $Ra > 97$. Recently Sezai [87] investigated convection in a cube for $Ra$ up to 1000. He identified ten steady flow patterns of which five show oscillatory behaviour in some Rayleigh-number range. Further Sezai found that two of the steady solutions are stable for $Ra$ up to 900, whereas an earlier study by Kimura *et al.* [43] had suggested that oscillations begin at $Ra$=575.

**Numerical Methods**

Numerical methods are required for the solution of equations (1), (2), (9) and (10), together with an equation of state (EOS) that includes accurate thermodynamic properties of water, and allows for the possibility of boiling i.e., the development of two-phase conditions.

Work began in the 1970s on numerical techniques for geothermal reservoir simulation [7],[11],[19],[20],[26],[48],[52],[54],[74], [92]. A brief discussion of this development is given in the review by the author [63]. A code comparison project organized by the US Department of Energy in 1980 [88] showed that several simulators produced similar results for a suite of challenging test problems. The key numerical techniques required to model the phase of transitions and strong advection that occur in geothermal flows are fully implicit time differencing and upstream weighting of interface quantities. The well-known geothermal simulators such as TOUGH2 [78], FEHM [105], STAR [73] and TETRAD [96] all use these techniques.

Upstream weighting provides a robust numerical technique but unfortunately also adds numerical dispersion and the smearing of sharp fronts [58]. Some work has been done on higher order methods [75],[76] but they are difficult to generalize for unstructured three-dimensional finite volume grids. Similarly good results have been achieved with an Euler-Lagrange approach for single-phase 2D flow [13] but more work is required to implement the method for two-phase or 3D flows.

Reservoir simulation has now been applied to setting up models of many geothermal systems [60],[62]. One of the most fully studied systems is Wairakei, New Zealand. Because of the ready availability of data Wairakei was used early on as a test case by

researchers developing geothermal reservoir simulators [51],[70],[71],[72],[54]. The author and co-workers have worked on a sequence of models that have increased in complexity as data have become available and as the capacity of the hardware has improved [6],[49],[50],[61],[63]. Our latest model [65] has ~10,000 blocks and another with ~30,000 blocks is under development. This is still small compared with oil and gas reservoir models which may contain $\sim 10^6$ blocks.

**Model Calibration**

The most difficult task in geothermal reservoir modelling is model calibration. In mathematical terms model calibration is an inverse problem which requires the choice of model parameters such as permeability and porosity so that a best fit is obtained between the model results and measured data. Model calibration involves three stages: conceptual modelling, natural state modelling and production history matching [60].

Conceptual modelling is the term given to the process of collecting all the data (geological, geophysical, geochemical etc) and synthesizing it into two or three sketches of the hydro-geological structure of the system.

In natural state modelling the conceptual model is used to set up the permeability structure of a computer model. Also the locations and magnitudes of the deep sources of heat and mass at the base of the computer model are determined from the conceptual model.

The computer model is run until stable steady-state conditions are achieved and then the model results are compared with data. In particular, measured down-hole temperature profiles are compared with the model results and the locations of surface outflows of heat and mass in the model are compared with the actual locations of hot springs and steaming ground. This aspect of model calibration is different for geothermal reservoirs than for oil and gas reservoirs because of the dynamic, convective nature of geothermal systems. The convective system in a geothermal system is affected by the permeability structure and thus influences the sub-surface temperature distribution.

If the model temperature distribution does not produce a good match to the measured down-hole temperatures, then the permeability structure is adjusted and the model is run again. Many iterations of this process may be required. A plot of the model results for one well at Wairakei are shown in Fig. 3.
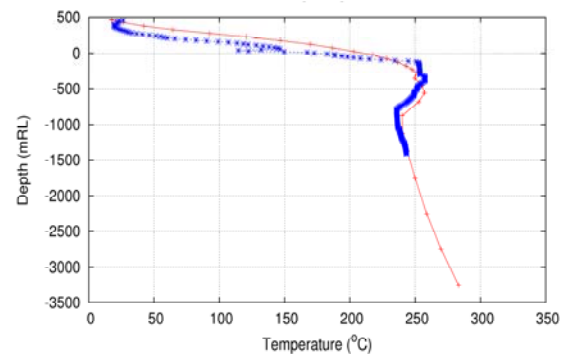


Figure 3. Natural state temperatures in one well for the Wairakei model (data - blue symbols, model - red line)

Once a reasonable natural-state model has been obtained the results are used as the initial conditions for a simulation of the production and injection history, with the measured mass flows being assigned to the appropriate model blocks. Then the pressure and enthalpy changes predicted by the model are compared to the data, and adjustments are made to the permeabilities and porosities to improve the model. Typical

results for the Wairakei-Tauhara model [65] are shown in Figs. 4 and 5.

