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Autocorrelation Functions and the Determination of Integral Length with Reference to Experimental and Numerical Data

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Abstract

Results from a particle image velocimetry (PIV) investigation of grid turbulence were recently compared with results from a direct numerical simulation (DNS) of homogeneous isotropic turbulence [5]. The comparison highlighted the difficulty of specifying an adequate spatial domain for numerical and experimental studies so that an accurate determination of integral length is obtained. Integral length is determined from the autocorrelation function, but the autocorrelation function is incorrect if the spatial domain is not sufficiently large. In general, the spatial domain in an experimental or numerical investigation is limited by restrictions on the experimental equipment or computational resources. Using the available spatial domain, an autocorrelation function is obtained from which the integral length is determined. The accuracy of the integral length is usually evaluated by comparing it to the spatial domain. If the spatial domain is sufficiently larger than the integral length, the integral length is judged to be accurate. However, how many times larger than the integral length does the spatial domain need to be?

Introduction

Defining a length or time period that is characteristic of the largest scales in a turbulent flow is of importance both in defining a suitable area or volume for experimental and numerical investigations, and also to understanding the process of energy production and dissipation in the flow. In some cases a suitable scale can be defined by the physical constraints of the flow domain. For example, in pipe flow the diameter of the pipe is of the order of the largest eddies in the flow, and the ratio of the pipe diameter to mean velocity along the pipe is a good estimate of the time period required to describe the flow. In other cases where the largest scale is not obvious from the flow geometry, an integral scale can be defined that is a measure of the longest connection or correlation distance between two points in the flow that are separated either by distance or time [3]. In this paper, we determine the integral length scale of the velocity defined by:

$$\Lambda = \int_0^\infty R_{ii}(r,t)dr \tag{1}$$

where the double-i subscript in $R_{ii}(r,t)$ indicates the *auto*correlation function (i.e. correlation of a velocity component with itself) defined by:

$$R_{ii}(r) = \frac{\langle u_i(x_i,t)u_i(x_i+r,t)\rangle}{\langle u_i^2\rangle} \tag{2}$$

and r is the distance between two points in the flow. The autocorrelation function is longitudinal if r is parallel to u_i, and transverse if r is perpendicular to u_i, where u_i is the root-mean-square velocity in the i-direction.

Results

Comparison of Experimental and Numerical Data

The experimental data cited in this paper is from a PIV investigation of grid turbulence. Details of the experiment can be found in [5]. The experimental results are compared to numerical results from DNS of homogeneous isotropic turbulence. Details of the DNS can be found in [6].

In the case of the PIV data, the size of the spatial domain depended on the image acquisition equipment that was available. There was a balance between obtaining a sufficiently large experimental domain and accurately resolving the particles in the acquired images. The spatial domain was of size 3λ where λ is the Taylor microscale determined from:

$$\lambda^2 = \frac{u_1^2}{\langle(\frac{du_1}{dx_1})^2\rangle} \tag{3}$$

where u_1 is the root-mean-square of the stream-wise component of velocity, and $\frac{du_1}{dx_1}$ indicates the streamwise gradient of the streamwise velocity. For the DNS data the Taylor microscale is calculated from:

$$\lambda = \sqrt{\frac{15\nu u^2}{\varepsilon}} \tag{4}$$

The kinetic energy dissipation (ε) is determined spectrally and ν is the kinematic viscosity. The side-length of the three-dimensional simulation volume was greater than 7λ, although the velocity field could be restricted to any spatial domain, including 3λ.

Figure 1 compares the atocorrelation function obtained from the PIV data to that obtained from the DNS data using the entire available domain, and also restricting the domain to different values, including 3λ. Both the longitudinal and transverse autocorrelation functions are shown. For the DNS data it would be expected that, once the spatial domain is large enough, there would be little change in the autocorrelation function when the spatial domain is increased. In the longitudinal case there is a significant difference between the autocorrelation function for a spatial domain of 4.5λ and that for 8λ, so it appears that a spatial domain of 4.5λ is not sufficient to accurately determine the longitudinal integral length in this case. In the transverse case the autocorrelation functions found for a spatial domain of 4.5λ and 8λ are quite similar, but that for 3λ is significantly different, so a spatial domain of 3λ appears to be insufficient for an accurate determination of transverse integral length in this case.

