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Dependence of Strouhal Number, Drag and Lift on the Ratio of Cylinder Diameters in a Two-Tandem Cylinder Wake

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Abstract

This paper presents the effect of the diameter of a cylinder and Reynolds number (*Re*) on time-averaged drag (C_D), rms drag (C_{Drms}), rms lift (C_{Lrms}) and Strouhal number (*St*) in the wake of a downstream cylinder. The *Re* range considered is $7\times10^3 \sim 3.3\times10^4$. The upstream cylinder diameter *d* investigated was 6, 10, 15, 20 and 25 mm, while the downstream cylinder diameter *D* was 25 mm, corresponding to *d/D* ranging from 0.24 ~ 1.0. The spacing ratio *L/d* (where *L* is the distance between the center of the upstream cylinder center and the leading stagnation point of the downstream cylinder) is 1, 2 and 5.5, covering different flow regimes. At *L/d* = 1, *d/D* = 1.0, increasing *Re* results a switching of a steady-reattachment to alternating-reattachment of the two free shear layers of the upstream cylinder on the downstream cylinder at *Re* = 23250, augmenting the width between the two free shear layers in the gap between the cylinders. As decreasing *d/D* corresponds to diminishing width between the free shear layers in the gap, the steady-reattachment of the shear layers occurred for *d/D* < 1.0 at *L/d* = 1 and for *d/D* < 0.8 at *L/d* = 2. While increasing *d/D* from 0.24 to 1.0 at *L/d* = 5.5 changes *St* from 0.203 to 0.12, that at *L/d* = 1 and 2 has little or no influence on *St*. An augmentation of *d/D* from 0 (isolated cylinder) progressively to 0.24, 0.4, 0.6 to 0.8 causes a reduction in C_D by about 12~15%, 24~30%, 67~72% and 87~94% at *L/d* = 1 and 6~20%, 36~38%, 73~77% and 86~91% at *L/d* = 2, respectively. As *d/D* grows from 0.24 to 1.0, C_D at *L/d* = 1 and 2 decays faster with *d/D* than at *L/d* = 5.5. *d/D* = 0.24~0.6 in the reattachment regime (i.e., at *L/d* =1 and 2) may generate streamwise vibration of the cylinder at $f_v/f_{n1} \approx 0.5$ and 1, in addition to at $f_v/f_{n1} = 2$, where f_v and f_{n1} are the vortex-shedding and first-mode-natural frequencies of the cylinder. At *L/d* = 5.5, C_{Drms} and C_{Lrms} grow with *d/D*. On the other hand, at *L/d* = 1 and 2, C_{Drms} is more or less constant and C_{Lrms} diminishes with *d/D*.

Nomenclature

ν	Kinematic viscosity of air
C_D	Time-averaged drag coefficient
C_{Drms}	Root-mean-square (rms) drag coefficient
C_{Lrms}	Root-mean-square (rms) lift coefficient
d	Diameter of the upstream cylinder (Fig. 1)
D	Diameter of the downstream cylinder (Fig. 1)
f_n	Natural frequency of cylinder system
f_{n1}	First-mode natural frequency of cylinder system
f_{n2}	Second-mode natural frequency of cylinder system
f_v	Vortex shedding frequency
HT1	Hotwire 1 placed in the gap between the cylinders
HT2	Hotwire 2 placed behind the downstream cylinder
L	Distance between the center of the upstream cylinder center and the leading stagnation point of the downstream cylinder (Fig. 1)
Re	Reynolds number ($=U_\infty D/\nu$)
St	Strouhal number ($=f_v D$(or d)$/U_\infty$) depending on whether it is for downstream or upstream cylinders
t	Time
U_∞	Free-stream velocity

1. Introduction

The study of aerodynamic interference between two closely separated cylinders is of both fundamental and practical significance. In engineering, fluid forces and Strouhal numbers are the major factors considered in the design of multiple slender structures subjected to cross flow, e.g., chimney stacks, tube bundles in heat exchangers, overhead power-line bundles, bridge piers, stays, masts, chemical-reaction towers, offshore platforms and adjacent skyscrapers. The simplest configuration of multiple slender structures is two cylinders in either tandem or side-by-side arrangement.

Flow around two tandem cylinders of identical diameters is in general classified into three major regimes [1]: (i) the extended-body regime (L/d < 0.7), where the two cylinders are so close to each other that the free shear layers separated from the upstream cylinder overshoot the downstream one, and the flow in the gap of the cylinders is stagnant; (ii) the reattachment regime (L/d = 0.7~3.5), where the shear layers separated from the upstream cylinder reattach on the downstream cylinder and the flow in the gap is still insignificant; (iii) the co-shedding regime (L/d > 3.5), where the shear layers roll up alternately in the gap between the cylinders and thus the flow in the gap is significant. From the fundamental point of view, aerodynamic interference between two closely separated cylinders may give rise to flow separation, reattachment, vortex impingement, recirculation and quasi-periodic vortices, involving most generic flow features associated with multiple structures. Thus, flow around two tandem cylinders provides a good model to understand the physics of flow around multiple cylindrical structures.

