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First Trace for Irregular Shock Wave Process in Weak Mach Reflection

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Abstract

Referring to von Neumann's words, transition from a two- to a three-shock reflection configuration in the weak domain presents some very considerable theoretical difficulties. It nonetheless remains an observed fact that such a transition does indeed occur in the real world. This paper describes the novel experimental technique and the results which for the first time did yield a "footprint" that something quite out-of-the-ordinary is taking place within the base of the reflected wave where the latter butts the point of shock confluence.

The results of the work presented here remained puzzling initially, for they seemed to contradict some preconceived outcome. It was only years later that the specific detail which was first considered to be misleading could be interpreted. It disclosed that the flow through the reflected shock is being forced to deviate from the classic Rankine-Hugoniot shock transformation process. This departure then enables the unyielding boundary conditions to be fulfilled and three-shock reflection to get established. This occurs with some lag past detachment of RR, this interval being required for the properties to adjust to the imposed conditions. The hypothesis of the departure from classic shock theory has been verified in the wind tunnel and was confirmed.

Introduction

Two basic types of shock wave reflection configurations are known to take place. Regular Reflection (RR) with only two shock waves takes place at steep ramp angles (small angles of incidence ω_{i0}). Mach Reflection (MR) which features three shock waves, occurs for shallow ramp inclinations (large angles of incidence ω_{i0}, see figure 1). In MR, the point of shock confluence is seen to lift away from the reflecting surface and at said common point, the two shock waves of RR are being supplanted by a single shock of higher strength which extends to and makes contact with the ramp surface. The change-over criteria from RR to MR (and vice versa) has been the subject of numerous investigations over the past 65 years. Von Neumann [14] found that with regards to transition, the phenomenon needed to be subdivided into a weak and a strong reflection domain. The characteristics that delineate the two have been discussed by Hornung [11] and Henderson [10] among others. The weak domain presents an interesting and highly challenging impasse for the onset of weak MR (WMR) and this represents the subject that is being dealt with in this paper.

In the range of RR, the reflected shock wave behaves and agrees quite accurately with the theory of Rankine-Hugoniot (RH). While the most comprehensive analysis of the two main shock reflection configurations has been provided by von Neumann [14], a diagram of RR which covers the entire range of physically meaningful shock strengths (from the acoustic limit up to infinitely strong incident shock waves and based on ideal gas behaviour) may be found in Courant and Friedrichs ([3], see figure 55, p. 328). At the terminating point of RR the reflected shock has reached its maximum flow deflecting capacity. This location is commonly known as the detachment condition in reference to the point where an oblique shock detaches from the tip of a wedge. A pregnant description of the characteristics of oblique shock solutions is found in section 16.5 of Shapiro [16]. The subject of the impasse is that at this point, the classic shock theory is confronted with a analytic singularity (the determinant of a quadratic polynomial equation becomes equal to zero). Beyond this point there simply does no longer exist any RH based solution that could fulfil the boundary conditions and provide for a smooth RR to WMR transition. Wherefore, beyond RR the problem is deadlocked by over-determination. This means that the requirements of conservation which the RH theory is based on are being challenged for a path that will enable an extension of the solution domain to be opened up.

In summary, for an ideal gas, the weak reflection domain is characterised by the following handicapping property:

Considering the shock polar representation in the pressure vs. flow deflection plane, at termination of RR, the point of tangency of the reflected shock with the pressure axis is situated *inside* the polar of the incident shock wave (see figure 4a of [10], figure 10 of [11], or figure 4a of [20]). Interpreting this shock polar disposition, past detachment of RR, the RH shock theory fails and is thus incapable of providing a real solution for any reflection configuration for the following two reasons:

1) A two-shock reflection pattern is precluded because there no longer exists any real oblique shock solution that could meet the required flow deflection requirement.
2) With regards to a three-shock reflection pattern, the higher compounded pressure jump across the incident/reflected shock pair compared to the pressure across a fictitious incipient Mach stem standing normal to the ramp surface inhibits growth of the latter.

