

Evaluation of Two-phase Turbulence Closure models modifications in Near Wall Region of Boundary Layer

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Abstract

Previous study of authors showed that $k-\tau$ turbulence model has a good accuracy for prediction of hydrodynamic behaviour of near wall turbulence in a gas-solid boundary layer flow. In that study two-phase $k-\tau$ model was derived from Yokomine et al. modified $k-\varepsilon$ model. In the present study, a new version of two phase $k-\tau$ turbulence model has been presented. In this model we use Adeniji et.al. modifications to account for the effect of solid particles both on the turbulence kinetic energy and its dissipation rate. The new modified $k-\tau$ has been compared with the later version of $k-\tau$ for prediction of flow behaviour in near wall region of two-phase boundary layer. The governing equations for two-phase boundary layer flow with Eulerian-Eulerian approach by two different modified $k-\tau$ turbulence models have been solved numerically using finite volume method. In comparison with available experimental data, the obtained results by modified $k-\tau$, based on Yokomine et al. modifications, have better agreement than modified $k-\tau$ based on Adeniji et al. model.

Introduction Many problems associated with industrial and environmental pollution require studying hydrodynamic characteristics of gas-solid flows. Gas-fine solid flows are intended to be a particular class of two-phase flows in which small particles are suspended in a gas. Although most gas-solid systems encountered in practice are turbulent, the case of laminar and turbulent boundary layer motion of gas-solid on a flat plate was discussed by Soo [8]. In many instances, the loading of particles is such that the particles carry a significant fraction of the mass and momentum in the flow yet occupy a negligible volume fraction. In such cases, particle collisions are rare and the flow field can still be represented by the Navier-Stokes equations with some modifications to account for the drag of the particles.

A boundary layer order of magnitude analysis for a particulate suspension for laminar flow was carried out by Chamkha and Peddison [3]. They found out that a variety of outcomes are possible depending on the order of magnitude assumptions selected. They sought the effect of particle diffusion for a plane steady flow past a flat surface. They indicated that changes in fluid-particles suspension models could lead to significant qualitative change in predictions.

For the case of turbulence, many researchers have performed experimental and computational studies. To name a few; Squires and Eaton [10], Yokomine and Shimizu [12], Adeniji-Fashola and Chen [1], Taniere, Oesterle and Foucaut [11] and recently similar simulation has preformed by using $k-\tau$ turbulence closure model by Gharraei, Esmaeilzadeh and Basirat [6], Esmaeilzaedeh and Gharraei [5]. Previous study of the authors showed that $k-\tau$ turbulence model has a good accuracy for prediction of hydrodynamic behaviour of near wall turbulence in

a gas-solid boundary layer flow. In that study two-phase $k-\tau$ model was derived from Yokomine et al. modified $k-\varepsilon$ model. In the present study, a new version of two phase $k-\tau$ turbulence model has been presented. In this model we use Adeniji et.al. [1] modifications for k and ε , that based on Chen and Wood [4] well known gradient type diffusion model. The new modified $k-\tau$ has been compared with the latter version of modified $k-\tau$ for prediction of flow behaviour in near wall region of two-phase boundary layer. The governing equations for two-phase flow with Eulerian-Eulerian approach by two different modified $k-\tau$ turbulence models have been solved numerically using finite volume method. In comparison with available experimental data, the obtained results by modified $k-\tau$ based on Yokomine et al. modifications have better agreement than modified $k-\tau$ based on Adeniji et al. model.

Analysis and turbulence models

The transport equations for the two-phase gas particle flows can be derived assuming that the carrier gas and the dispersed particles are two separate interpenetrating continua. Application of Reynolds decomposition and time-averaging can yield the mean flow transport equations for the two phases. To further simplify the problem, the following assumptions are made:

- (i) The dispersed particle phase is very dilute so that the volume concentration of the gas-phase can be regarded as approximately unity;
- (ii) The particle material density ρ_{mp} is far larger than the carrier gas, but its bulk density (the product of its material density and its volume concentration) $\bar{\rho}_p$ is relatively very small;
- (iii) Particles are monosized spherical ones and the two-phase flow is steady.