Pannell *et al*. [2] measured the combined drag force acting on the two parallel circular wires in a tandem arrangement for L/d < 4.5. They surprisingly stated "it is interesting to notice that the minimum drag on two wires in contact is only 40% of the drag on one wire alone." This was resulted from the fact that the existence of the downstream wire improved the streaming of the upstream wire. Similar experiment was extended by Biermann & Herrnstein [3] up to L/d = 7.5 (Re = 1.05×10^5). They measured only C_D of the individual cylinders; therefore, more investigation was needed to clarify the other parameters, such as, C_{Drm}, C_{Lrms}, St, surface pressures, wakes, boundary layer characteristics around the cylinders, etc. Time-averaged pressure measurements on the surfaces of the cylinders were conducted by Hori [4] (Re = 8×10^3), Zdravkovich & Pridden [5] (Re = 6×10^4), Gu & Sun [6] (Re = 2.2×10^5 ~ 3.3×10^5) and Alam *et al.* [7] (Re = 6.5×10^4). The results showed that for L/d < 3 a negative pressure on the front surface of the downstream cylinder was generated instead of positive pressure, exceeding that on the rear surface. In case of the upstream cylinder, the pressure only on rear surface was affected by the presence of the downstream cylinder. Nevertheless, beyond L/d = 3, the pressure distribution around the cylinders is comparable to that around a single isolated cylinder. Fluctuating surface pressure and behavior of boundary layer on the cylinders for L/d < 8.5 were investigated by Alam *et al.* [7]. C_{Drms} and C_{Lrms} were measured by Arie *et al.* [8] (Re = 1.57×10^5) and Alam *et al.* [7]. In the reattachment regime, the later authors established a correlation between C_{Lrms} and reattachment position on the downstream cylinder of the shear layers from the upstream cylinder; as the reattachment position moves toward the forward stagnation line, C_{Lrms} on the downstream cylinder increases and vice versa. Novak [9] (Re = 1.05×10^4), Kiya *et al.* [10] (Re = 1.58×10^4) and Igarashi [11, 12] (Re = 8.7×10^3~5.2×10^4) measured St and showed that the two cylinders shed vortices at the same frequency at least up to L/d = 10. Wakes characteristics of the cylinders could be found in [13] (Re = 1.5×10^3~1.5×10^4) and [14] (Re = 850~1350). Phase-averaged flow structure, momentum and heat transport in the wake were measured by Zhou & Yiu [15], St by Kiya *et al.* [10], Alam *et al.* [7], Xu & Zhou [16] and downstream evolution of wake by Yiu *et al.* [17].

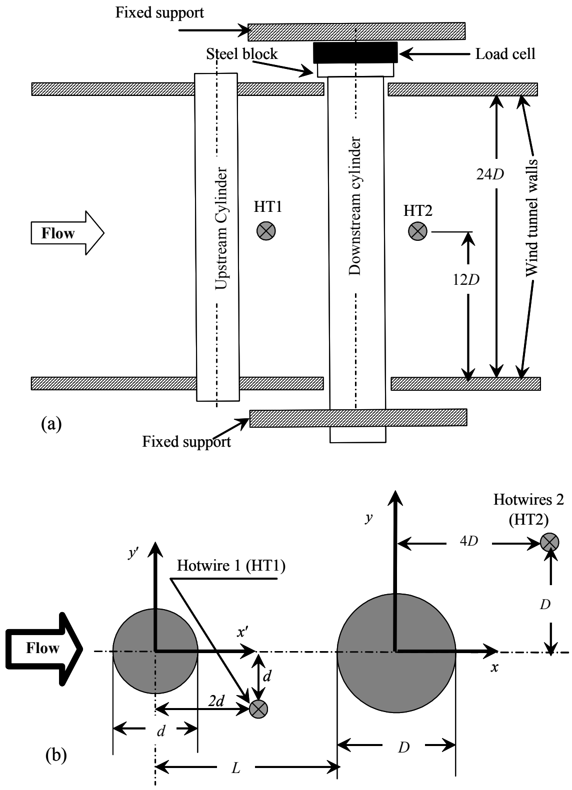

Fig. 1. (a) Experimental set-up, (b) definition of symbols

The reported studies were concerned with two cylinders of an identical diameter. There has not been extensive research on forces, Strouhal numbers and flow around two cylinders of different diameters. The structures in group or at close proximity are not always of the same diameters. For example, a piggyback pipeline normally comprises of one (or more) small pipe (or pipes) and a large pipe. The small and the large pipes could either be in contact or separate, depending on design and installation requirements. The application demands engineers to know the answer of the following questions. What is the effect of the upstream cylinder diameter on C_D, C_{Drms}, C_{Lrms}, St and the wake of the downstream cylinder? How are these parameters dependent on Re and L/d?

This work aims to study experimentally C_D, C_{Drms}, C_{Lrms}, St and the wake of the downstream cylinder of two tandem circular cylinders of different diameters. The upstream cylinder diameter (d) is varied, with the downstream cylinder diameter (D) unchanged, so that the ratio d/D varies from 1.0 to 0.24. The investigated L/d and Re are 1.0 ~ 5.5 and 7×10^3 ~ 3.3×10^4, respectively.

2. Experimental Details

Measurements were conducted in a low-speed, close-circuit wind tunnel with a 2.4-m-long test section of 0.60 m × 0.60 m. See [18] for the detail of the tunnel. Two cylinders were mounted in tandem in the horizontal mid plane of the working section. Figure 1 shows schematically experimental setup and the definitions of coordinates (x', y') and (x, y), with the origins defined at the upstream and downstream cylinder centers, respectively. All cylinders were made of brass. The upstream cylinder was solid and fixed-mounted at both ends at the wind tunnel walls. On the other hand, the downstream cylinder was hollow (outer diameter D = 25 mm and inner diameter 21 mm) and fixed-mounted at external rigid supports detached from the wind tunnel, as forces being measured by load cell. d was 25, 20, 15, 10 and 6 mm, respectively, and the corresponding d/D was 1.0 ~ 0.24, resulting in a maximum blockage of about 2.4%, and a minimum aspect ratio of 24. U_∞ was varied from 4.22 to 20 m/s, corresponding to variation of Re from 7×10^3 to 3.3×10^4 based on the downstream cylinder.

Two tungsten wires HT1 and HT2, each 5 μm in diameter and approximately 2 mm in length, placed at x'/d = 2, y'/d = - 1 and x/D = 4, y/D = 1 (Fig. 1), were used to measure the frequencies of vortex shedding from the upstream and downstream cylinders, respectively. The wires were operated at an overheat ratio of 1.8 on constant temperature circuit. The hotwire-probe holder was placed perpendicular to the wake-center plane to minimize the disturbance to flow.