It was noted in [5] that the autocorrelation functions obtained from the PIV data were different from previous experimental and numerical results. It appears from Figure 1 that the limited spatial domain of the PIV investigation may account for much of the deviation from past results. Also shown in 1 is the theoretical autocorrelation functions obtained by Townsend [8] for isotropic turbulence with uniform size structures and turbulence with a wide range of structure sizes.

Velocity Integral Length

The determination of the integral scale from equation (1) is not straight-forward [1]. The form of the autocorrelation function is such that it generally decreases rapidly to its first zero crossing, after which it may become negative and proceed to oscillate

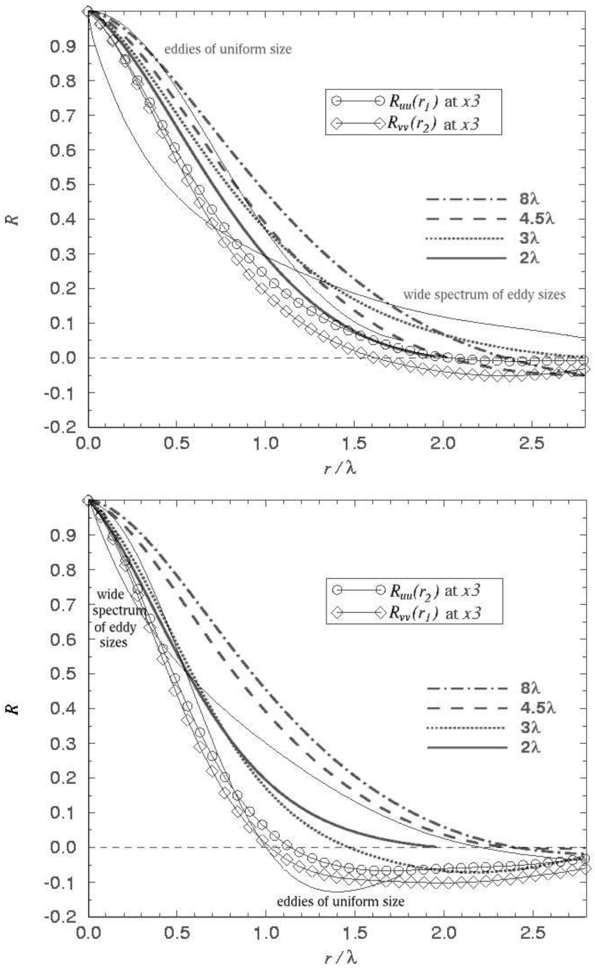

Figure 1: Longitudinal and transverse velocity autocorrelation functions obtained from DNS data by progressively restricting the spatial domain ($Re_\lambda = 26$) compared to experimental data obtained using a spatial domain of 3λ ($Re_\lambda = 23$). Top: longitudinal autocorrelation; bottom: transverse autocorrelation.

about zero. While equation (1) involves the determination of the integral over an infinite domain, the domain of the autocorrelation function from experimental or numerical data is finite, and there is some uncertainty on how best to define the integration domain. For example, Tritton [9] described how, in the case of transverse autocorrelations, one might observe negative correlations, and further that the shape of the autocorrelation function following the first zero crossing may contain information about the structure of the turbulence. On the other hand, Yaglom [10] found that while the oscillations in the autocorrelation function may reflect the turbulence structure, they can also be described as "spurious" if a small number of data points are used to determine the autocorrelation function, and the error exceeds the quantity being estimated.

The integration domain for the determination of the integral length as a representative length scale of the turbulence can be specified in a number of ways. In this study we investigate the following four methods:

1. integrate over the entire available domain;
2. if the autocorrelation function has a negative region, integrate only up to the value where the autocorrelation function is a minimum [9];
3. integrate only up to the first zero-crossing [4]; or
4. integrate only up to the value where the autocorrelation function falls to $1/e$ [9].

