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Velocity-Velocity Difference Correlations and the Sweeping Decorrelation Hypothesis

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Abstract

The correlation, $\rho_{u^k,(\delta u)^l}$ between the streamwise velocity $u(x)$ and the velocity difference $\delta u[\equiv u(x+r)-u(x)]$ in the streamwise direction, is often used to represent the interaction between the energetic and inertial scales. At infinite Reynolds number, local isotropy requires the energetic and inertial scales to be statistically independent. By writing $\rho_{u^k,(\delta u)^l}$ in terms of the second and fourth-order velocity structure functions, it is shown that the correlation can never be zero in the inertial range and is therefore an inaccurate representation for the energy-inertial scale interaction. The implication of this result on the sweeping decorrelation hypothesis is discussed.

Introduction

The assumption of local isotropy (LI), a key ingredient of the hypotheses in Kolmogorov [13](hereafter K41), essentially implies that the dissipative scales are statistically independent of the energy containing scales. In the limit of infinite $R_\lambda[\equiv \langle u^2\rangle^{1/2}\lambda/\nu; \lambda = \langle u^2\rangle^{1/2}/\langle(\partial u/\partial x)^2\rangle^{1/2}]$ the inertial range (IR),$\eta \ll r \ll L_u$, the region separating the energy and dissipative scales, is often assumed to also be independent from large scale forcing. Here $\eta[\equiv \nu^3/\langle\varepsilon\rangle$ where $\langle\varepsilon\rangle$ is the mean energy dissipation rate] and L_u is a large-scale parameter. The assumption of statistical independence is contentious and various arguments have been proposed in order to adequately describe the energy–inertial scale (EIS) interaction process. In mean shear, [4] suggested that LI will never be satisfied, while [22] emphasised that for flows containing large-scale anisotropy, there must be a non-vanishing influence on the small scales – even at infinite R_λ.

While Kolmogorov's equation [12] may be used to distinguish between the dissipative and inertial scales in physical space (a cross-over between the dissipative and inertial scales exists at approximately $r^* = 12$ [1],where an asterix indicates normalisation by η), the equation cannot be used to measure the effect of the large scales on the small scales since it neglects large scale forcing. The following section will show that the correlation that is used to represent the EIS interaction is incorrect. This will be discussed with reference to the sweeping decorrelation hypothesis (SDH)[18] in the final section.

Energy-Inertial Scale Correlations

To test SDH, Praskovsky et al. [16] [hereafter P93], proposed a correlation function that represents the interaction between the energy, $(\equiv u^k, k \geq 1)$ and inertial scales $[\equiv (\delta u)^l = [u(x+r)-u(x)]^l$, for r within the IR and $l \geq 2]$. They proposed that if u^k and $(\delta u)^l$ are statistically independent then SDH is satisfied, since the small scales are not distorted by the energy containing large scales. Note that while statistical independence requires that the correlation be zero, the converse is not necessarily true [19]. In general, the correlation function $\rho_{u^k,(\delta u)^l}$ is defined [P93]

$$\rho_{u^k,(\delta u)^l} = \frac{\left\langle \left(u^k - \langle u^k\rangle\right)\left((\delta u)^l - \langle(\delta u)^l\rangle\right)\right\rangle}{\sigma_{u^k}\sigma_{(\delta u)^l}} \tag{1}$$

where $\sigma_{\alpha^m} = \langle(\alpha^m - \langle\alpha^m\rangle)^2\rangle^{1/2}$ is the standard deviation of α^m. For $m=1, \sigma_\alpha = \langle\alpha^2\rangle^{1/2}$ and for $m=2, \sigma_{\alpha^2} = \langle\alpha^2\rangle(F_\alpha - 1)^{1/2}$. Here, $F_\alpha = \langle\alpha^4\rangle/\langle\alpha^2\rangle^2$ is the flatness factor and $\langle.\rangle$ denotes ensemble averaging.