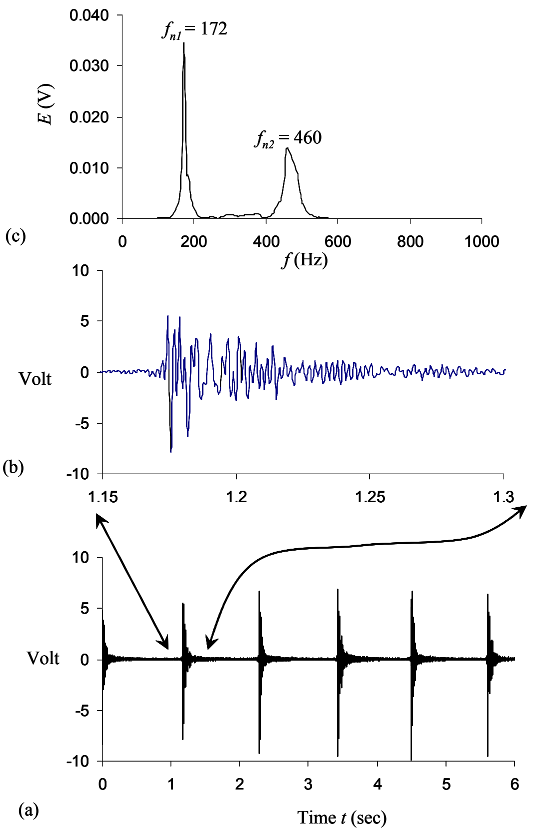

Fig. 2. (a) Signal from load cell when striking the cylinder, (b) enlarged view of a portion of the signal, (c) power spectrum of signal (a).

A three-component strain-gauge load cell (KYOWA Model LSM-B-500NSA1), characterized by high response, resolution and stiffness, was installed at one end of the downstream cylinder to measure the fluid forces. The load cell measures instantaneous integral fluid forces acting on the length of the cylinder exposed in the wind tunnel. The signals from the hot wires and load cell were offset, amplified and then digitized using a 12-bit A/D board at a sampling frequency of 2.0 kHz. The sampling duration was about 20 seconds and at least three sample signals in one test were obtained at each measurement point. The overall uncertainty in the measurements of C_D, C_{Drm}, C_{Lrms} and St,was estimated to be about 3%, 7%, 5% and 2%, respectively.

3. Results and Discussion

3.1. Natural Frequency of the Cylinder System Comprising Load Cell

Being an important parameter when flow-induced force on the cylinder is measured using load cell, natural frequency f_n of the cylinder system comprising load cell is identified. In order to determine f_n, the middle of the cylinder was hit slightly several times by the handle of a screw driver and signal from the load cell was captured. The gap between two successive hits was arbitrary, about one second, but should be long enough compared to $1/f_n$. A typical load cell signal is presented in Fig. 2(a). An expanded view of a part of the signal is given in Fig. 2(b). Power spectrum (Fig. 2c) of the signal 2(a) is made displaying two peaks at 172 and 460 Hz. They correspond to the first- and second-mode natural frequencies (f_{n1} and f_{n2}), respectively.

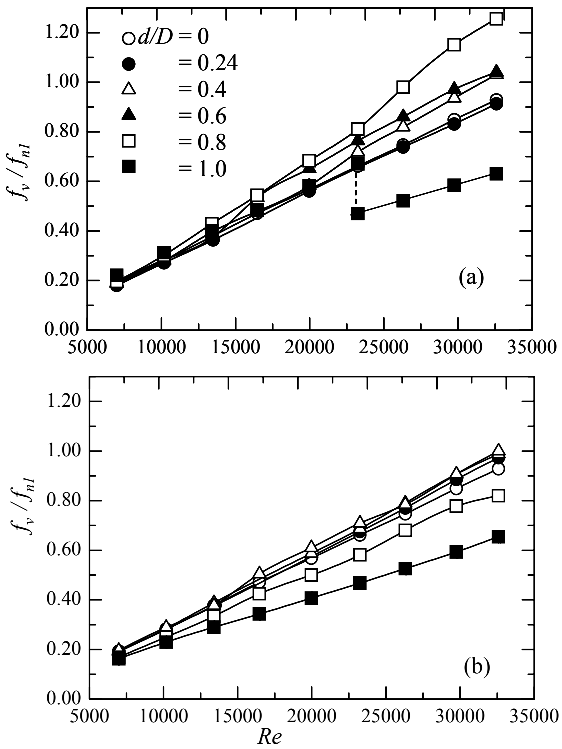

Fig. 3. Dependence on Re of f_v/f_{n1} of the downstream cylinder at (a) L/d = 1, (b) L/d = 2.

3.2. Dependence of f_v/f_{n1} and St on Re

Vortex shedding frequency f_v is normalized by f_{n1}, being less than or around f_{n1} for the Re range examined. Figure 3 shows variation of f_v/f_{n1} with Re at L/d = 1 and 2, where f_v was obtained from power spectral analysis of HT2 signal. As we see at L/d = 1 (Fig. 3a), f_v/f_{n1} for d/D = 0~0.6 closes to ≈0.5 and ≈1.0 at about Re = 16460 and 33000, respectively. d/D = 0 defines a single isolated cylinder. f_v/f_{n1} for d/D = 1.0 is ≈0.5 at Re = 20000 and 26300; it does not reach 1.0 for the Re range examined. Nevertheless, that for d/D = 0.8 reach 1.0 at Re = 26300. For d/D = 1.0, f_v/f_{n1} displays a sudden drop at Re = 23250, marked by a dashed line. Note that f_v/f_{n1} for this d/D corresponds to a St of about 0.21 for $Re \leq 23250$ and 0.14 for $Re \geq 23250$, as will be shown later.