To investigate the effect of the spatial domain on the integral length determination, each of the four integration domains listed were used to determine the integral length from the longitudinal autocorrelation function of velocity. The spatial domain of the velocity field was progressively restricted. The results are shown in Figure 2. Note that measurements are with respect to a DNS grid size of 2π, chosen so that wavenumbers are integer values.

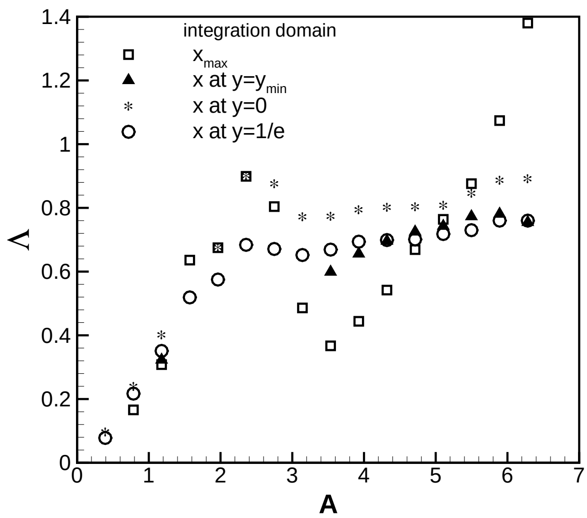

Figure 2: Integral length (Λ) determined from DNS data ($Re_\lambda = 26$) using four different integration domains and progressively restricting the spatial domain (A).

It is expected that the effect on integral length of increasing the spatial domain would be to increase integral length up to a limit where further increasing the spatial domain has little or no effect, at which point the integral length is judged to be accurate. From Figure 2 this appears to be what happens for three of the four integration domains, but not for the case where the integration domain includes all the available data (i.e. integrating from $x = 0$ to $x = x_{max}$). The reason for this can be seen by considering Figure 3 which shows the autocorrelation function obtained for the entire available spatial domain. The boundary conditions for the DNS are periodic, and the effect of this can be seen in the form of the autocorrelation function, with very high values (close to 1) being obtained when the distance between correlated points is close to the side length of the grid. While this is a true representation of the autocorrelation function from the DNS data, it is not a fair representation for homogeneous isotropic turbulence, where the autocorrelation function would be expected to decay to zero within a sufficiently large spatial domain. It also leads to an erroneous measure of integral length if all the data is used in the evaluation, and that error can be seen in the continued increase in the integral length observed using this integration domain in Figure 2. The effect of the periodic boundary condition can also be seen in the fact that the integral length does not follow a gently increasing trend as the spatial domain is increased up to approximately 3, but increases significantly then reduces. For these reasons this integration domain is not used in the determination of integral length in the remainder of this study.

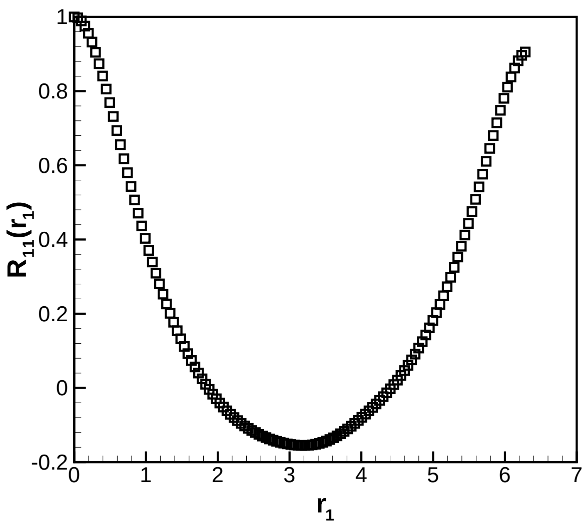

Figure 3: Longitudinal autocorrelation function obtained using the entire spatial domain available ($Re_\lambda = 26$).