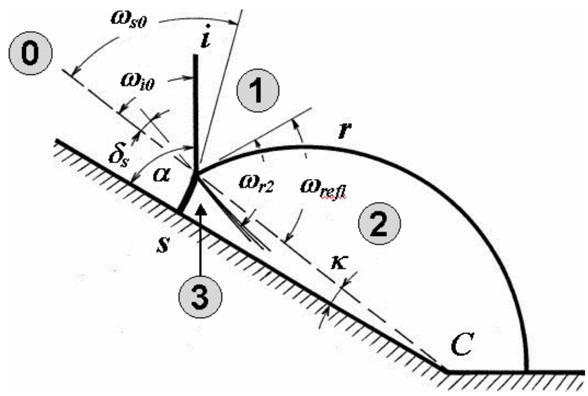

Figure 1: Nomenclature used for Mach reflection (three-shock configuration). The numbers in the grey circles denote the four flow regions which are separated by three shock waves (i-, r- and s-shocks) and one contact discontinuity. The flows in regions 0 and 1 are constant and uniform throughout. Since the Mach stem (s-shock) and the reflected shock (r-shock) are curved, the flows in areas 2 and 3 are non-uniform and always subsonic for WMR.

In air (diatomic gas, $\gamma = 1.402$), these facts limit the weak domain to pressure ratios p_1/p_0 of the incident shock wave to less than 2.3115 or, to undisturbed flow Mach numbers M_0 less than 2.2039 (these values pertain to conditions at detachment of RR). A pertinent display of this limiting condition has first been provided by Pantazopol et al. [15] (see also the corresponding diagram, figure 2 in [20]).

The theoretical solution for transition which does exist along the strong shock branch ought not to remain unmentioned here. In the past, this has sometimes been regarded as the correct solution for WMR. However, a detailed examination reveals that this would call for a highly discontinuous transition jump from the two- to the three-shock reflection configuration (the more so as the incident shock gets weaker within the weak domain; see the curves labelled C on figures 1 and 2 of [19] as well as the turquoise points on figures 4a and 4b of [20]). Such behaviour has never been observed to occur in experiments. This solution is misleading and may therefore be disregarded.

On contrasting the discordance between the RH theory and observations made beyond RR in the weak domain, the results of experiments appear to stand in quasi-contradiction to the classic understanding of shock waves. Moreover, the failure of said theory for transition to WMR as opposed to the comparatively successful agreement for strong incident shock waves also stands out in striking disparity. These two conflicting elements prompted Birkhoff [2] to take up this phenomenon in his review of paradoxes in fluid mechanics. A salient description of the dilemma has also been provided by Griffith [7].

Most observations made in laboratories that were reported and known before the experiments presented here could be interpreted, did already reveal that a two-shock RR-like configuration appears to persist to some extent beyond termination of RR (see Smith [21] and [22], Harrison and Bleakney [8], Kawamura and Saito [13], Henderson and Siegenthaler [9] and figure 3 of [20]). In other words, while theory fails to provide any realistic solution past RR detachment, experiments reveal a steady increase in the angle of reflection ω_{r2} as the angle of incidence ω_{i0} is incremented (see the diagrams, figures 1 and 2 of [19]). An angle of incidence is then reached where the emergence of a Mach stem begins to appear, thus transition to Mach reflection is occurring.

The novel experimental technique presented in this paper became instrumental in providing new insights in the underlying process just described. The motivation for developing the procedure originated more out of curiosity than anything else. The aim was to obtain a succinct graphical record of the disparate flow velocities that occur on either side of the slip stream in the wake of MR. This approach unexpectedly conceded a hint that something quite out-of-the-ordinary must be taking place past detachment of RR. This then is the process which opens the path to the establishment of the three-shock configuration (WMR). The correct interpretation of the feature so recorded, though ambiguous at first and thus protracted, did provide the key to some entirely new understanding.

It ought perhaps to be recollected that many of the explanations provided herein represent concepts which evolved over a time span of many years. In search for an answer, a number of exploratory calculations were performed. A point was reached where a viable postulate which appeared to genuinely mimic the experimental data was found. However, the assumption made looked rather far-fetched, so the hypothesis needed to be verified through pertinent testing in the laboratory (see Siegenthaler and Madhani [18]). As the model was thus confirmed, the theory of transition to WMR, although yet endowed with some significant questions as to the underlying physical process, was refined and presented by the author at the 25th ISSW in 2005 [19].