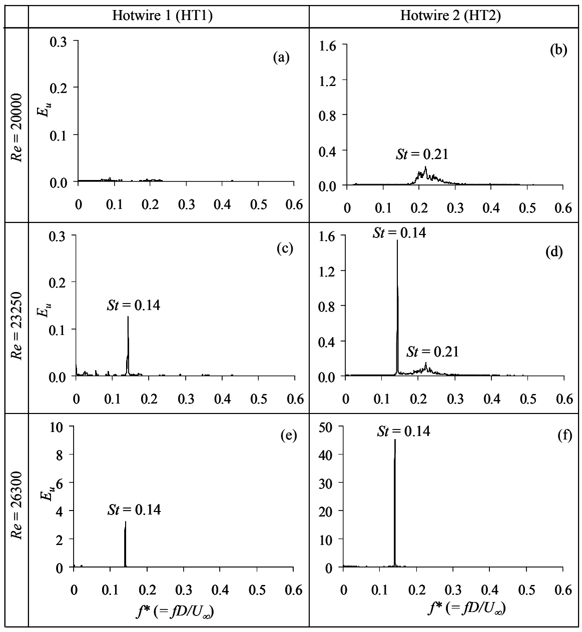

Fig. 4. Power spectral density function E_u of the streamwise fluctuating velocity u obtained from HT1 and HT2 at d/D = 1.0, L/d = 1.

Power spectrum results at Re = 20000, 23250 and 26300 (Fig. 4) explore that (i) at Re = 20000 power spectrum of HT1 signal does not display any peak, however that of HT2 signal displays a small peak at St = 0.21 corresponding to f_v/f_{n1} = 0.57, (ii) at Re = 23250 power spectrum of HT1 signal exhibits a peak at St = 0.14 and that of HT2 signal displays two peaks at St = 0.14 and 0.21, respectively, corresponding to f_v/f_{n1} = 0.47 and 0.67, and (iii) at Re = 26300 both power spectrums of HT1 and HT2 signals display quite strong peak at St = 0.14. A deep observation on these points directs that the two shear layers emanating from the upstream cylinder reattach steadily on the downstream cylinder for Re < 23250, and those reattach alternately for Re > 23250, as sketched in Fig. 5. At a given Re, Alam *et al.* [7, 19] (Re = 65000, 55000) observed that an increase in L/d from 0.6 to 3.5 results an alternating-reattachment flow changing to steady-reattachment flow at L/d = 2.5. Xu & Zhou [16] at L/d = 1.5 observed a drastic change of St from 0.218 to 0.18 when Re was increased from 10000 to 15000. However, flow-structures associated with the two St were not discussed. Their Re for the drastic change is slightly lower than the present one; this may be due to difference in L/d.

It is plausible that, for steady reattachment flow, HT1 would not confront any alternating vortex shedding, hence no peak in the power spectrum result. The power spectrum at Re = 23250 seems the result of a superimposition of power spectrum at Re = 20000 onto that at Re = 26300. Re = 23250 is the critical where both steady- and alternating-reattachment flows persist. However, at a given time both flow patterns could not co-exist, but may appear individually. From the power spectrum results (Fig. 4d), it is not clear whether the two frequencies (i.e., St) co-exist or emerge individually. In order to explore the conundrum, wavelet analysis of the HT2 signal is done and the result of the analysis is presented in Fig. 6. Morlet wavelet with wave number 6.0 was used as the mother wavelet in the analysis (see [20] for details).

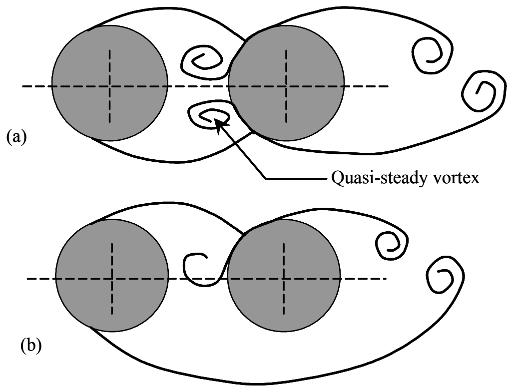

Fig. 5. Sketch of flow at d/D = 1.0, L/d = 1: (a) steady-reattachment flow, Re < 23250, (b) alternating-reattachment flow, Re > 23250.

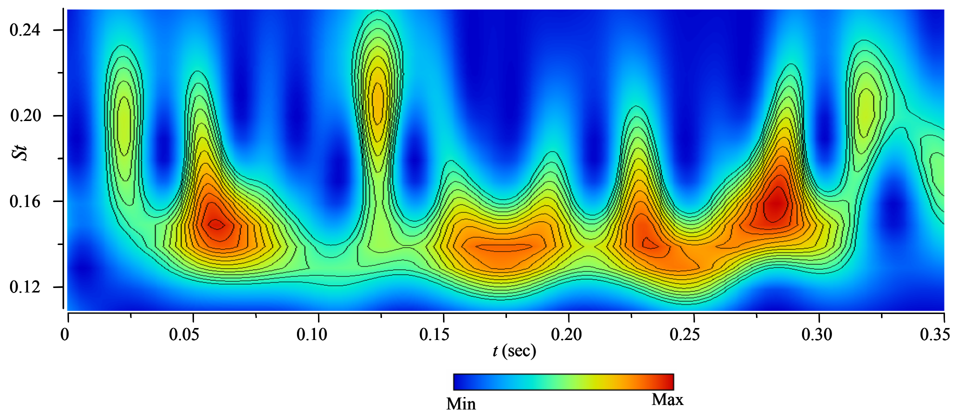

Fig. 6. Wavelet scalogram of HT2 signal at Re = 23250, d/D = 1.0, L/d = 1.

