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A Correct Model of Variance, Skewness, Kurtosis in Boundary Layer with Turbulent External Layer

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Abstract

Boundary layer intermittency is one of the most important and interesting fluid mechanics research topics. Intermittency refers to the fact that at each point in a flow field, the boundary layer is alternately present or absent. The presence percentage of boundary layer at a point is called the intermittency factor I at that point. The I distribution versus distance from the wall is generally described in literature by the erf function. If the external layer is turbulent, at each point of the flow statistical quantities values of fluctuation velocity can be defined (Variance, Skewness, Kurtosis) which are characteristic either of the instants when the boundary layer is present or of the instants when the external layer is present. Moreover, direct measurements of these same statistical quantities give values that depend on previous ones, through not simple expressions. In this paper these expressions have been fully derived. The possibility of using simplified expressions of linear combination, giving as weights I and its complement to one, has also been shown. Finally these simplified expressions have been compared with the more complex general expressions, showing errors in the region of 10% maximum.

Introduction

The boundary layer flow is characterised by intermittency that originates from the presence of an indented interface, continuously variable, separating the whirling turbulent boundary layer fluid from the external fluid flow. Consequently, at each point of the flow, the boundary layer can be alternately present or absent. The fraction of total time for which the boundary layer is present at the considered point is defined as intermittency factor I at that point: it can vary between 0 and 1 [8][3][9][1][7].

Experimental methods based either on thermal effects or on instantaneous velocity gradients measurements have been devised in order to reveal intermittency. Nevertheless, thermal effects are difficult to use in water flows, and gradient measurements cannot be interpreted easily when the external flow is also turbulent. Therefore these methods cannot be used in water boundary layer situations with turbulent external layer.

Recently the authors, who have been studying this type of flow for some time, devised an indirect method to measure intermittency, based on measurements of local mean, Variance, Skewness, Kurtosis of instantaneous velocities and longitudinal integral length scales [6]. One of the hypotheses of the indirect method was that each statistical quantity associated with fluctuation velocity can be expressed at any point of the flow as a weighted mean of its values at that same point, in the presence or absence of a boundary layer; and that the weights to be considered are respectively I and its complement to one, that is:

$$Z = Z_i I + Z_\infty(1 - I) \qquad (1)$$

where Z is a turbulence statistical quantity of fluctuation velocity (Variance, Skewness, Kurtosis and longitudinal integral length scale) at a point in the boundary layer, Z_i is its value when the boundary layer is present and Z_∞ is its value when the boundary layer is absent. Moreover the authors proposed that the I distribution can be given through a modified erf function, with the presence of a logarithmic function, to better reproduce the real behaviour of the intermittency.

Aim of this paper

Henceforth longitudinal integral length scale will no longer be taken into account, due to its particular nature compared with those of the other statistical quantities, all of which are statistical moments of fluctuation velocity. Model (1), in relation to statistical moments of fluctuation velocity, is exact if the moment Z is defined in the same way either when the boundary layer is present or when it is absent (that is Z_i has the same definition as Z_∞, even though they concern different instants: the first one when the boundary layer is present, the second one when the boundary layer is absent). In particular, at each point of the flow, the fluctuation velocity (v'), should be obtained starting always from the general local mean velocity (v): this value is different from the local mean velocity relative only to instants when the boundary layer is present (v_i), and from the local mean velocity relative only to instants when the boundary layer is absent (v_∞). Among these three local mean velocities the following relation holds:

$$v = v_i I + v_\infty (1 - I) \quad (2)$$

In fact it is much more interesting to obtain the relation corresponding to (1), but relative no more to Z_i and Z_∞, (computed as previously mentioned), but to new values Z_{ii} and $Z_{\infty\infty}$ computed on the basis of fluctuation velocities v'_i and v'_∞ obtained initially from local mean velocities v_i and v_∞ respectively.

Now it is clear that model (1) is correct if it concerns Z_i and Z_∞, but is only a simplified one if it concerns Z_{ii} and $Z_{\infty\infty}$. Obviously the model (1) is the less simplified the more v_i e v_∞ values are alike.

Previous papers by the authors concerned Z_{ii} and $Z_{\infty\infty}$ (which are in fact more scientifically interesting), and model (1) was however applied, in order to avoid the complexity of the complete expressions [1], which would make the study more difficult at the first stage.