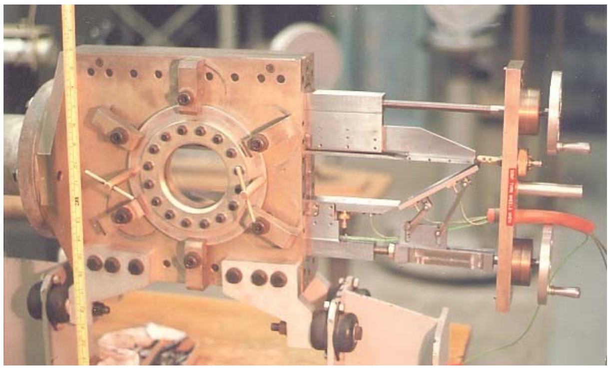

Figure 2: View of the shock tube's test section . The reflecting ramp and its associated adjusting mechanisms have been pulled out for modification work. The fine green leads connect the piezoelectric pressure transducers, two each being mounted on the ramp and two more on the lower horizontal plate. The injection equipment for the helium plume is not installed on this view. Mirrors of the schlieren system are visible in the background.

The flow visualisation technique described in this paper was developed nearly thirty years ago. A qualitative description of the first experimental results was originally provided in [17]. They were also referred to at the 15th Mach Reflection Symposium in Aachen, Germany (September 15-19, 2002).

The Shock Tube and its Set-up

The experiments were performed in a shock tube of conventional design (see figure 2). The size of the test section was 25.4 mm width by 68.3 mm height between the parallel upper and lower solid plates. The shock reflecting ramp fit snugly between the side walls and its inclination was continuously adjustable with a pair of jacking screws. A pair of optical grade windows of 97 mm clear diameter with their inside surface lined up flush with the test section walls, enabled the observation of the flow. The shock timing transducers, the pressure recording equipment as well as the schlieren system used, were the same as described in Henderson and Siegenthaler [9]. The duration of the stabilised spark discharge used as light source amounted to $^1/_3$ microsecond. The test gas was air at ambient pressure and temperature (state of the gas ahead of the incident shock wave).

The thought of resorting to a flow tracer technique in the shock tube originated from a latent desire to devise an experiment which would provide a visual record of the difference in wake velocities that prevail on either side of the slip-stream behind a Mach reflection process. The idea was to "insert" a "quasi-weightless" vertically oriented pencil into the test gas at rest before firing the tube. This pencil would present no resistance to deformation and would let the shock front propagate through it. It would then be swept along by the wake flow and be longitudinally modulated in accordance with the intensity of the flow velocities encountered at different levels above the ramp. This pencil would consist of a gas of dissimilar density with respect to the test gas and, due to the different index of optical refraction, would enable it to "become visible" with the schlieren system. In order to minimise the gas dynamic disturbance (change of acoustic impedance) thus inserted into the path of the propagating shock waves, the spatial extent of this pencil would need to be kept as confined as possible.

This concept was realised by introducing a lighter gas (helium) that would rise through the quiescent test gas (air) by buoyancy and thus form a tracer prior to the incident shock wave reaching the ramp. A small nozzle of 0.5 mm diameter and 2 mm length was mounted in the middle of the ramp width and placed at a distance of about $^1/_3$ up from the dihedral corner. The axis of this nozzle pointed vertically and its exit face was lined up flush with the surface of the ramp. Helium was introduced with minute over-pressure. A micro-metering valve enabled fine adjustment of the flow rate. This needed to be sufficient for the plume to show up on the schlieren system. Yet it should not be too strong, otherwise the jet might become turbulent and the test gas would get unduly contaminated. In order to obtain the desired freezing effect, a further requirement was to endeavour to keep the plume ascending velocity negligible compared to the flow velocities experienced behind the shock waves. This ratio is estimated to have been of the order of one to twenty at most for an incident shock pressure ratio of two to one.

Helium being seven times lighter than air, thus provided for the formation of a narrow laminar plume rising gently in the test gas. Upon being subjected to the shearing action of the slip-stream which originates at the triple point of MR, the streamwise modulation recorded some instant after the passage of the shock waves would represent a "witness" of the variable sweeping velocities.

The sequence of events to perform an experiment was first to open the helium supply line and simultaneously start an electronic timer. The latter would be pre-set to fire the shock tube within an adjustable delay of 3 to 5 seconds. The spark light source itself was timed as usual by the pressure signals provided by transducers mounted along the driven tube section. For these experiments, the schlieren apparatus needed to be set at near maximum sensitivity (high cut-off) and fast film (3000 ASA) was used.