The figure 6 divulges unequivocally that the two frequencies do not co-exist but appear individually, i.e., the vortex shedding occurs at $St \approx 0.21$ at time t = 0.02~0.03, 0.11~0.14 and 0.31~0.33 sec and at $St \approx 0.14$ at t = 0.03~0.11 and 0.14~0.31 sec, implying that the steady- and alternating-reattachment flows switch from one to the other. Duration of vortex shedding and energy intensity at $St \approx 0.21$ are significantly small compared to those at $St \approx 0.14$, consistent with the peak at $St \approx 0.21$ being tiny (Fig. 4d). For other d/D, f_v/f_{n1} increases almost linearly with Re, having different slope depending on d/D. At L/d = 2 (Fig. 3b), f_v/f_{n1} for d/D = 0 ~ 0.6 is 0.5 at Re = 16460 and that for d/D = 0.8 and 1.0 at Re = 20000 and 26300, respectively. Resonance speed (f_v/f_{n1} = 1) is reached for d/D = 0~0.6 at Re = 33000, but not reached for d/D = 0.8 and 1.0. This information will be helpful in describing characteristics of force on the cylinder.

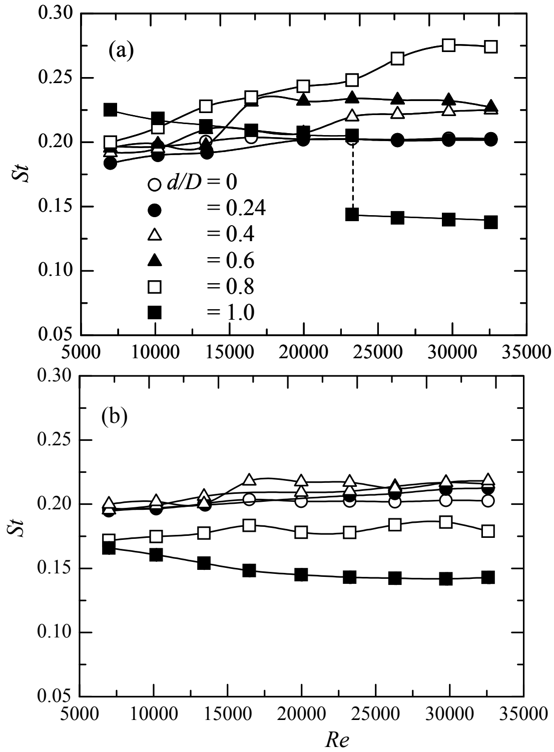

Fig. 7. Dependence on Re of St of the downstream cylinder at (a) L/d = 1, (b) L/d = 2.

f_v is in general normalized to St known extensively. Fig. 7 shows dependence of St on Re. At L/d = 1, St for d/D = 0 ~ 0.6 is slightly dependent on Re, displaying values between 0.19 ~ 0.225, while that for d/D = 0.8 and 1.0 changes significantly. St for d/D = 0 is obtained as ≈0.2. As discussed earlier, St for d/D =1.0 is about 0.21 and 0.14 for Re lower and greater than 23250, respectively, corresponding to the steady- and alternating-reattachment flows. The St distributions advocate that flow over the cylinders for d/D = 0.24~0.8 at the whole range of Re is similar to that for d/D = 1.0 at Re < 23250, i.e., a steady-reattachment flow takes place for d/D = 0.24~0.8. This is reasonable because, as d/D decreases from 1.0, the width between the two free shear layers in the gap between the cylinders reduces, hence steady-reattachment flow is prone to be generated.

One question may arise, for d/D =1.0, why does the steady-reattachment flow change to the alternating-reattachment flow at Re = 23250 when Re increases? Indeed, the width between the free shear layers in the gap is dependent on Re. As found in literatures, for a single cylinder the width at Re = 18000~10000 is higher than that at 1500~18000 [21, 22]. Thus we can enunciate that for d/D = 1.0 the change of the steady-reattachment flow to alternating reattachment is due to an increase in the width with increaseing Re. For d/D < 1.0, the width is smaller being directly proportional to d, hence only steady-reattachment flow occurs. St for d/D < 1.0 increases slightly with increasing Re due to change in reattachment position on the downstream cylinder. At L/d = 2, flow around the cylinders at d/D = 0.8 and 1.0 is alternating-reattachment and at others d/D is steady-reattachment, evidenced by power spectrum results (not shown).

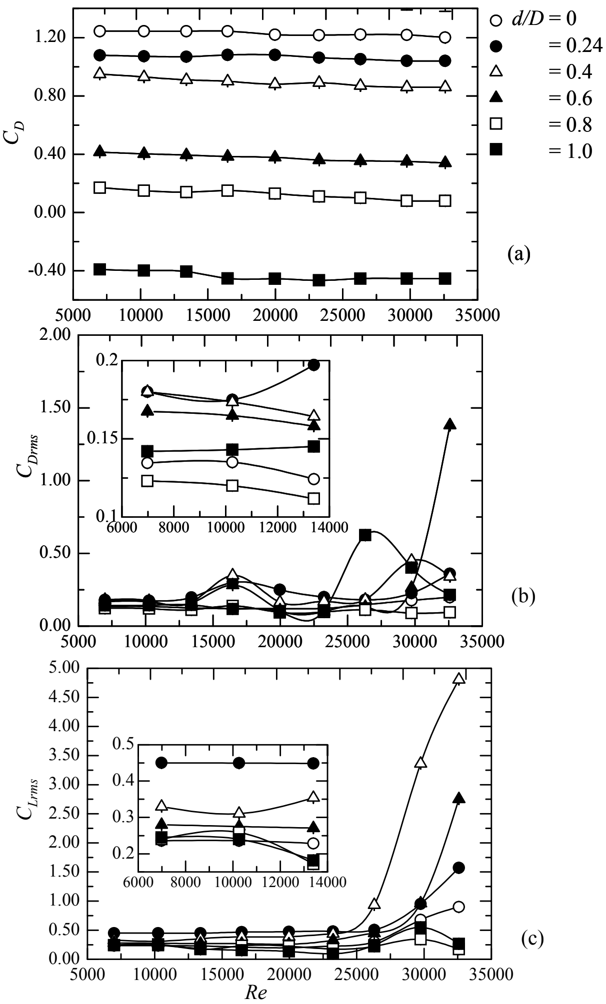

Fig. 8. Dependence on Re of forces on the downstream cylinder at L/d = 1: (a) C_D, (b) C_{Drms}, (c) C_{Lrms}.