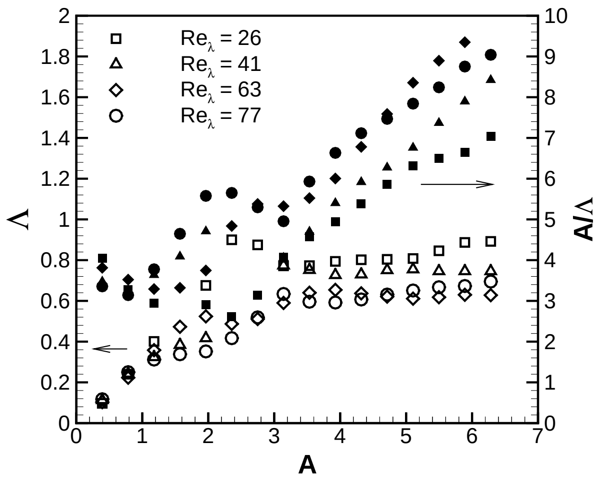

Figure 4: Integral length calculated from the longitudinal autocorrelation function over a range of spatial domains (A). Also shown is the ratio of the spatial domain to integral length (A/Λ). Results are shown for a range of Re_λ.Hollow symbols indicate Λ, filled symbols indicate A/Λ.

Of the three remaining integration domains, two of the methods give similar results (integration from zero to x at $y = y_{min}$ and from zero to x at $y = 1/e$). The first of these methods (zero to x at $y = y_{min}$) has a disadvantage in that it often cannot be evaluated. If the autocorrelation function does not become negative and does not have a minimum value, the integration domain cannot be specified. The second of these two methods (from zero to x at $y = 1/e$) discounts a large portion of the autocorrelation function and is expected to underestimate the integral length. For the remainder of the results presented in this study, the integration domain used in the evaluation of integral length is from $x = 0$ to the first zero crossing of the autocorrelation function.

Having selected an integration domain, the question is: what is the minimum spatial domain that will ensure an accurate determination of integral length? In order to help answer this question, Figure 4 compares the integral length determined from the longitudinal autocorrelation function found for a range of Re_λ using a range of spatial domains. For all the Re_λ shown in Figure 4 it appears that increasing the spatial domain A above ~ 3.2 only has a small effect on the evaluation of Λ. However, the number of integral lengths in the spatial domain for this determination is different for each of the four Reynolds numbers, the spatial domain being between ~ 4 and ~ 5.5 times greater than the integral length (Λ). It is also noted that in one case an underestimate of integral length appears to be obtained, even though the spatial domain is approximately six times greater than the integral length. It appears from these results that the number of integral lengths required in the spatial domain in order to obtain the correct integral length may depend on the Reynolds number of the flow under investigation. A spatial domain greater than six times the integral length appears to be just sufficient for the worst case represented in Figure 4.

Vorticity Integral Length

The results in the previous section indicate that a spatial domain equivalent to at least six integral lengths is required for determination of an accurate integral length from the longitudinal velocity autocorrelation function for the DNS results shown. Further investigations were undertaken to determine what spatial domain is required when determining the integral length scale for the vorticity.

Using the same method for obtaining integral length, that is integrating up to the x-position of the first zero-crossing of the autocorrelation function, results are obtained for integral length of the vorticity from the transverse autocorrelation function. The results are presented in Figure 5. It appears that the spatial domain required for an accurate determination of vorticity integral length is smaller than needed for velocity integral length, being approximately 1.5 compared to 3.2. However, for all the Re_λ the number of multiples of vorticity integral length required in the spatial domain is larger, being between 4.5 and 7 times larger than the integral length compared to between 4 and 5.5 times larger for the velocity integral length. It is likely, however, that a spatial domain chosen to accurately resolve velocity integral length will also accurately resolve vorticity integral length.

Conclusions

Integral length is important in characterising the structure of turbulence. It is a measure of the longest correlation distance between the flow velocity (or vorticity, etc) at two points in the flow field [3]. It is possible to extract an integral length from a numerical or experimental investigation of turbulence, it is however not possible to determine if the integral length so obtained is accurate.