To sum up, the purpose of this plume of helium served as an indicator which yielded an integrated record of the variable flow velocities it is being subjected to after being overtaken by some configuration of reflecting shock waves. In order to obtain appreciable displacement of the tracing plume, substantial wake flow velocities were required. A pressure ratio of about 2 to 1 for the incident shock was used to satisfy this requisite, (as mentioned above, at p_1/p_0 = 2.3115 the weak domain ends and changes to the dual-solution case whereon the need for an irregularly non-RH behaving reflected shock wave vanishes, this at least with regards to the RR to MR transition requirements).

Except for a major difference in scale, the technique is reminiscent of the full scale field tests performed by Dewey [4]. A description of the latter is also found in chapter 13.1 of Ben-Dor et al. [1]. In a more recent series of experiments carried out in a straight test section, Jacobs [12] studied what happens when a gas cylinder of disparate density is being overtaken by a very weak shock wave (M = 1.095). The intricate nature of the unfolding deformation imposed by the mechanisms at work are shown with amazing clarity in that work. More recently, Fabre et al. [5] modelled and analysed the process of a shock wave propagating through a cylindrical entropy spot.

Discussion

Figure 3 is a schlieren photograph of a plume of helium as it formed in the test gas before any shock passed it. The time interval between opening the helium supply line and the instant this photo was taken was somewhat overdrawn (over 7 seconds). This resulted in a cloud of air/helium mixture to accumulate under the upper plate. The vertical arrow which is shown on each of the four photographs, indicates the location where helium is being injected through the ramp plate.

On each of the following three pictures (figures 4 to 6) the flow is shown photographically "frozen" at an instant Δt_1 after the incident shock passed the corner of the ramp, respectively at an instant Δt_2 after the incident shock passed the helium plume injection nozzle. The shock waves propagate from the right to the left. The main characteristics of each experiment are summarised

Figure	**P [-]**	**θ_w [°]**	**Δt_1 [µs]**	**Δt_2 [µs]**
4 RR	2.04	54°44'	87	50
5 PRR	2.04	43°24'	137	74
6 WMR	2.04	36°44'	128	50

P: Strength (pressure ratio p_1/p_0) of the incident shock wave.

θ_w : Angle of inclination of the ramp with respect to the horizontal axis of the test section.

Δt_1: Time elapsed between the instant the incident shock passed the ramp corner and the instant the photograph was taken.

Δt_2: Time elapsed between the instant the incident shock passed the He-plume nozzle and the instant the photo was taken.

in the table. At the indicated shock strengths P, termination of RR is known to occur at an inclination angle of θ_w = 47°40'.

For RR in figure 4, the corner signal (expansion wave) clearly shows up. It is lagging well behind the point of shock reflection and demonstrates that the flow downstream of it is supersonic. The plume of helium was swept along uniformly and presents nothing unusual. The positive aspect is that there is hardly any smearing of helium along the ramp surface taking place. This suggests that the ascending velocity of the tracer gas is sufficiently low for the application striven for to be successful. This also shows that the boundary layer which develops along the ramp surface remains negligibly thin.

The turbulence which did develop in the lower portion of the plume on figure 5 is thought to be due to a transient instability in the helium feeding line. Also, as the time interval Δt_2 of this photo was nearly 50% longer than for figures 4 and 6, it is factual that the kind of unfolding disturbance induced by a shock wave that propagates through a plume of dissimilar density as investigated independently in [12] as well as in [5] was granted more time to develop in this shot. Notwithstanding this flaw, the striking feature is depicted by the bending-over of the plume's lower end. Considering the potential smearing effects that would be produced by either a boundary layer of substantial thickness, or by injecting the helium gas too forcefully, this kink is perceived as pointing in the opposite direction, or the "wrong way around".

In classic MR, a well defined step-like difference in velocity would have been expected to show up across the slip-stream. This would have translated into the plume getting severed squarely by shearing action. Looking at figure 6 however, the first conspicuous deduction suggested by the plume's shape is provided by the near non-existence of any velocity disparity across said discontinuity. This first appraisal will prove to be hastened and superficial. The other aspect which is clearly revealed by the schlieren optics is the very crisp appearance of the slip-stream. Notwithstanding the apparent lack of velocity disparity, this latter observation however suggests that a substantial density gradient nevertheless prevails. The lack of any appreciable velocity disparity which is suggested by the plume's trace is further supported by the fact that the slip-stream appears to remain laminar (very thin) over most of its length.