3.3. Dependence of Forces on *Re*

Figures 8 and 9 show variation of C_D, C_{Drms} and C_{Lrms} with increase in Re at L/d = 1 and 2. First at L/d = 1, it is observed that C_D is slightly dependent on Re for higher d/D values. For d/D = 0, C_D is about 1.22 which is close to the well-known value 1.2. Increase in d/D from 0 to 0.8 causes a reduction in

C_D, by 12~15%, 24~30%, 67~72% and 87~94% for d/D = 0.24, 0.4, 0.6 and 0.8, respectively, the counterpart is C_D at d/D = 0. However, for d/D = 1, C_D is negative, about -0.40 for Re < 15000 and 0.45 for Re > 15000. At the same L/d and d/D, Biermann & Herrnstein [3], Hori [4], Zdravkovich & Pridden [5] and Alam *et al.* [7] observed C_D of -0.45 (Re = 6.5×10^4), -0.43 (Re = 8×10^3), -0.53 (Re = 3.1×10^4) and -0.42 (Re = 6.5×10^4), respectively, consistent with our result. On other hand, C_{Drms} and C_{Lrms} (Fig. 8b, c) change vigorously with increase in Re, especially for Re > 15000. C_{Drms} distribution for d/D = 0.24 ~ 0.6 forms a peak at Re = 16460 and that for d/D = 1.0 at Re = 26300. Furthermore, C_{Drms} for d/D = 0.24 ~ 0.6 becomes larger again for Re > 27500. The formation of these peaks or the larger values of C_{Drms} results from the fact that f_v/f_{n1} is either 0.5 or 1.0 at those Re (Fig. 3), as discussed earlier.

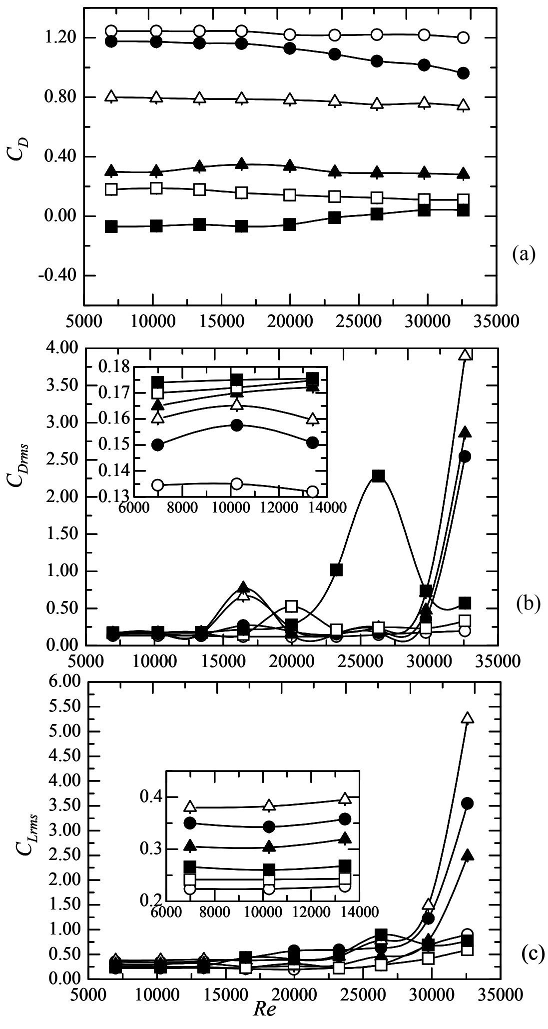

Fig. 9. Dependence on Re of forces on the downstream cylinder at L/d = 2: (a) C_D, (b) C_{Drms}, (c) C_{Lrms}. Symbols as in Fig. 8.

It is well known that vortex excited vibration of a single isolated cylinder in the cross-flow and stream-wise directions occurs at f_v/f_{n1} ≈ 1 and 2, respectively, because the frequencies of the oscillating lift and drag on a stationary cylinder are equal to and twice f_v [23]. But the present results show that for tandem cylinders with d/D = 0.24~0.6, C_{Drms} intensifies significantly when f_v/f_{n1} ≈ 0.5 and 1. Thus d/D = 0.24~0.6 may generate vortex excited streamwise vibration at f_v/f_{n1} ≈ 0.5 and 1, in addition to at f_v/f_{n1} ≈ 2.

For each d/D, C_{Lrms} is more or less constant for Re < 25000. Here there is no peak at Re = 16460 for d/D = 0.24~0.6 as is in C_{Drms} distributions, however a peak forms at Re = 30000 for d/D = 0.8 and 1.0 each, corresponding to f_v/f_{n1} ≈1.0 and 0.5, respectively, implying that C_{Lrms} is not sensitive to the value f_v/f_{n1} =0.5 at lower Re, but may be at higher Re. C_{Lrms} magnifies significantly for d/D = 0.24~0.6 at Re = 33000 due to resonance of vortex shedding and cylinder natural frequencies. However, the resonance effect for other d/D is relatively feeble. As f_v/f_{n1} for Re < 15000 is less than 0.5 for all d/D, it is expected that values of C_{Lrms} and C_{Drms} for Re < 15000 is free of influence of f_v/f_{n1}. Hence a small figure for Re < 15000 with reduced vertical scale is inserted to show obviously the effect of d/D on C_{Lrms} and C_{Drms}. From the inserted figures, it is observed that d/D = 0.24 ~ 0.6 yields a higher C_{Lrms} and C_{Drms} than other d/D.