In this paper two aims will be attained:

1) the complete expressions of Variance, Skewness and Kurtosis, referring to Z_{ii} and $Z_{\infty\infty}$;

2) the discrepancy between the expressions of the correct model and the corresponding ones of the simplified model will be computed.

The correct model

In order to define the complete expressions of Variance, Skewness and Kurtosis the following should be considered:

First of all it is clear that among v, v_i, v_∞, v', v'_i and v'_∞ the following relations hold:

$$v'_i = v' + (v - v_i) \quad (3)$$

$$v'_\infty = v' + (v - v_\infty) \quad (4)$$

Moreover, the following expression will be used:

$$a = v_\infty - v_i \quad (5)$$

Through (5), (2) and the I definition, it is possible to replace (3) and (4) by the following expressions:

$$v'_i = v' + (1 - I)\, a \quad (6)$$

$$v'_\infty = v' - I\, a \quad (7)$$

Once these expressions have been stated, the general rules required to define the complete expressions of Z moments (Variance, Skewness and Kurtosis), referring to the corresponding quantities concerning instants when the boundary layer is present or absent (Z_{ii} and $Z_{\infty\infty}$ respectively), are:

1st) for each statistical quantity it is necessary to start from equation (1), and afterwards to substitute Z_i and Z_∞ with suitable definitions based on v' (equivalent in form but calculated at different instants);

2nd) in equations obtained from the previous point, v' must be replaced by its expression obtained through (6), referring to Z_i, and by its expression obtained through (7), referring to Z_∞;

3rd) the expressions obtained must be algebraically elaborated;

4th) the expressions obtained in the 3rd point must be algebraically elaborated further in order to express Z_{ii} e and $Z_{\infty\infty}$;

5) finally the expressions obtained in the 4th point must also be algebraically elaborated once more in order to obtain an expression similar to (1) but referred to Z_{ii} and $Z_{\infty\infty}$ and, consequently, also the subsequent corrective terms of the correct model in respect of the simplified model.

Through these general rules the following expressions of Variance, Skewness and Kurtosis were obtained.

Variance of fluctuation velocity

The general rules, developed up to the 4th point, give:

$$V = I\, V_{ii} + (1 - I)\, V_{\infty\infty} + I(1-I)\, a^2 \quad (8)$$

In this case it is not necessary to develop the 5th point, because in (8) the expression of the simplified model and the subsequent corrective terms of the correct model are already clear.

Skewness of fluctuation velocity

The general rules, developed up to the 4th point, give:

$$S = \frac{\overline{v'^3}}{\overline{v'^2}^{\frac{3}{2}}} = I\frac{\overline{v'^2_i}^{\frac{3}{2}}}{\overline{v'^2}^{\frac{3}{2}}} S_{ii} + (1 - I)\frac{\overline{v'^2_\infty}^{\frac{3}{2}}}{\overline{v'^2}^{\frac{3}{2}}} S_{\infty\infty} + I(1 - I)\left[\frac{3a\left(\overline{v'^2_\infty} - \overline{v'^2_i}\right)}{\overline{v'^2}^{\frac{3}{2}}} + \frac{a^3(2I - 1)}{\overline{v'^2}^{\frac{3}{2}}}\right] \quad (9)$$

In this expression v' has been replaced by (6) and by (7) only in the numerator, for the sake of simplicity.

At the 5th point, the general rules give:

$$S = I S_{ii} + (1-I) S_{\infty\infty} + I S_{ii}\left(\frac{\overline{v'^2_i}^{\frac{3}{2}}}{\overline{v'^2}^{\frac{3}{2}}} - 1\right) + (1-I) S_{\infty\infty}\left(\frac{\overline{v'^2_\infty}^{\frac{3}{2}}}{\overline{v'^2}^{\frac{3}{2}}} - 1\right) + I(1-I)\left[\frac{3a\left(\overline{v'^2_\infty} - \overline{v'^2_i}\right)}{\overline{v'^2}^{\frac{3}{2}}} + \frac{a^3(2I-1)}{\overline{v'^2}^{\frac{3}{2}}}\right] \quad (10)$$

Kurtosis of fluctuation velocity

The general rules, developed up to the 4th point, give:

$$K=\frac{\overline{v'^4}}{\overline{v'^2}^2}=I\frac{\overline{v_i'^2}^2}{\overline{v'^2}^2}K_{ii}+(1-I)\frac{\overline{v_\infty'^2}^2}{\overline{v'^2}^2}K_{\infty\infty}+$$

$$+I(1-I)\left[\frac{4a(\overline{v_\infty'^3}-\overline{v_i'^3})}{\overline{v'^2}^2}+\frac{6\overline{v_i'^2}a^2(1-I)+6\overline{v_\infty'^2}a^2 I}{\overline{v'^2}^2}+\frac{a^4(1-3I+3I^2)}{\overline{v'^2}^2}\right] \quad (11)$$