Based on these assumptions, the resulting mean flow equations for the continuity and momentum of each phase can be written tensorially for isothermal turbulent two-phase flows as follow:

Continuity and momentum

$$\frac{\partial \rho U_{fi}}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \bar{\rho}_p U_{pi}}{\partial x_i} + \frac{\partial \overline{\rho'_p u'_{pi}}}{\partial x_i} = \frac{\partial}{\partial x_j} \left[\rho_p D_p \left(\frac{\partial U_{pi}}{\partial x_j} \right) \right] \quad (2)$$

$$\frac{\partial \rho U_{fi} U_{fj}}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(-\overline{\rho u'_{fi} u'_{fj}} + \mu \frac{\partial U_{fi}}{\partial x_j} \right) + \overline{F_{pi}} \quad (3)$$

$$\frac{\partial \overline{\rho_p} U_{pi} U_{pj}}{\partial x_j} = \frac{\partial}{\partial x_j} \left(-U_{pi} \overline{\rho_p' u_{pj}'} - \overline{\rho_p} \overline{u_{pi}' u_{pj}'} \right) + U_{pj} \overline{\rho_p' u_{pi}'} + \frac{\partial}{\partial x_j} \left(\rho_p D_p \frac{\partial U_{pi}}{\partial x_j} \right) - \overline{F_{pi}} \quad (4)$$

D_p is the Brownian diffusivity of particles and equal to

$$D_p = \frac{KT}{3\mu d_p} \quad (5)$$

where K is the Boltzman constant .

The drag force between the two phases, F_{pi} in Eqs. (3), (4) is given by

$$F_{pi} = \frac{\alpha_p \rho_p}{\tau_p^*} (u_{pi} - u_{fi}) f_p \quad (6)$$

f_p is a correction factor, and is given by Boothroyd [2]:

$$f_p = \begin{cases} 1 + 0.15 \text{Re}_p^{0.687}, & 0 < \text{Re}_p \leq 200 \\ 0.914 \text{Re}_p^{0.282} + 0.0135 \text{Re}_p, & 200 < \text{Re}_p \leq 2500 \\ 0.0167 \text{Re}_p, & \text{Re}_p > 2500 \end{cases} \quad (7)$$

where the particle Reynolds number is defined as

$$\text{Re}_p = \frac{d_p |u_{pi} - u_{fi}|}{\nu} \quad (8)$$

τ_p^* is the characteristic response time scale of the particle to the change in the fluid motion, defined by

$$\tau_p^* = \frac{\rho_p d_p^2}{18\mu} \quad (9)$$

The time averaging of F_{pi} yields:

$$\overline{F_{pi}} = \frac{\overline{\rho_p} \overline{f_p}}{\tau_p^*} (U_{pi} - U_{fi}) + \frac{\overline{f_p}}{\tau_p^*} \overline{\rho_p' (u_{pi}' - u_{fi}')} \quad (10)$$

Two-phase Turbulence models

As mentioned before, in this study we use the $k-\tau$ turbulence model for prediction of near wall turbulence behavior of gas-particle boundary layer . Speziale et al. [9] introduce the formal change of dependent variables $\varepsilon = k/\tau$ and transform the standard $k-\varepsilon$ model to $k-\tau$ one. We use the similar transformation to transform Adeniji et al. [1] refined $k-\varepsilon$ model to a two-phase $k-\tau$ turbulence model.