Considering figure 6 more closely, it is clear that the bending-over of the plume within the layers next to and above the slip-stream (in the wake of the reflected shock wave) may be regarded as qualitatively similar to the kink observed at the base of the plume's trace in the PRR configuration (figure 5). The analogy between the plume's trace in PRR and in WMR is twofold:

The bent-over kink is pointing upstream and is perceived as being generated within the low end (origin) of the reflected shock wave. This peculiarity is believed to be a sequel to the stalemate situation that afflicts said shock past its point of RR-detachment.

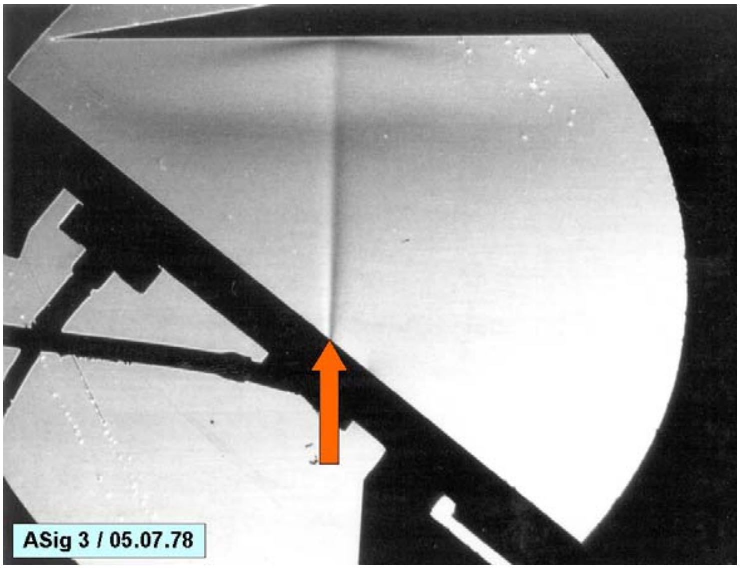

Figure 3: Still Life

Schlieren photograph of the tracing plume injected through a vertical nozzle (dia. 0.60 mm) mounted about 40 % up along the reflecting ramp. Helium is blown with minimal over-pressure into the quiescent test gas (air) in order for a laminar plume to form by buoyancy. The spark light source was triggered 7 to 8 seconds after initiating the flow of helium. Notice the accumulated "cloud" of helium which formed under the upper plate; this resulted from the spark light having been somewhat over-delayed.

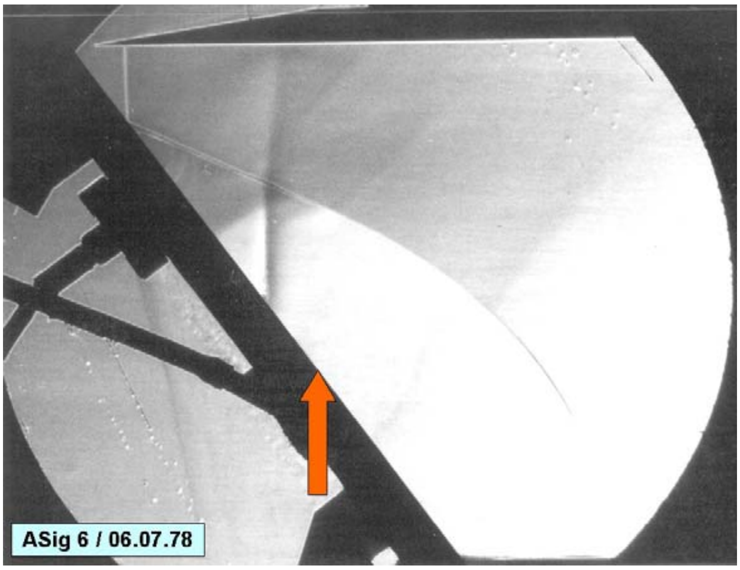

Figure 4: RR - Regular Reflection

Schlieren photograph of the plume of helium taken after it has been overtaken by a propagating shock reflection process of the RR configuration. The red arrow shows the location where helium is being injected vertically. The displacement of the plume generated by the sweeping wake flow reveals no anomaly in this case. Notice the corner signal (expansion wave) which is lagging well behind the point of shock confluence, the flow downstream of the reflected shock being supersonic.