Author	(k,l) pairs $\equiv \rho_{u^k,(\delta u)^l}$
P93[16]	$(2,2);(4,2);(6,2)$
Katul et al. (1997)[9]	$(1,1)$
Katul et al. (1995)[10]	$(2,2);(4,4)$
Katul et al. (1995)[11]	$(1,1);(2,2)$
Xu et al. (2001)[21]	$(1,1);(2,2)$

Table 1: Combinations of $\rho_{u^k,(\delta u)^l}$ used for determining the EIS interaction.

Tests for SDH using $\rho_{u^k,(\delta u)^l}$ have been carried out in large R_λ mixing layer and channel flow turbulence [P93], the atmospheric surface layer (ASL) [9, 10, 11] and in moderately high R_λ axisymmetric jet turbulence [21]. These results all show that $\rho_{u^k,(\delta u)^l} \neq 0$ for $r > 0$ with k and $l \leq 4$. The various combinations used are shown in Table 1. All authors concluded that SDH is not exactly valid. Nevertheless, P93 and [10] suggested that there is a tendency for $\rho_{u^l,(\delta u)^k}$ to approach zero in the IR and hence SDH will be satisfied in the limit of infinite R_λ.

By writing $\rho_{u^l,(\delta u)^k}$ in terms of simpler, non mean-subtracted correlations between $u^s(x)$ and $u^t(x+r)$

$$B_{L_1\ldots L_s, L_1\ldots L_t} = \frac{\langle u^s(x)u^t(x+r)\rangle}{\langle u^2\rangle^{s/2}\langle u^2\rangle^{t/2}} \tag{2}$$

and non-dimensional structure functions

$$D_{L_1\ldots L_m} = \langle[u(x+r)-u(x)]^m\rangle/\langle u^2\rangle^{m/2}, \tag{3}$$

we will show that the correlation $\rho_{u^k,(\delta u)^l}$ is incorrect for representing the EIS interaction and is not an appropriate test for SDH. However, prior to expanding (1), some insight into the validity of $\rho_{u^k,(\delta u)^l}$ can be established. Clearly not all correlations in Table 1, using different k and l, can represent the actual EIS interaction. In addition, Hill & Wilczak [8] have suggested that $\rho_{u,\delta u}$ is an incorrect representation for the EIS interaction. They indicated that since δu is defined as the difference $u(x+r)-u(x)$, then $u(x)$ must contain a non-zero correlation with (δu). A similar observation was made by Sreenivasan & Stolovitzky [17]. By expanding (1), we also arrive at the same conclusion for the higher-order correlations. Since P93 used correlations of the form $\rho_{u^2,(\delta u)^l}$ with $l \geq 2$, we will concentrate on the simpliest, $\rho_{u^2,(\delta u)^2}$.

Expansion of $\rho_{u^2,(\delta u)^2}$

For $m=2$, we can write

$$\rho_{u^2,(\delta u)^2} = \frac{F_u + B_{LL,LL} - 2B_{LLL,L} - D_{LL}}{(F_u-1)^{1/2}(F_{\delta u}-1)^{1/2} D_{LL}}, \tag{4}$$

using equations (2) and (3). To obtain (4), no assumptions concerning the large or small-scale structure have been made. In the limit of large r, (4) approaches $[(F_u-1)/(2F_u+2)]^{1/2}$ ($\equiv 1/2$ when $F_u = 3$), this is in agreement with the data of [10],[11],[16] and [21]. Approximations to (4) may be carried out using various relations between fourth-order correlation functions. The simplest and most accurate is that defined by

$$B_{L,LLL} = B_{LLL,L}. \tag{5}$$

Assuming a joint-gaussian distribution (JGA) between $u(x)$ and $u(x+r)$, Batchelor [2] used a moment generating function to show that