Adeniji et al. [1] two-phase $k-\varepsilon$ model equations are given as:

$$U_{fi} \frac{\partial k}{\partial x_i} = \tau_{ij} \frac{\partial U_{fi}}{\partial x_i} - \varepsilon + \frac{\partial}{\partial x_i} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right] + S_k^P \quad (11)$$

$$U_{fi} \frac{\partial \varepsilon}{\partial x_i} = C_{\varepsilon 1} \frac{\varepsilon}{k} \tau_{ij} \frac{\partial U_{fi}}{\partial x_i} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_i} \left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_i} \right] + S_\varepsilon^P \quad (12)$$

where

$$S_k^P = \overline{F_{pi} u_{fi}'} = -\frac{\overline{\rho_p' u_{fi}'}}{\tau_p^*} \overline{f_p} (U_{fi} - U_{pi}) - \frac{2k}{\tau_p^*} \rho_p \left[1 - \text{Exp} \left(-\frac{\tau_p^*}{2\tau} \right) \right] \quad (13)$$

$$S_\tau^P = \frac{2}{\tau_p^*} \overline{\rho_p} + \frac{S_k^P}{\tau} \quad (14)$$

By definition and transformation of above equations and imposing the speziale et. al. [9] corrections in $k-\tau$ model equations we have:

$$U_{fi} \frac{\partial k}{\partial x_i} = \tau_{ij} \frac{\partial U_{fi}}{\partial x_i} - \frac{k}{\tau} + \frac{\partial}{\partial x_i} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right] + S_k^P \quad (15)$$

$$U_{fi} \frac{\partial \tau}{\partial x_i} = (C_{\varepsilon 1} - 1) \frac{\tau}{k} \tau_{ij} \frac{\partial U_{fi}}{\partial x_i} + (C_{\varepsilon 2} f_2 - 1) + \frac{\partial}{\partial x_i} \left[\left(\nu + \frac{\nu_t}{\sigma_{\tau 2}} \right) \frac{\partial \tau}{\partial x_i} \right] - \frac{2}{\tau} \left(\nu + \frac{\nu_t}{\sigma_{\tau 2}} \right) \frac{\partial \tau}{\partial x_i} \frac{\partial \tau}{\partial x_i} + \frac{2}{k} \left(\nu + \frac{\nu_t}{\sigma_{\tau 1}} \right) \frac{\partial k}{\partial x_i} \frac{\partial \tau}{\partial x_i} + S_\tau^P \quad (16)$$

Model coefficients are given as [9]:

$$\sigma_k = \sigma_{\tau 1} = \sigma_{\tau 2} = 1.36$$

$$C_{\varepsilon 1} = 1.44$$

$$C_{\varepsilon 2} = 1.83 [1 - (2/9) \exp(-\text{Re}_t^2/36)] \quad (T)$$

Also, ν_t is the turbulent or eddy viscosity. The damping functions f_μ and f_2 are introduced by Schwab and Lakshminarayana [7]. Hence

$$\nu_t = f_\mu C_\mu k \tau \quad (18)$$

$$f_\mu = (1 + 3.45/\sqrt{\text{Re}_t}) \text{Tanh}(y^+/70) \quad (19)$$

$$\text{Re}_t = \frac{k^2}{\nu \varepsilon} = \frac{k}{\nu \tau} \quad \text{with } \tau = \frac{k}{\varepsilon} \quad (20)$$

$$f_2 = [1 - \exp(-y^+/A_2)]^2 \quad (21)$$

where

$$C_\mu = 0.09, A_2 = 4.9$$

The gradient hypothesis [4] is used to model the density fluctuations of $\overline{\rho_p' u_{pi}'}$ and $\overline{\rho_p' u_{fi}'}$, i.e.,

$$-\overline{\rho_p' u_{pi}'} = \frac{\nu_T}{Sc_f} \frac{\partial \overline{\rho_p}}{\partial x_i} \quad (22)$$

$$-\overline{\rho_p' u_{fi}'} = \frac{\nu_p}{Sc_p} \frac{\partial \overline{\rho_p}}{\partial x_i} \quad (23)$$

The Reynolds stress in the gas momentum equation is modelled using $k-\varepsilon$ eddy viscosity-diffusivity model. Hence,

$$-\overline{u_{fi}' u_{fi}'} = \nu_t \left[\frac{\partial u_{fi}}{\partial x_j} + \frac{\partial u_{fj}}{\partial x_i} \right] - \frac{2}{3} k \delta_{ij} \quad (24)$$