At L/d = 2 (Fig. 9), C_D decreases by 6~20%, 36~38%, 73~77% and 86~91% for d/D = 0.24, 0.4, 0.6 and 0.8, respectively, the counterpart is that at d/D = 0. d/D = 1.0 results C_D between -0.07 and 0.04 depending on Re. Biermann & Herrnstein [3], Zdravkovich & Pridden [5] and Alam *et al.* [7] at this L/d and d/D observed a C_D of 0.003 (Re = 6.5×10^4), 0.05 (Re = 3.1×10^4) and 0.002 (Re = 6.5×10^4), respectively. C_{Drms} distribution displays peaks at three Re = 16460 (d/D = 0.24 ~ 0.6), 20000 (d/D = 0.8) and 26300 (d/D = 1.0). These peaks are due to coincidence of f_v at the half of f_{n1}. Both C_{Lrms} and C_{Drms} for d/D = 0.24 ~ 0.6 at Re = 33000 is amplified by about 20~30 and 12~24 times, respectively, compared to those for d/D = 0. They for d/D = 0 are more or less constant at about 0.12 and 0.22, respectively except for Re > 26300.

3.4. Dependence of Forces on *d/D*

Figure 10 presents C_D, C_{Drms} and C_{Lrms} for L/d = 1, 2 and 5.5 at a Re = 10200 where the effect of f_v/f_{n1} on forces is negligible for all d/D values. In the previous sections, results for L/d = 5.5 are not presented because the upstream as well as the downstream cylinder at this L/d sheds vortices like a single isolated cylinder (i.e., the co-shedding regime), hence forces were insensitive to Re for the range of Re examined. As d/D grows from 0 to 1.0, C_D reduces progressively from 1.22 to -0.4, -0.07 and 0.34 at L/d = 1, 2 and 5.5, respectively, indicating a greater decreasing C_D with d/D occurring at L/d = 1 and 2. The cause of the greater decreasing C_D at L/d = 1 and 2 is that the free shear layers of the upstream cylinder cover a larger frontal-area of the downstream cylinder for larger d/D, thus lower pressure prevails in a larger area of the front surface, resulting in a lower C_D. At L/d = 5.5, the decrease of C_D with increasing d/D is relatively small and the mechanism of the decreasing C_D at this L/d is apparently different from that at L/d = 1 and 2. Here the increasing d/D is linked to increasing wake width behind the upstream cylinder and hence to decreasing approaching flow velocity (i.e., local initial flow velocity) to the downstream cylinder, resulting in lowering C_D.

It is likely that L/d = 5.5 is associated with a greater reduction in C_D for d/D < 0.4 and L/d = 1 or 2 for d/D > 0.4, suggesting that the upstream cylinder with d/D < 0.4 has a greater role in suppressing C_D in the co-shedding regime and that with d/D > 0.4 in the reattachment regime. C_{Drms} for L/d = 1 and 2 is almost independent on d/D, because increasing d/D though increases the size of vortex between the cylinders, the vortex being quasi-steady (Fig. 5a) does not increase C_{Drms}. On the other hand, at L/d = 5.5 C_{Drms} rises progressively with d/D. In fact, larger d/D is associated with larger-size and higher-strength vortices shed alternately from the upstream cylinder; the vortices alternately strike the downstream cylinder during their convections causing increasing C_{Drms} [7, 24-28]. C_{Lrms} (Fig. 10c) also increases for the same reason. On the contrary, for L/d = 1 and 2, C_{Lrms} is higher at smaller d/D.

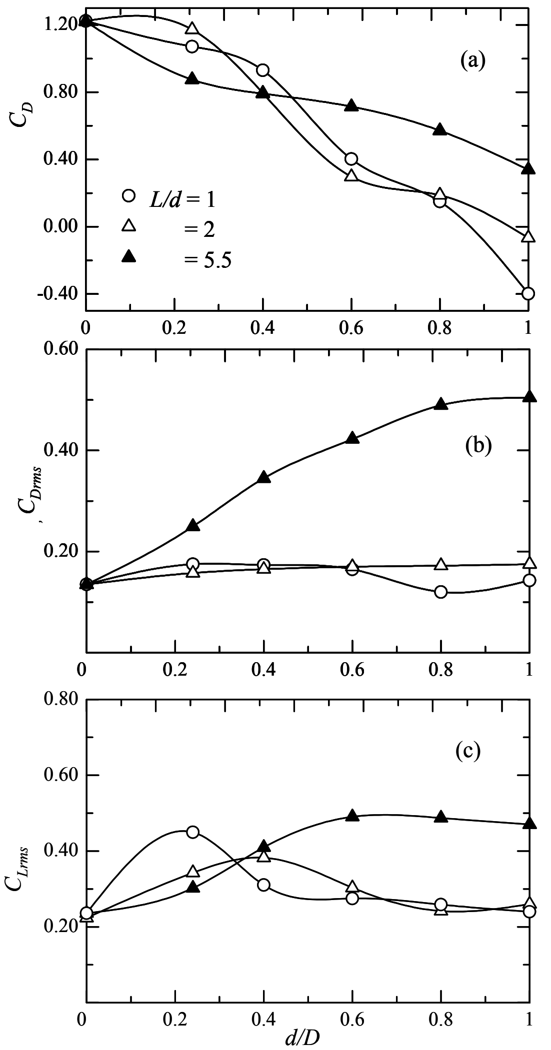

Fig. 10. Dependence on d/D of forces on the downstream cylinder at Re = 10200: (a) C_D, (b) C_{Drms}, (c) C_{Lrms}.