$$B_{LL,LL} = 1 + 2[B_{L,L}]^2. \tag{6}$$

Similarly, Hill [7] obtained

$$B_{LLL,L} = F_u B_{L,L} \tag{7}$$

by modifing the JGA generating function. Equation (5) is also derived using cyclic permutations of (7). Therefore JGA, when applied to all fourth-order velocity statistics, uses $F_u = 3$ and the equations (5),(6) and (7). The limitations of JGA, however are well known, since it predicts that all odd-order structure functions are zero. Also JGA gives $D_{LLLL} = 3D_{LL}^2$. Consequently, it is incompatible with K41 theory for odd-order moments of (δu), and with turbulence models (e.g. [6]) that are used to account for small-scale intermittency. However, as a first approximation to (4), we note that JGA predicts

$$\rho_{u^2,(\delta u)^2} = D_{LL}/4. \tag{8}$$

Compared with JGA, better approximations to $\rho_{u^2,(\delta u)^2}$ are obtained using assumptions (5) and (7). With (5)

$$\rho_{u^2,(\delta u)^2} = \frac{4F_u - 4B_{LLL,L} + D_{LLLL} - 6D_{LL}}{6(F_u-1)^{1/2}(F_{\delta u}-1)^{1/2} D_{LL}}. \tag{9}$$

We now attempt to simplify (9), while retaining D_{LLLL}. Using (7)

$$\rho_{u^2,(\delta u)^2} \approx \frac{D_{LLLL} + D_{LL}(2F_u-6)}{6(F_u-1)^{1/2}(F_{\delta u}-1)^{1/2} D_{LL}}, \tag{10}$$

and by noting that $D_{LLLL} \gg (2F_u-6)D_{LL}$, then

$$\rho_{u^2,(\delta u)^2} \approx \frac{1}{6(F_u-1)^{1/2}} \frac{[D_{LLLL}]^{1/2}}{(1-1/F_{\delta u})^{1/2}}. \tag{11}$$

Here, $(1-1/F_{\delta u})^{1/2}$ is a weakly varying function with r, since it is approximately 1 at $r=0$ and decreases to about $4/5$ for large r. Consequently, $\rho_{u^2,(\delta u)^2}$ varies approximately as $[D_{LLLL}]^{1/2}$. This result, along with that obtained with JGA confirms that $\rho_{u^2,(\delta u)^2}$ cannot be zero in the IR.

The effect of the different approximations to $\rho_{u^2,(\delta u)^2}$ may be assessed using relatively high R_λ plane-jet data. These data, obtained on the jet centreline, were used in [21] for $\rho_{u^2,(\delta u)^2}$ and $\rho_{u,\delta u}$. Taylor's hypothesis is used to convert data from a time increment to a spatial increment r. The value of $R_\lambda [= 1170]$ is of similar order to that used by [10],[11] and [16]. Figure 1 shows the ratio of (4), (8), (9) and (11) relative to (1). Also shown are the exact relation and (11). While there is a relatively large discrepancy at small r, the ratios approach 1 within the IR. The IR is identified by the region over which $D_{LLL} \sim r$ (not shown). Of the two simplest approximations, equation (11) is marginally more accurate than (8) within the IR. These results indicate that $\rho_{u^2,(\delta u)^2}$ will never be zero, even at infinite R_λ. They also support the conclusions of [8], that $\rho_{u^k,(\delta u)^l}$ is an inaccurate test for statistical independence between the energy and inertial scales.