Similarly, the Reynolds stress for the particle phase is modelled by

$$-\overline{u_{pi}' u_{pj}'} = \nu_p \frac{1}{2} \left(\frac{\partial u_{pi}}{\partial x_j} + \frac{\partial u_{pj}}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \left(-\frac{1}{2} \overline{u_{pl}' u_{pl}'} + \nu_p \frac{\partial U_{pi}}{\partial x_i} \right) \quad (25)$$

where, the particle viscosity term can be defined [4]:

$$v_p = \frac{v_t}{1 + \tau_p / t_e} \quad (26)$$

$$t_e = C_t \tau \quad (27)$$

$$C_t = 0.125$$

In the previous modified $k-\tau$ model, presented by the authors, Yokomine et al. [12] modifications were applied for turbulence kinetic energy and its dissipation rate and the particle source terms were defined as [5]:

$$S_k^p = S_{k1}^p + S_{k2}^p \quad (28)$$

$$S_{k1}^p = -\frac{2k\rho_p}{\rho\tau_p^*} \left[1 - \text{Exp}\left(-\frac{\tau_p^*}{2\tau}\right) \right] \text{Exp}\left(\frac{-C_{p2}d_p}{2C_\mu k^{1/2}\tau}\right) \quad (29)$$

$$S_{k2}^p = \frac{3\mu\rho_p \text{Re}_p}{4\rho\rho_{mp}d_p^2} (C_D - C_{D,St}) |U_{fi}(U_{fi} - U_{pi})| \quad (30)$$

$$S_\tau^p = C_{\varepsilon 1} \frac{\tau}{k} S_{k2}^p \quad (31)$$

where

$$C_{p2} = 0.1$$

Boundary Conditions

The following boundary conditions are used:

$$y = 0, x > 0; U_f = V_f = V_p = 0, k = 0, \tau = 0 \quad (32)$$

$$y \rightarrow \infty; U_f = U_p = \rho_p = 1, \frac{\partial k}{\partial y} = 0, \frac{\partial \tau}{\partial y} = 0 \quad (33)$$

and at $y=0$, the Eq. (2) and crosswise component of Eq. (3) give

density and velocity of the particulate phase:

$$\frac{\partial(\rho_p U_p)}{\partial x} = D_p \left(\frac{\partial^2 \rho_p}{\partial y^2} \right) \quad (34)$$

$$U_p \frac{\partial U_p}{\partial x} = \frac{D_p}{\rho_p} \left[\frac{\partial}{\partial y} \left(\rho_p \frac{\partial U_p}{\partial y} \right) \right] - \frac{U_p}{\tau_p^*} \quad (35)$$

Equations have been simplified by applying boundary layer approximations and have been solved numerically by finite volume method (Hybrid scheme). The equations due to discretization have been solved by TDMA method. Iterations have been continued until the maximum residual of all variables has been less than 10^{-4} .

Results and Discussion

The accuracy of any numerical method will be determined by comparison with experimental results. Adeniji et al. [1] modifications are based on Chen and Wood [4] model that presence of the dispersed phase is presented as extra dissipation of both k and ε . On the other hand in the Yokomine et al. [12] model, particle source terms have been added to standard $k-\varepsilon$ model that takes both of the turbulence energy enhancement due to wakes generated behind particles and the turbulence attenuation by sympathetic vibration into account.

Figures 1 and 2 show the numerical results of $k-\tau$ turbulence model, with two different modifications, compared with experimental results of Taniere et al. [6]. The turbulence intensity

is assigned a value of 1% at inlet boundary. Taniere et al. [6] have presented two cases; glass beads with diameter of $60 \mu\text{m}$ and density of 2500 kg/m^3 and PVC beads with diameter of $130 \mu\text{m}$ and density of 1430 kg/m^3 .