Most models are calibrated by heuristic manual methods with the experience and skill of the modeller determining the quality of the model.
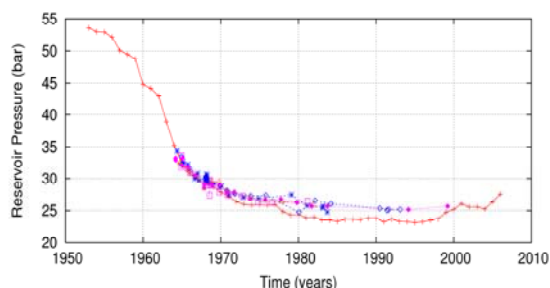


Figure 4. Pressure decline in the western borefield for the Wairakei model (data - blue symbols, model - red line)
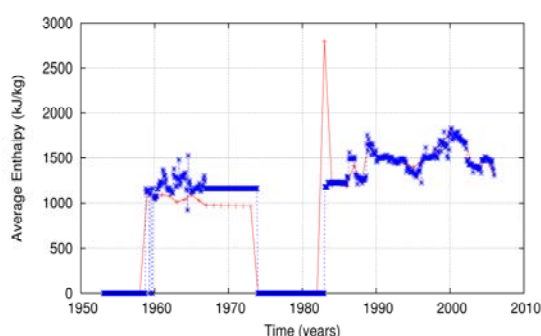


Figure 5. Enthalpy changes for one well in the Wairakei model (data—blue symbols, model—red line)

## Future directions in geothermal modelling

This topic was discussed by the author at the TOUGH Symposium in 2009 [64] and is briefly summarised again here.

### Wellbore – reservoir interaction

Many geothermal wells have more than one feed zone and the proportion from each feed may vary as the reservoir pressures change. To represent these processes a combined well-bore/reservoir simulator is required. A few researchers have experimented with this approach [5],[33], but it has not been very successful as the maximum time-step achievable is controlled by the well-bore flow and is often very small. An alternative approach using a multi-layer deliverability option is available in TOUGH2 but it assumes an approximate hydrostatic pressure profile in the well. A method that is more accurate than this but which is more computationally efficient than a full well-bore/reservoir model is required.

### Larger-deeper models

Very few models of geothermal systems include the whole of the large-scale convective system. Thus, the base boundary condition must include some input of very hot water, corresponding to the upflow zone of the convective plume. It would be better to make the model large enough so that the whole convective system is contained in the model, in which case the permeability structure has to be compatible with the flow and temperature structure. Recently the model of Wairakei-Tauhara developed by the author and co-workers has been extended by adding extra layers, so that it is now 4 km deep. Probably more layers, extending the model down to 6–7 km, should be added and a larger area included.

The use of deeper models leads to the need for a thermodynamic EOS that can handle higher pressures and temperatures. Croucher and O'Sullivan [14] have implemented the IAPWS-97 thermodynamic formulation [97], including a supercritical capability valid for pressures up to 100 MPa and temperatures up to 800˚C. This improvement allows for models of high temperatures and pressures to be set up, provided that the fluid can be approximated as pure water. It would be very useful for models of other fields (such as Ohaaki and Ngawha in New Zealand) to have an EOS for mixtures of water, carbon dioxide, and sodium chloride that is accurate for temperatures and pressures ranging from atmospheric up to supercritical (pure water) conditions. This would require the extension of the range of validity of the ECO2N fluid property module [78] in TOUGH2, which has been used for modelling carbon sequestration [80],[81].

### Near surface behaviour

The development of geothermal systems usually affects surface features such as geysers, hot springs and steaming ground. For example the exploitation of Wairakei caused the liquid-fed features like geysers and hot springs to disappear but caused an increase in the activity of steam heated features.

In order to model the interaction between the deep reservoir and surface activity in the model of Wairakei, an air-water EOS is used and the model is extended up to the ground surface [65]. Some other geothermal models take the water table as the top of the model. The inclusion of the unsaturated zone in the model works satisfactorily, but the movement of the water table is not tracked very accurately as the minimum layer thickness is 25 m. It would be useful to be able to handle the movement of the water table in a geothermal model similarly to the way unconfined aquifers are included in groundwater models. However a more sophisticated approach is required as the surface, where the pressure is atmospheric, may be partly boiling. Nevertheless, having the top surface of either water or steam able to move up or down through a grid-block would be a very useful advance for accurately representing near-surface behaviour, such as the development of large areas of steaming ground.