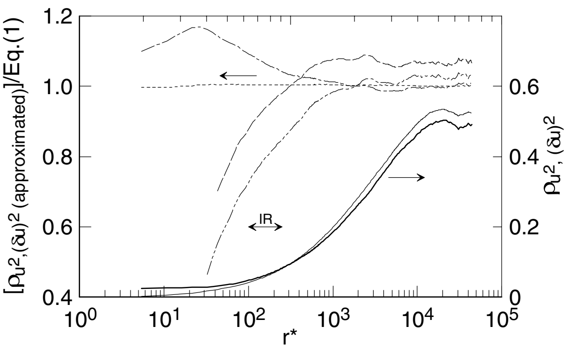

Figure 1: Comparison between different approximations to $\rho_{u^2,(\delta u)^2}$ using the plane jet ($R_\lambda = 1170$) data.
— Eq.(1), — Eq.(11), - - - Eq.(4)/Eq.(1),— - - — Eq.(8)/Eq.(1), — - — Eq.(9)/Eq.(1), – – Eq.(11)/Eq.(1).

Expansion of $\rho_{u,\delta u}$

Subsequent to P93, [9, 10, 11, 21] used $\rho_{u,\delta u}$ as a simpler measure of the EIS interaction. These results indicated both a plausible dependence on R_λ and atmospheric stability and the implications for the EIS interaction were discussed. However, we show by expanding $\rho_{u,\delta u}$ and using the identity $2\langle u(\delta u)\rangle = -\langle u^2\rangle D_{LL}$, that

$$\rho_{u,\delta u} \quad = \quad \frac{(B_{L,L}-1)}{[D_{LL}]^{1/2}} = -\frac{[D_{LL}]^{1/2}}{2} \tag{12}$$

The second expression in (12) is obtained by assuming streamwise homogeneity and approaches a limit of $-1/\sqrt{2}$ for large r. Clearly, no conclusions regarding the EIS interaction can be drawn from this correlation.

Application to the Sweeping Decorrelation Hypothesis

The sweeping decorrelation hypothesis (SDH) proposed by Tennekes [18], assumes that the small scale eddies are advected past a fixed point by energy containing eddies without dynamic distortion. The basic tenet of SDH implies that the inertial range scales are uncorrelated with the energy containing scales (Chen & Kraichnan[3]). Tests for SDH have been carried out using spectral methods [3, 15, 20]. P93 suggested that a more rigorous test may be provided by higher-order structure functions. In order to test SDH using $\rho_{u^k,(\delta u)^l}$, we expand the higher-order structure functions proposed by P93 and apply K41 scaling, and also its refinement [14](K62), to moments of (δu) which appear in the expansion.

Higher-order Velocity Structure Functions

The term 'Higher-order velocity structure functions', to be used here in the context of SDH, was initially proposed P93. It is based on a definition provided by [20] for higher-order spectra. The higher-order structure functions represent the normalised, second-order moment of the velocity increment (δu^m), for integer $m > 1$, *viz.*

$$D_{LL}^{(m)}(r) = \langle [u^m(x+r) - u^m(x)]^2 \rangle / \langle u^2 \rangle^m, \tag{13}$$

these structure functions were not considered in K41. After writing in terms of (δu), and expanding and collecting like terms, then for $m = 2$, (the general expansion for $m \geq 2$ is given in P93)

$$D_{LL}^{(2)} = 4\left[\frac{\langle u^2(\delta u)^2\rangle}{\langle u^2\rangle^2} + \frac{\langle u(\delta u)^3\rangle}{\langle u^2\rangle^2} + \frac{D_{LLLL}}{4}\right]. \tag{14}$$

Equivalently, expanding (13)[for $m = 2$] and assuming streamwise homogeneity,

$$D_{LL}^{(2)} = 2\left(F_u - B_{LL,LL}\right). \tag{15}$$

P93 proposed that, in order to satisfy SDH in the IR, $D_{LL}^{(m)} \sim r^{2/3}$, where $\sim$ denotes "scales as". This condition is related to the original observation of Dutton and Deaven[5] and the subsequent confirmation by Van Atta & Wyngaard[20] that scaling of the higher-order spectra is not based on the dimensional analysis implicit in K41 (this would predict that $D_{LL}^{(m)} \sim r^{m/3}$). When the condition required by K41 is relaxed to one of $D_{LL}^{(m)} \sim D_{LL}$, then in order to satisfy SDH, two consequences are implied from (14).
i) Only the first term is non-zero and u^2 and $(\delta u)^2$ must be uncorrelated with each other, or
ii) The sum of all terms scale as D_{LL}.
We will first discuss i), as proposed by P93 and subsequently show, at least when the approximations to $\rho_{u^2,(\delta u)^2}$ are used, that $D_{LL}^{(m)} \nsim D_{LL}$.