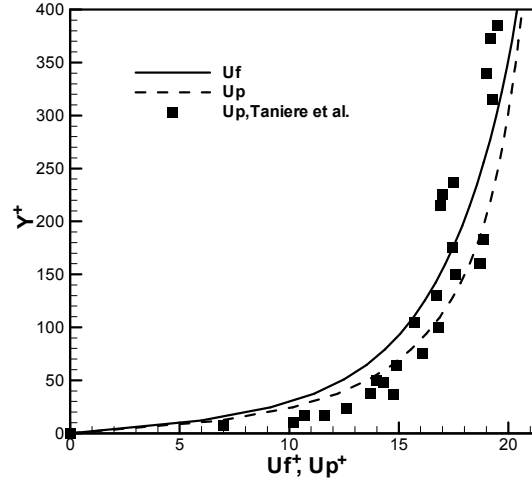


Figure1: Fluid and particle phase velocity, with Yokomine et al. modifications, ($d_p=60 \mu\text{m}$)

The experimental results have been presented at $x=5.06\text{m}$ and 5.15m , where x is the distance from the leading edge of the flat plate. The free stream flow velocity was 10.6m/s and particles were injected at $x=3.17\text{m}$ and loading factor was about $\beta = 0.1$.

These figures show that $k-\tau$ model, based on Yokomine et al. modified $k-\varepsilon$ [12], has very good agreement with experiments in comparison to $k-\tau$ model derived from Adeniji et al. modified $k-\varepsilon$ [1]. However, converging the numerical solution of $k-\tau$ model based on Adeniji et al. modified $k-\varepsilon$ [1] is very easier than $k-\tau$ with Yokomine et al. [12] modifications.

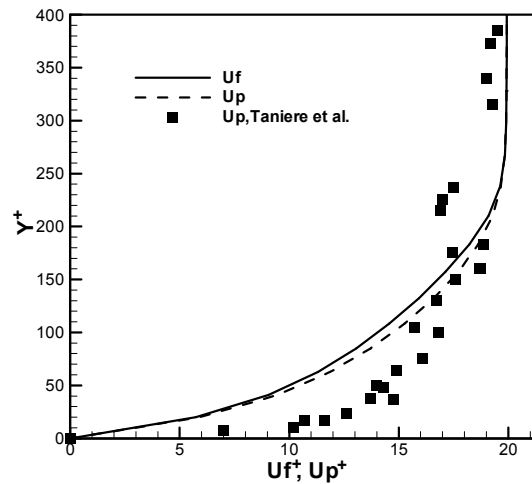


Figure2: Fluid and particle phase velocity, with Adeniji et al. modifications, ($d_p=60 \mu\text{m}$)

Figures 3 and 4 show a similar comparison for case 2 of Taniere et al. [11] experiments, $d_p=130 \mu\text{m}$. These comparisons lead to similar results as figures 1 and 2, but the accuracy lessens for $130 \mu\text{m}$ case for both modifications. The $60 \mu\text{m}$ particles greatly disperse toward the low speed side ($Y>0$) and prediction is much easier with the two-fluid model. In contrast, the $130 \mu\text{m}$ particles

tend to move straight, being less affected by the gas phase and hard to capture with the two-fluid model.

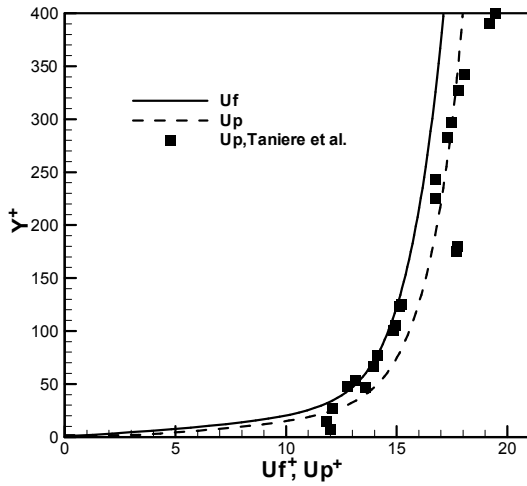


Figure3: Fluid and particle phase velocity, with Yokomine et al. modifications, ($d_p=130 \mu m$)