### Fluid-rock interaction

One of the great hopes of the geothermal industry is the development of enhanced (or engineered) geothermal systems (EGS) – also known as hot dry rock projects (HDR). The idea with an EGS is to drill one well, use hydraulic fracturing to create a permeable zone and then drill a second well to intersect the new man-made permeability [103]. Then heat can be extracted from the fractured rock by circulating water down one well and up the other. It is a simple idea, but although there have been several attempts to establish pilot projects none of them have become commercially successful. However there are very large resources of hot, low permeability, rock available worldwide [94] and EGS are currently under development in many countries (e.g. [30]), including Australia [9],[12],[31],[32],[83].

The main difficulty with an EGS is in establishing a large enough fracture network to give a satisfactory heat exchange system underground. In order to model the development of the fractures created by hydraulic fracturing and to model their performance as a heat exchanger, during the life of the EGS, it is necessary to be able to model the combined fluid flow, heat flow and rock mechanics problem [33].

The author and others [77],[86] have considered the much easier but related problem of modelling subsidence in a geothermal system. By using temperature and pressure charges calculated with TOUGH2 as input for a rock mechanics simulation using the ABAQUS package [1] we have had some success in

matching the occurrence of the subsidence bowls at Wairakei-Tauhara [101]. The result for the Wairakei subsidence bowl is shown in Fig. 6.

The EGS problem is much more challenging than the subsidence problem and remains to be solved, although some progress was made in a study of the Hijiori hot dry rock site [93], carried out with the FEHM code [105].
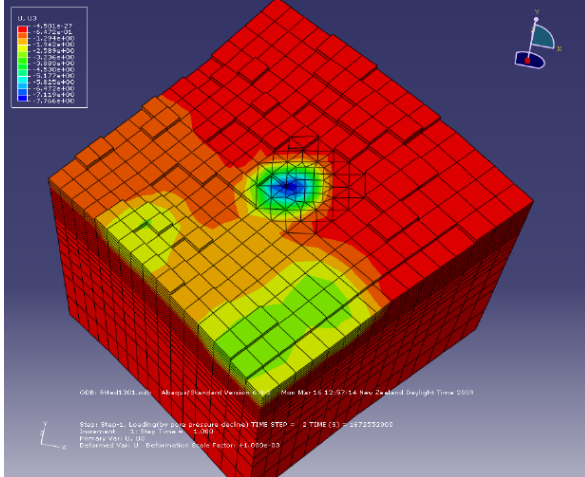


Figure 6. Model results for the surface deformation at the Wairakei subsidence bowl.

Some interesting fluid-rock interaction studies of mass transfer through the ductile zone, below geothermal systems, have been carried out by Fussels *et al.* [28] and Regenauer-Lieb *et al.* [82].

Model calibration

The major challenge facing the geothermal modelling community is automatic model calibration. The three approaches currently being investigated at the University of Auckland (and elsewhere) are: heuristic manual methods, inverse modelling and statistical sampling based on the multi-chain Monte Carlo method (MCMC).

To speed up manual calibration and to make it less dependent on the modeler, an "expert system" approach may be useful—and this is one of our current research themes. The idea is to codify the various strategies followed by a modeller and to apply them in a systematic fashion. For example, in a natural state model, if block I is too hot, then the following steps should be followed:

- Check flow directions for all connections between block I and other blocks.

- For flows into block I, if the neighboring block J is hotter, then decrease the permeability of block J. If block J is colder than block I, then increase the permeability of block J.

- Repeat for all blocks sending fluid into block I.

Several rules of this kind are currently used by modellers, but need to be formalized. There are many challenges to overcome in order to make such an expert system work, and there are several unanswered questions. For example: will it converge to a good solution in a reasonable time?

The inverse modelling approach using nonlinear optimization methods has been applied using software such as iTOUGH2 [21],[23],[24],[25],[25] and PEST [18].

The difficulty with using iTOUGH2 (or any inverse modeling code) in calibrating a geothermal model is the choice of the variable parameters. At one extreme, each block in the model could be assigned different x, y, z permeabilities and porosities. This would result in a huge number of unknown parameters and

is currently impractical. The simpler approach, and that which is usually used with iTOUGH2, is to assign a relatively small number of rock types and then use the permeabilities and porosities of a subset of these rock-types as the parameters to be optimized [45],[69]. However, even if the optimal values for all parameters, for all rock-types, are determined by iTOUGH2, the resulting model is probably not going to be the best possible.