Expansion of $D_{LL}^{(2)}$

In (14), the first two terms are expanded using equation (1) *viz.*

$$D_{LL}^{(2)} = 4D_{LL}\Bigg[1 + (F_u - 1)^{1/2}(F_{\delta u} - 1)^{1/2}\rho_{u^2,(\delta u)^2} + \frac{\sigma_u \sigma_{(\delta u)^3}}{\langle u^2\rangle\langle(\delta u)^2\rangle}\rho_{u,(\delta u)^3} + \frac{D_{LLLL}}{4D_{LL}}\Bigg]. \tag{16}$$

If the test for SDH is initially applied according to P93, (where $\rho_{u^2,(\delta u)^2}$ and $\rho_{u,(\delta u)^3} = 0$ in the IR), then

$$D_{LL}^{(2)} = 4D_{LL}[1 + D_{LLLL}/4D_{LL}].$$

To satisfy SDH, $D_{LL}^{(2)} \sim D_{LL}$, this requires either

$$D_{LLLL} \sim D_{LL} \tag{17}$$

or

$$\frac{D_{LLLL}}{4D_{LL}} \ll 1. \tag{18}$$

While it is well established that (17) will never be satisfied, equation (18) is valid in the limit of infinite R_λ. We demonstrate this by writing (18) using K62 and assuming that $L_u/\lambda \sim R_\lambda$ [19], *viz.*

$$\frac{D_{LLLL}}{4D_{LL}} \sim \left(\frac{r}{\lambda R_\lambda}\right)^{2/3-[\mu(4)-\mu(2)]}, \tag{19}$$

where $\mu(2)$ and $\mu(4)$ equal the departure the scaling exponent of the 2nd and 4th-order structure functions from their respective K41 predicted values. For $R_\lambda > 500$, $[\mu(4) - \mu(2)] \approx 0.1$ and $D_{LLLL}/D_{LL} \sim R_\lambda^{-0.56}$. This ratio will approach zero at sufficiently large R_λ, leading to $D_{LL}^{(2)} \sim D_{LL}$. We note that this result is only correct when all $\rho_{u^k,(\delta u)^l}$ are zero, i.e. the D_{LL} dependence of $D_{LL}^{(2)}$ is a forced condition.

It is possible to carry out a similar analysis without assuming that either $\rho_{u^2,(\delta u)^2}$ or $\rho_{u,(\delta u)^3}$ are zero. Assuming equation (6)

$$2\langle u(\delta u)^3\rangle = -\langle u^2\rangle^2 D_{LLLL}, \tag{20}$$

then (14) becomes

$$D_{LL}^{(2)} = 4\frac{\langle u^2(\delta u)^2\rangle}{\langle u^2\rangle^2} - D_{LLLL}.$$

Writing $D_{LL}^{(2)}$ in terms of correlations gives,

$$D_{LL}^{(2)} = 4D_{LL} + 4\frac{\sigma_{u^2}\sigma_{(\delta u)^2}}{\langle u^2\rangle^2}\rho_{u^2,(\delta u)^2} - D_{LLLL}. \tag{21}$$