In some of the archival literature a domain equivalent to at least two to three times the measured integral length is recommended for the accurate determination of integral length (eg. [2]). However a more recent citation states that a reasonable lower limit on the domain is eight integral length scales [7]. This recommendation is consistent with the results shown here, where it is found that, in the worst case, the spatial domain must be at least six times larger than the integral length. In the same reference [7] it states that the effect of limiting the spatial domain has not been studied systematically. In this paper a systematic approach to determining the effect of limiting the spatial domain is presented. The results have suggested that a spatial domain at least six times larger than the integral length is required for the two Reynolds numbers presented, however the results indicate that there may be a Reynolds number effect that should be considered. Further investigations may show that specifying a

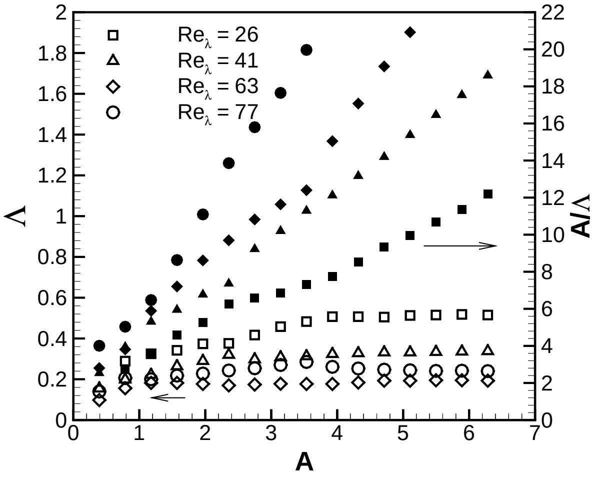

Figure 5: Integral length calculated from the autocorrelation function of vorticity over a range of spatial domains (A). Also shown is the ratio of the spatial domain to integral length (L/Λ). Results are shown for a range of Re_λ.Hollow symbols indicate Λ, filled symbols indicate A/Λ.

simple relationship between integral length and spatial domain is inappropriate.

In the case of the integral length of the vorticity it appears that the spatial domain required for determination of an accurate integral length for velocity will also be sufficient for determination of an accurate integral length for vorticity.

Acknowledgements

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References

[1] Dias, N.L., Chamecki, M., Kan, A. and Okawa, C.M.P., A Study of Spectra, Structure and Correlation Functions and their Implications for the Stationarity of Surface-Layer Turbulence, *Boundary–Layer Meteorology*, **110**, 2004, 165–189.

[2] Eswaran, V. and Pope, S.B., Direct Numerical Simulations of the Turbulent Mixing of a Passive Scalar, *Physics of Fluids*, **31**(3), 1988, 506–520.

[3] Hinze, J.O., Turbulence, *McGraw–Hill*, 1975

[4] Katul, G.G. and Parlange, M.B., Analysis of Land Surface Heat Fluxes using the Orthonormal Wavelet Approach, *Water Resource Research*, **31**, 1995, 2743–2749.

[5] O'Neill, P.L., Nicolaides, D., Honnery, D. and Soria, J., Velocity, Velocity Gradient and Vorticity Statistics of Grid Turbulence Obtained using Digital Cross-Correlation PIV, submitted to *Experiments in Fluids*, 2004.

[6] O'Neill, P.L. and Soria, J., The Relationship Between the Topological Structures in Turbulent Flow and the Distribution of a Passive Scalar with an Imposed Mean Gradient, submitted to *Fluid Dynamics Research*, 2004.

[7] Pope, S.B., Turbulent Flows, *Cambridge University Press*, 2000.

[8] Townsend, A.A., The structure of turbulent shear flow, *Cambridge University Press*, 1976.

[9] Tritton, D.J., Physical Fluid Dynamics, Second Edition, *Oxford University Press*, 1988.

[10] Yaglom, A.M., Correlation Theory of Stationary and Related Random Functions, Volume 1: Basic Results, *Springer Verlag*, 1987.