It might be possible to produce a better model by subdividing the zone assigned, say, to rock-type IGNIM into two new zones, labelled IGNIA and IGNIB, for example. Then iTOUGH2 could be re-run optimizing the parameters for IGNIA and IGNIB independently. We have had some success with this technique, but what is required is a more systematic approach with, say, an outer XiTOUGH2 code that controls the re-assignment of rock-types and calls iTOUGH2 to optimize parameter values for each new rock-type structure.

There are some fundamental problems with inverse modeling based on a least-squares-error approach (see Fox [27]). One difficulty is that a global optimum or even a local optimum found by a nonlinear optimization technique may not be a "good solution," in the sense that the optimal parameter values may not be what a reservoir engineer expects or finds acceptable. What is really required is to identify a region of the multidimensional parameter space where good solutions are likely to be found.

Recently Cui [15] has used statistical sampling based on the MCMC method to calibrate a geothermal model. Statistical sampling methods have the potential to produce better models as they provide statistics of the parameters rather than just permit estimates. As well as calibrating a 3D model Cui compared his MCMC results with the results obtained using iTOUGH2 for a simple model consisting of a uniform layer feeding a geothermal well. Measurements of pressure and production enthalpy for an extended well test lasting 130 days were used to calibrate the model. The parameters determined by calibration were permeability, porosity, initial reservoir pressure, initial vapour saturation, relative permeability parameters.

A comparison of the MCMC and iTOUGH2 results is shown in Fig. 7. It shows the probability distributions for porosity and permeability (log scale) obtained using MCMC sampling and point estimates obtained with iTOUGH2.
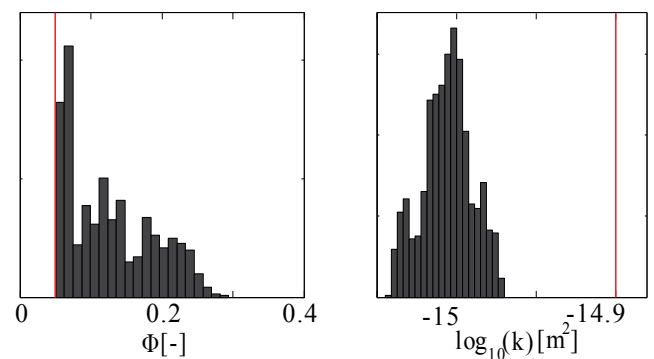


Figure 7. Statistical distributions of porosity and permeability obtained using MCMC sampling [15] compared with point estimates (red) obtained using iTOUGH2 [22].

The results show that the point estimates for porosity and permeability obtained by Finsterle *et al.* [22] using iTOUGH2 probably correspond to a local minimum and they are statistically poor results.

Further advances with the MCMC technique are required to make it practically useful for calibrating geothermal models. Currently, we are investigating the use of a hierarchy of models ranging from a coarse grid to a fine grid, and we are investigating

adaptive delayed acceptance algorithms [10],[15]. As with inverse modelling techniques (e.g. iTOUGH2), MCMC is ideal for implementation on a cluster of computers in a distributed memory configuration. In the future, it may be possible to use a cluster of multi-core processors, each running a parallelized version of TOUGH2 [55],[103], to calibrate large complex geothermal models.

## Conclusions

The basic theoretical problems in geothermal fluid dynamics have received much attention during the last 70 years with many studies carried out on the onset of convection and on the large amplitude post-onset behaviour. The problem of convection in a porous medium heated from below is one of simplest examples of hydrodynamic instability but nevertheless it exhibits interesting complex behaviour, with several modes of convection possible each of which changes from steady to oscillatory flow at different values of *Ra*. Further research is required to fully describe all the transitions.

Since their first development in the 1970s techniques for modelling flows in geothermal systems have developed considerably but more remains to be done. We would like to be able to run bigger and better models of geothermal fields; we would like to be able to calibrate them better and more quickly; and we would like the models to be able to incorporate more complex reservoir behaviour.

Improved calibration methods. All three of the calibration methods discussed above (heuristic manual methods, inverse modelling, statistical sampling) require further research to make them work well.

Improved numerics. Our highest priority item in this category is the tracking of the movement of the water/steam-table. This is probably achievable, and Euler-Lagrange methods [13] could be implemented in TOUGH2. We are less optimistic about implementing front tracking into general 3D unstructured grids commonly used in geothermal models.

New EOSs. There are no fundamental impediments to the development of EOS modules that can handle mixtures of water and $CO_2$ (and perhaps NaCl) over a wide temperature and pressure range, although working out the details could be time-consuming (e.g., Kissling *et al.* [46]).

Fluid-rock interaction. The problem of predicting the spread of a fracture zone in an HDR (EGS) project is challenging. More research is required.

Some of these aims can be met with current techniques, but others require more research.

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