Substituting (10) into (21), an approximation to $D_{LL}^{(2)}$ is

$$D_{LL}^{(2)} = \frac{4F_u D_{LL} - D_{LLLL}}{3}. \tag{22}$$

Instead, when (11) is used,

$$D_{LL}^{(2)} = \frac{12D_{LL} - D_{LLLL}}{3}. \tag{23}$$

This relation is identical to that obtained by [8]. When JGA is used on (15) and (23)

$$D_{LL}^{(2)} = 4D_{LL} - [D_{LL}]^2 = 2D_{LL}(1 + B_{L,L}). \tag{24}$$

Comparison between the various approximations to $D_{LL}^{(2)}$ are shown in Figure 2. In contrast with individual approximations

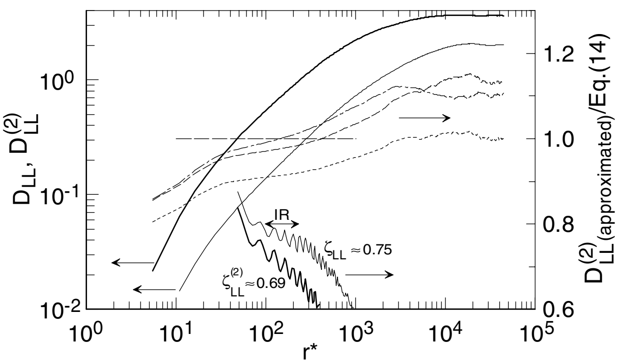

Figure 2: Comparison between approximations to $D_{LL}^{(2)}$ using plane jet data. Also shown is D_{LL}, and the local slopes to $D_{LL}^{(2)}$ and D_{LL}, represented by $\zeta_{LL}^{(2)}$ and ζ_{LL} respectively.
—, Eq.(14); —,D_{LL}; - - -, Eq.(22)/Eq.(14); – – , Eq.(23)/Eq.(14); — - —, Eq.(24)/Eq.(14) ; a long dash line equal to 1 is shown for reference.

to $\rho_{u^2,(\delta u)^2}$, the approximation to $D_{LL}^{(2)}$ using JGA is marginally better within the IR than that for either (22) or (23). Also, it is not possible to determine if SDH is exactly satisfied when using

(22),(23) or (24), since $D_{LL}^{(2)}$ cannot be written entirely in terms of D_{LL}.

Comparing ζ_{LL} and $\zeta_{LL}^{(2)}$–the scaling exponents for the measured values of D_{LL} and $D_{LL}^{(2)}$ respectively–provides the most accurate test for establishing whether SDH is satisfied. Using the local slope estimate of the scaling exponent, e.g. $\zeta_{LL} = d[\ln D_{LL}]/d[\ln r^*]$, the dependence of $D_{LL}^{(2)}$ relative to D_{LL} can be determined. Figure 2 shows that $D_{LL}^{(2)}$ does not have the same scaling exponent (≈ 0.69) as that obtained for $D_{LL}(\approx 0.75)$. In addition, the width of the approximate scaling range for $D_{LL}^{(2)}$ is neglible compared with that for D_{LL}. Both of these differences suggest that SDH is not satisfied in this flow. Therefore, only by comparing the local slopes of $D_{LL}^{(2)}$ and D_{LL} is it possible to establish whether SDH is satisfied.

Conclusions

The relations proposed by P93, have been used to determine the EIS interaction and test SDH, this requires that the energy and inertial scales be statistically independent. This has previously been quantified through the correlation $\rho_{u^k,(\delta u)^l}$, for various values of k and l. By expanding the higher-order streamwise structure function $D_{LL}^{(2)}$ (containing two correlation terms), it is shown that neither of the correlations are zero. Consequently, while $\rho_{u^k,(\delta u)^l}$ represents the correlation between u and δu, it is not a precise measure of the statistical dependence between the energy and inertial scales. This result is in agreement with the conclusion of [8], which was arrived at by a different approach. We also conclude that, since these correlations will never equal zero, it is not possible to establish if SDH is satisfied when writing $D_{LL}^{(2)}$ in terms of the correlations. It is suggested that the local slope be used to determine the scaling behaviour of $D_{LL}^{(2)}$.

Acknowledgement

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