3.5. Dependence of *St* on *d/D*

Figure 11 illustrates the dependence of St on d/D. At L/d = 1 and 2, St measured by HT1 and HT2 was the same, hence one symbol is used for both. On the other hand, at L/d = 5.5, the two cylinders shed vortices at two different frequencies, hence different symbols are used for the two cylinders. St at L/d = 1 and 2 is almost the same and shows almost no dependence on d/D, except somewhat smaller St at d/D = 0.8 and 1.0 for L/d = 2 where an alternating reattachment flow transpires, as discussed before. This implies that d/D in the reattachment regime generating steady-reattachment flow may has a very little or no influence on St. However, that generating alternating-reattachment flow yields a lower St. At L/d = 5.5, the upstream cylinder St is smaller for a smaller d/D. It is likely that the downstream cylinder acts to block the flow. This effect is more significant at small d/D, which may prolong the period of vortex shedding from the upstream cylinder, thus decreasing St. The downstream cylinder has two St, one is associated with local initial flow condition around the cylinder and the other one appearing for $d/D \geq 0.4$ was equal to the St of the upstream cylinder. Hence the later one was not presented in the figure. The presented downstream cylinder St associated with local initial flow decreases with increasing d/D. At d/D = 1.0, St is rather small, 0.12, because the approaching flow velocity at the downstream cylinder is significantly lower than that at the upstream cylinder or a single isolated cylinder. With increasing d/D, the upstream cylinder wake becomes wider and the flow velocity approaching the upstream side of the downstream cylinder decreases, resulting in a decrease in St.

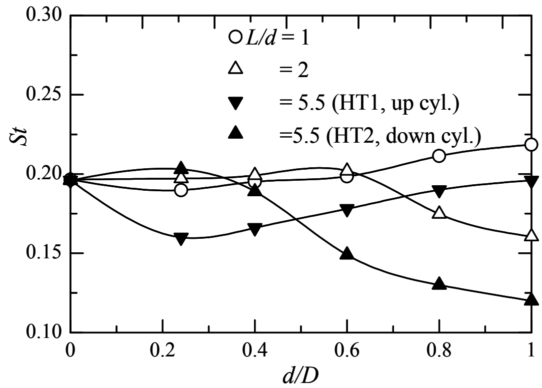

Fig. 11. Dependence of St on d/D at Re = 10200.

4. Conclusions

f_v/f_{n1}, St, C_D, C_{Drms}, C_{Lrms} and the wake of a fixed circular cylinder and their dependence on Re and d/D in the presence of an upstream cylinder are investigated. Both reattachment (L/d = 1 and 2) and co-shedding (L/d = 5.5) regimes are examined. The diameter ratio d/D is varied from 0 to 1.0 and Re from 7×10^3 to 3.3×10^4 based on the downstream cylinder. The preliminary investigation leads to following conclusions.

(i) f_v/f_{n1} varies almost linearly with Re giving almost a constant St at each d/D for the Re range examined, except at d/D = 1.0, L/d = 1 where St of 0.21 and 0.14 are observed for $Re \leq 23250$ and ≥ 23250, respectively. A steady-reattachment flow yields the St of 0.21 and an alternating-reattachment flow the St of 0.14. The Re = 23250 is critical where the steady- and alternating-reattachment flows switch from one to the other. Decreasing d/D results a decrease in width between the two free shear layers in the gap, hence only steady-reattachment flow is generated for lower d/D (i.e., for d/D < 1.0 at L/d = 1 and d/D < 0.8 at L/d = 2).

(ii) d/D in the reattachment regime generating steady-reattachment flow may has a very little or no influence on St. However, that generating alternating-reattachment flow yields a lower St. At L/d = 5.5, increasing d/D makes the upstream-cylinder wake wider and hence decreases the flow velocity approaching the upstream side of the downstream cylinder, causing a decrease in *St* from 0.203 to 0.12 with increase in *d/D* from 0.24 to 1.0.

(iii) Increase in *d/D* from 0 to 0.8 causes a reduction in C_D, by 12~15%, 24~30%, 67~72% and 87~94% at *L/d* = 1 and 6~20%, 36~38%, 73~77% and 86~91% at *L/d* = 2 for *d/D* = 0.24, 0.4, 0.6 and 0.8, respectively, the counterpart is that at *d/D* = 0. It is well known that vortex excited lateral and streamwise vibrations of a single isolated cylinder occur at $f_v/f_{n1} \approx 1$ and 2, respectively. For *d/D* = 0.24~0.6 in the reattachment regime, $f_v/f_{n1} \approx 0.5$ and 1 intensifies C_{Drms} significantly, by about 12~24 times at $f_v/f_{n1} \approx 1$ at *L/d* = 2 for example. Thus *d/D* = 0.24~0.6 may generate vortex excited streamwise vibration at $f_v/f_{n1} \approx 0.5$ and 1 in addition to at $f_v/f_{n1} \approx 2$. On the other hand, C_{Lrms} intensifies at $f_v/f_{n1} \approx 1$ only.

(iv) C_D, C_{Drms} and C_{Lrms} are highly dependent on *d/D*. As *d/D* grows from 0.24 to 1.0, C_D in the reattachment regime (i.e., at *L/d* = 1 and 2) decays faster with increasing *d/D* than in the co-shedding regime (i.e., *L/d* = 5.5). However, the reverse is true for C_{Drms} and C_{Lrms}. In co-shedding regime, they both grow with *d/D*. On the other hand, in the reattachment regime C_{Drms} is more or less constant and C_{Lrms} becomes higher at smaller *d/D*.

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