Referring to v', the previous remark is still valid.
At the 5th point, the general rules give:

$$K=IK_{ii}+(1-I)K_{\infty\infty}+IK_{ii}\left(\frac{\overline{v_i'^2}^2}{\overline{v'^2}^2}-1\right)+(1-I)K_{\infty\infty}\left(\frac{\overline{v_\infty'^2}^2}{\overline{v'^2}^2}-1\right)+I(1-I)$$

$$\left[\frac{4a(\overline{v_\infty'^3}-\overline{v_i'^3})}{\overline{v'^2}^2}+\frac{6\overline{v_i'^2}a^2(1-I)+6\overline{v_\infty'^2}a^2 I}{\overline{v'^2}^2}+\frac{a^4(1-3I+3I^2)}{\overline{v'^2}^2}\right] \quad (12)$$

Differences between correct and simplified model

In order to verify the differences between the correct and simplified models, data from the authors' previous work were employed.

All the experimental data were obtained from a boundary layer developing along a smooth flat bottom (flat plate) of a rectangular channel coming from a sluice gate. In this plant the external flow was naturally turbulent: but two other levels of turbulence could be obtained by use of grids in the sluice gate (a 1,25 cm and a 2,5 cm mesh grids were used, which caused in that order increasing levels of turbulence). Instantaneous velocity measurements were performed in four test sections of the boundary layer, at distances 0.15 m apart. In each test section more than 20 points along the vertical were tested, including also points well out of the boundary layer. At each point the instantaneous longitudinal component of velocity was measured through an LDA device, so that, always at each point, more than 200.000 sample data were collected at time intervals of 0,005 s apart. A more detailed description of these tests is to be found in [4].

In each test section, the boundary layer thickness was calculated through a best fit method between experimental data of mean local velocities and fluctuation velocity distributions, and the corresponding distribution laws proposed in [10].

That being stated, the comparison between the correct and simplified models can be developed in the following way.

Referring to Variance, Skewness and Kurtosis, the following data are now available:

1) distributions of experimental data of the Z moments, relative to the aforementioned test sections and to the three different turbulence levels in the external layer (for Variance see [10], for Skewness and Kurtosis see [6], through suitable elaboration);

2) experimental values of $Z_{\infty\infty}$ moments in the external layer, relative to the different test sections and to the three different turbulence levels [4];

3) distributions of Z_{ii} moments, calculated through the simplified model. These are obviously independent of the turbulence level in the external layer (for Variance see [10] through suitable elaboration, for Skewness and Kurtosis see [6]).

Finally, the I distribution, calculated through the simplified model is available in [6].

In particular:

The distance from the plate, made non dimensional through the boundary layer thickness, will be called Y.

The I distribution is:

$$I=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\ln Y}{H}} e^{-\frac{\eta^2}{2}}d\eta$$

where the parameter H (0,29), was calibrated through the simplified model, as already mentioned.

Fig.1 shows the Z_{ii} distributions, calculated through the simplified model. Respectively they are: Variance, made non dimensional through the friction velocity v*; Skewness; and Kurtosis.

Referring to the thickness of the boundary layer, it is necessary to point out that a physically based thickness definition proposed by the same authors in [5] has been chosen. This definition appears to be particularly suitable when the external layer is turbulent. In any case, it is suitable to recall that in the case of an external layer without turbulence, this definition gives a thickness value 40% greater than the corresponding classical definition (Coles [2]), and about 50% greater than the corresponding one calculated with the 99% rule.

Starting from the aforementioned data, differences have been computed between the exact Z_{ii} values, as can be computed from the complete equations (8), (10), (12), and the Z_{ii} values of fig. 1. These differences are shown in figs. 2, 3, 4, with regard to the three Z moments respectively. In each figure three diagrams are shown, relative to the three aforementioned turbulence levels in the external layer. Each diagram is a mean among the data of the different test sections.

It is clear that the diagrams of figs. 2, 3, 4, only roughly represent the values of errors due to the simplified model, mainly as a result of the roughness of the I distribution.