$\xi_i = 0.49$ $\omega_{i0} = 35° 16'$ $\omega_{r2} = 29° 13'$

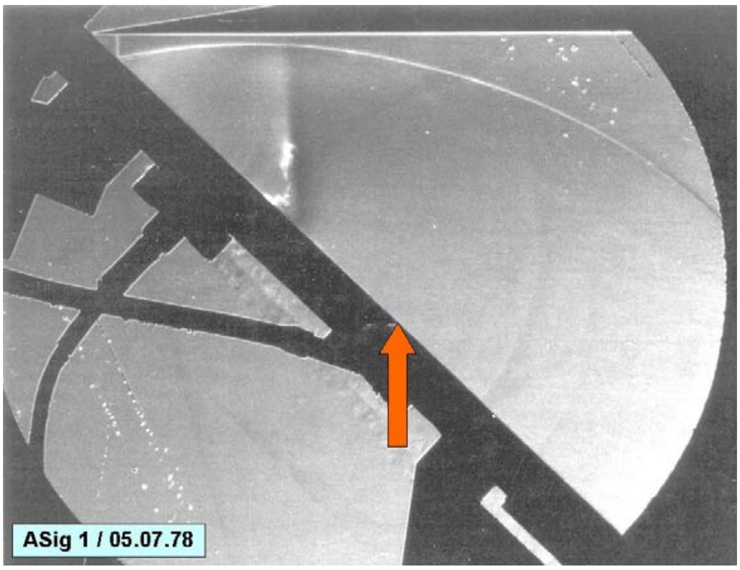

Figure 5: PRR - Pseudo-Regular Reflection

Schlieren photograph of the plume of helium taken after it has been overtaken by a propagating shock reflection process of the PRR configuration. The red arrow shows the location where helium is being injected. The displacement of the plume generated by the sweeping wake flow reveals a *"curling up the wrong way around"* within the streamlines that border the ramp surface. This anomaly is interpreted as a velocity defect which tapers off rapidly as the crosswise distance with respect to the streamline that emerges from the point of shock confluence increases. The observed curling up is pointing in the direction opposite to the way which would occur if it was caused by the wall boundary layer.

$\xi_i = 0.49$ $\omega_{i0} = 46° 36'$ $\omega_{r2} = 56° 01'$

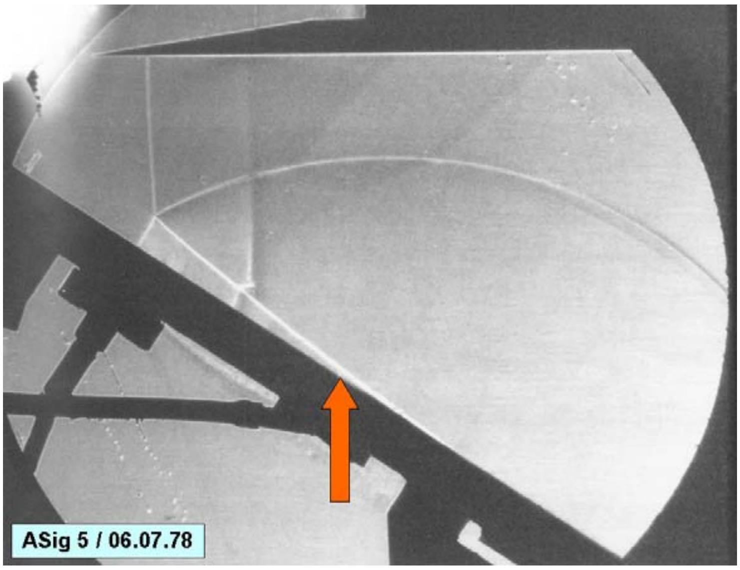

Figure 6: WMR - Weak Mach Reflection

Schlieren photograph of the plume of helium taken after it has been overtaken by a propagating shock reflection process of the WMR configuration. The red arrow shows the location where helium is being injected. Whereas the plume is shown to be swept along uniformly behind the Mach stem, a *"curling up the wrong way around"* similar to that seen in the PRR case is again being formed within the streamlines that emerge from the reflected shock, but this time the anomaly abuts the slip-stream. The curling up is strongest next to the slip-stream and tapers off with increasing crosswise distance. The striking feature here, is that the tracing plume suggests a near lack of velocity disparity to exist across the slip-stream. In contrast to this, the slip-stream itself appears very crisp which implies a substantial density difference to prevail between both sides of this discontinuity.