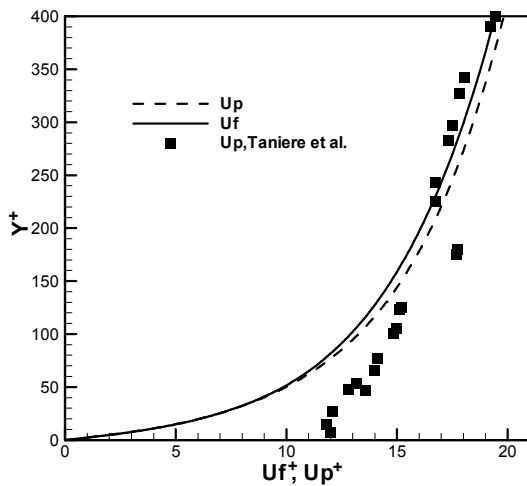


Figure4: Fluid and particle phase velocity, with Adeniji et al. modifications, ($d_p=130 \mu m$)

Figure 5 compares streamwise velocity fluctuations profile of current study results and experimental results of Taniere et al. (case 1). As can be seen, the $k-\tau$ model with Yokomine et al. modifications leads to more similar profile to experimental results in comparison with Adeniji et al. modifications.

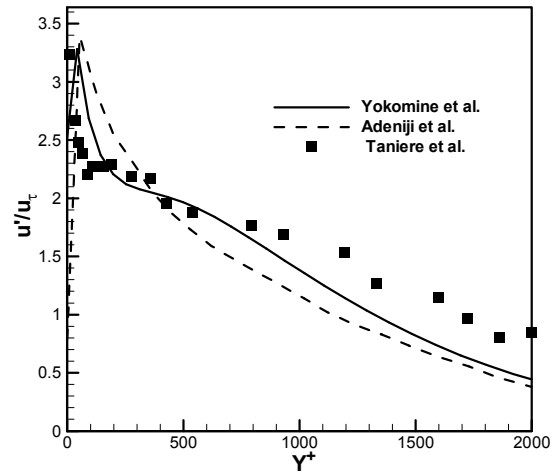


Figure5: fluid phase streamwise velocity fluctuations, ($d_p=60 \mu m$)

The crosswise velocity fluctuations is presented in figure 6. It is obvious that $k-\tau$ modified by Yokomine et al. model, gives better results in comparison with $k-\tau$ modified by Adeniji et al. model.

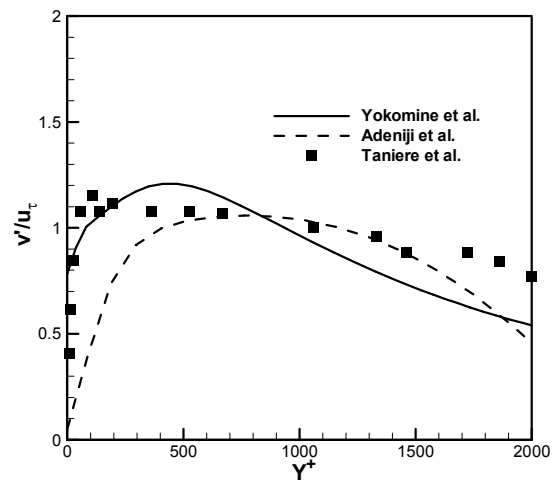


Figure6: fluid phase crosswise velocity fluctuations, ($d_p=60 \mu m$)

Conclusions

The governing equations of two-phase flow based on two-way coupling Eulerian-Eulerian approach are developed for the particulate suspension flow. The $k-\tau$ model with two different modifications for particles effect on turbulence kinetic energy and dissipation has been used as turbulence closure model. Comparisons of numerical and available experimental results show that $k-\tau$ turbulence model based on Yokomine et al. modifications has a better ability to predict the near wall turbulence behaviour in comparison with $k-\tau$ model based on Adeniji et al. modifications.

However, required time to converge the solution is very lower in $k-\tau$ model based on Adeniji et al. modifications, so it can be more economical to use.

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