Conclusions

In a boundary layer, the relationships between the statistical moments characteristic either of instants when the boundary layer is present or of instants when the external layer is present, and the same statistical quantities as can be measured, are quite complex. However it is possible to use simplified

expressions of linear combination that produce errors whose values are variable with the non dimensional distance from the plate, Y.
In particular the simplified model produces no error as far as Y ≈ 0,3, due to the fact that at small values of Y, I takes the value of unity so that statistical quantities measured and calculated through either a simplified or complex model are equivalent. Yet the simplified model produces no error also at Y ≈ 0,9 due to the circumstance that for high values of Y local mean velocity in the boundary layer is the same as the measured local mean velocity. The error is greatest at Y ≈ 0,7 (that is about 105% of the thickness of the boundary layer obtained through the rule of 99%). The errors in the Variance are alike in the three conditions of turbulence in the external layer, while the errors of Skewness and Kurtosis are greater the lower the turbulence level is in the external layer .
In the final analysis it is possible to state that the values of these errors (as compared with the total variation of the respective distributions) are about 1% for Variance, and about 10% for Skewness and Kurtosis, and consequently that these errors are significant but not excessive.

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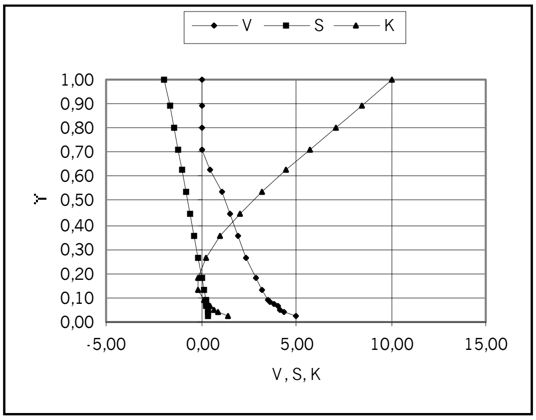

Fig.1 Variance, Skewness, Kurtosis distributions in a boundary layer with potential external flow

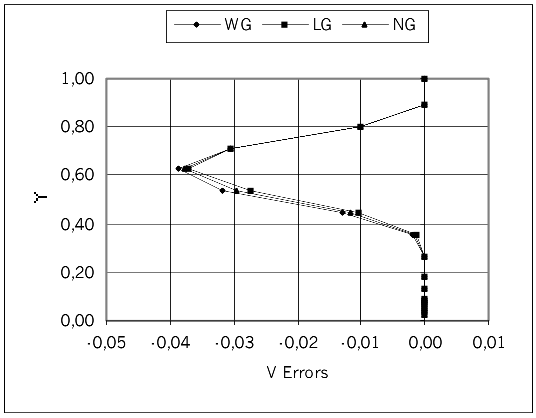

Fig.2 Variance errors distribution

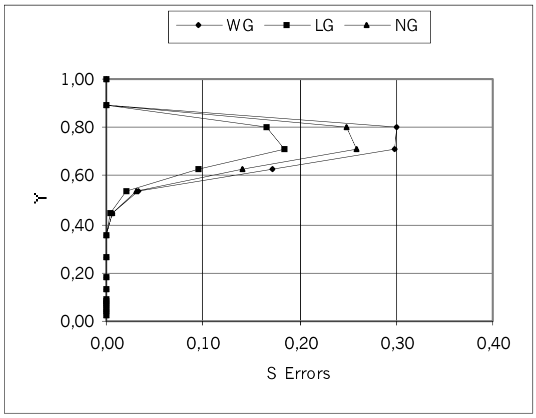

Fig.3 Skewness errors distribution

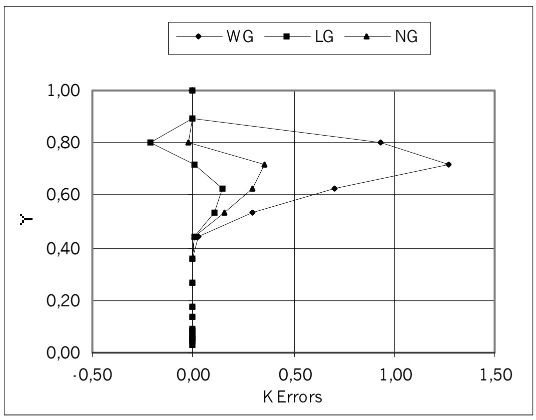

Fig.4 Kurtosis errors distribution