$\xi_i = 0.49$ $\quad$ $\omega_{i0} = 49° 46'$ $\quad$ $\omega_{r2} = 67° 07'$ $\quad$ $\kappa = 3° 30'$

The smoothness of the reflected shock's contour in the vicinity of the point of confluence, respectively the lack of any abrupt change in its slope is also apparent.

Recasting the attention onto figure 5, the appearance of that kink in the tracing plume represents a clear expression for the action of an anomalous effect which goes on accumulating along the streamlines in the wake flow of the reflected shock wave, as this stands in strong contrast to the standard behaviour seen in the wake of RR. We may therefore conclude, that this kink represents a "footprint" for the persistence of an irregular two-shock reflection configuration as was alluded to in the introduction. This irregular configuration is intercalated between detachment of RR and onset of WMR (see figure 3 of [20]). Going a step further, this footprint represents "proof" that a departure from RH is occurring within the base of the reflected shock wave. In this narrow range, the unyielding boundary conditions force the reflected shock to adjust its process of thermodynamic transformation. This departure from RH behaviour proceeds until a point is reached where the pressure equality stipulated under point 2 of the introduction can be fulfilled. And from this point onward, growth of an incipient Mach stem (third shock initially standing normal to the reflecting surface) becomes feasible.

This irregular two-shock reflection configuration has sometimes been referred to as Persistent RR in the past, but since it is known that the reflected shock is forced to depart appreciably from RH behaviour, the designation Pseudo-RR (PRR) is found to be more appropriate.

Conclusions

The tracer technique described here unexpectedly conceded a clue that a shock transformation process of quite unorthodox character is taking place in the problematic reflection domain where RH theory fails to provide an answer. The interpretation of the austere, but remarkable key feature so recorded, though ambiguous at first and thus protracted, opened the path to some entirely new insights. It revealed that the thermodynamic change of state which is forced to occur across the base of the reflected shock wave departs from the classic RH transformation process.

Although yet endowed with many open questions, a new theory of WMR, backed by experimental verification in the wind tunnel, has been elaborated on the basis of these results (see Siegenthaler and Madhani [18] as well as Siegenthaler [19] and [20]).

In short, the deduction from these experiments confirms that an irregular two-shock reflection pattern named PRR abuts the detachment limit of RR and that in this narrow range, the deviation from RH develops until a point is reached where the conditions for the establishment of WMR are fulfilled.

This deviation from classic RH compression process is atypical and translates into a drop of stagnation enthalpy (resp. stagnation temperature for an ideal gas). This is thought to take place within the confines of the shock thickness. While this does not directly affect the outflow velocity from the shock wave, the drop in stagnation temperature reduces the flow's critical speed of sound. Whence, on being subjected to the isentropic expansion which accelerates the wake flow, this anomaly results in the gas velocity increasing at a lesser rate along the streamlines that border the slip-stream than it does in the undisturbed flow field. The kink induced in the tracer plume is then the "footprint" of the dissimilar integration effects along neighbouring streamlines. With regards to transition, this departure from RH behaviour generates a twofold benefit: It raises the shock's deflection capacity and it also enhances its pressure ratio. Finally, upon further lowering the angle of the ramp, the intensity of this process of accommodation increases until such a point is reached where the growth of an incipient Mach stem becomes feasible, and this is where WMR is observed to develop.

Lastly, one may wonder why the addressed phenomenon (known as von Neumann paradox after [2]) has remained elusive for so long. One practical answer is simply that it does virtually not show up using standard optical flow visualising techniques. After being acquainted with the present results and looking back, there is however one record of old days by Fletcher et al. [6] which to this author does look suspicious and might reveal some commonality with the actual explanations (see figure 7b of [6]). The other aspect which probably played a significant role is that the outcome stands up against the mainstream understanding of more orthodox shock reflection problems.

S. D. G.

Acknowledgments

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