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Effect of Unequal Granular Temperature on Number of Collision and Dilute Viscosity of Kinetic Theory

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Abstract

Kinetic theory for particles of the same mass and the same granular temperature is well established. However, kinetic theory for particles of different mass and different granular temperature is yet to be established. In this paper we re-derived the equations of the number of collisions and dilute viscosity for a binary granular mixture with unequal granular temperature. Here particles are taken as of different size and density. The results indicate significant discrepancies when compared with the theory that is derived by assuming particles of equal mass and equal granular temperature.

Introduction

Computational Fluid Dynamics in multiphase flow has become a well-accepted and useful tool in modelling the gas/solid flow systems during recent years. The earlier models require a particular viscosity as an input into the models (Tsuo and Gidaspow, 1990; Kuipers et al., 1992; Lyczkowsky et al., 1993). However reliable measurements of such viscosities are scarce (Schuegel, 1971; Grace, 1982; Miller and Gidaspow, 1992).

Savage (1983) and his collaborators (Savage and Jeffrey, 1981; Jenkins and Savage, 1983) showed that dense phase kinetic theory can be applied to a granular flow of particles. This theory allows the computation of viscosity of particles from measurements of granular temperature (Gidaspow and Huilin, 1996; Gidaspow et al., 1994). Savage used the term granular temperature to quantify the random motion of particles about the mean velocity. Granular temperature was defined as the averages of the sum of the squares of the three fluctuating velocity components. In these theories all of the particles are assumed to be identical, characterised by a diameter, density and a coefficient of restitution. However, in real particle systems, particle of different size and density exists. The granular temperature might not be equal for particles of different size and density. Experimental data by Yang & Arastapoor (1996) have shown that particles of different diameters possess unequal turbulence energy in a riser of gas-solid dilute flow, indicating different granular temperature for particles of different size.

In this paper, the equations of the number of collisions and the particulate viscosity for a binary granular mixture with unequal granular temperature has been re-derived based on the kinetic theory for dense gases (Chapman & Cowling, 1970). The velocity distribution function and the definition of granular temperature has been taken from Jenkins and Mancini (1987). The granular temperature has been defined as the average of the three fluctuating energy components. The computed results indicate significant changes when compared with the results obtained under the assumptions of equal granular temperature.

Frequency of Collision

We considered two different types of species. Particles of each species are considered as smooth, elastic and homogeneous spheres. Let $\boldsymbol{c}_i$ and $\boldsymbol{c}_i^{/}$ be the velocities of particles of species i immediately before and after a collision, respectively. Again let $\boldsymbol{c}_j$ and $\boldsymbol{c}_j^{/}$ be the velocities of particles of species j immediately before and after a collision. It then follows that the relative velocity between the particles is $\boldsymbol{c}_{ij} = \boldsymbol{c}_i - \boldsymbol{c}_j$. Using conservation of momentum the velocity of the centre of combined mass, $\boldsymbol{G}$, can be defined as;

$$m_O \boldsymbol{G} = m_i \boldsymbol{c}_i + m_j \boldsymbol{c}_j \qquad (1)$$

where m_o is the combined mass of the particles.
The number of binary collisions per unit time per unit volume can be written as

$$N_{ij} = \iiint f^{(2)}(\boldsymbol{c}_i, \boldsymbol{r}, \boldsymbol{r} + d_{ij}\boldsymbol{k}, \boldsymbol{c}_j)\, \boldsymbol{c}_{ji} . \boldsymbol{k} d_{ij}^2 d\boldsymbol{k} d\boldsymbol{c}_i d\boldsymbol{c}_j \qquad (2)$$

where $\mathbf{k}$ is the unit vector along the line joining the centres of the particles. d_{ij} is the average diameter, defined as:

$$d_{ij} = \frac{d_i + d_j}{2} \qquad (3)$$

The probability of finding a pair of particles of species i and j in the volume $dr_i\ dr_j$ having velocities c_i and $c_i + dc_i$ and c_j and $\boldsymbol{c}_j + d\boldsymbol{c}_j$ can be expressed as:

$$f^{(2)} dc_i dc_j dr_i dr_j$$

The velocity distribution for particles of phase i can be written as

$$f_i = n_i \left(\frac{m_i}{2\pi\theta_i} \right)^{3/2} exp\left[-\frac{m_i (\boldsymbol{c}_\mathrm{i} - \boldsymbol{v}_\mathrm{i})^2}{2\theta_i} \right] \qquad (4)$$

where θ_i is the granular temperature, defined as

$$\theta_i = \frac{1}{3} m_i < C_i^2 > \qquad (5)$$

where, C_i is the fluctuating velocity.
It is assumed that each type of particles in the mixture has a different granular temperature. The pair distribution function was obtained by assuming chaos. Following the same procedure as followed in dense gas kinetic theory by Chapman & Cowling (1970), the complete pair distribution function can be expressed as the product of the spatial pair distribution function and the single particle velocity distribution functions (Savage and Jeffrey, 1981)

$$f^{(2)} = f_i f_j g_{ij} \qquad (6)$$

Based on the single solid phase model V. Mathiesen (2000) considered the binary radial distribution function as:

$$g_{ij} = \frac{N}{2}\frac{g_o}{(1-\varepsilon_g)}(\varepsilon_i + \varepsilon_j) \quad (7)$$

where

$$g_o = \left[1 - \left(\frac{\varepsilon_s}{\varepsilon_{s\,max}}\right)^{1/3}\right]^{-1} \quad (8)$$

Substituting the values of f_i, f_j , the pair distribution function becomes

$$f^{(2)} = g_{ij}\frac{n_i n_j}{(2\pi)^3}\left(\frac{m_i m_j}{\theta_i \theta_j}\right)^{3/2} \times$$

$$exp\left[-\frac{m_i(\mathbf{c}_i - \mathbf{v}_i)^2}{2\theta_i} - \frac{m_j(\mathbf{c}_j - \mathbf{v}_i)^2}{2\theta_j}\right] \quad (9)$$

Using the above equations the number of binary collisions per unit time per unit volume becomes

$$N_{ij} = \frac{\pi d_{ij}^2 g_{ij} n_i n_j}{(2\pi)^3}\left(\frac{m_i m_j}{\theta_i \theta_j}\right)^{3/2} \times$$

$$\iint c_{ji}\, exp\left[-\frac{m_i c_i^2}{2\theta_i} - \frac{m_j c_j^2}{2\theta_j}\right] d\mathbf{c}_i d\mathbf{c}_j \quad (10)$$

To evaluate this expression the variables of integration are changed from c_i, c_j to the variables G, g_{ji} and expanding it in a Taylor series, the integral can be written as

$$N_{ij} = 2d_{ij}^2 n_i n_j \left(\frac{m_i m_j}{\theta_i \theta_j}\right)^{3/2} \times$$

$$\int_0^\infty \int_0^\infty exp\left[-AG^2 - Dc_{ji}^2\right]\left[1 - 2B(G.C_{ji}) + \cdots\right] G^2 dG\, c_{ji}^3 dc_{ji} \quad (11)$$

where coefficients A, B, D are

$$A = \frac{m_i \theta_j + m_j \theta_i}{2\theta_i \theta_j} \qquad B = \frac{m_i m_j (\theta_i - \theta_j)}{2m_0 \theta_i \theta_j}$$

$$D = \frac{m_i m_j (m_i \theta_i + m_j \theta_j)}{2m_0 \theta_i \theta_j}$$

Carrying out the integral equation (11) becomes

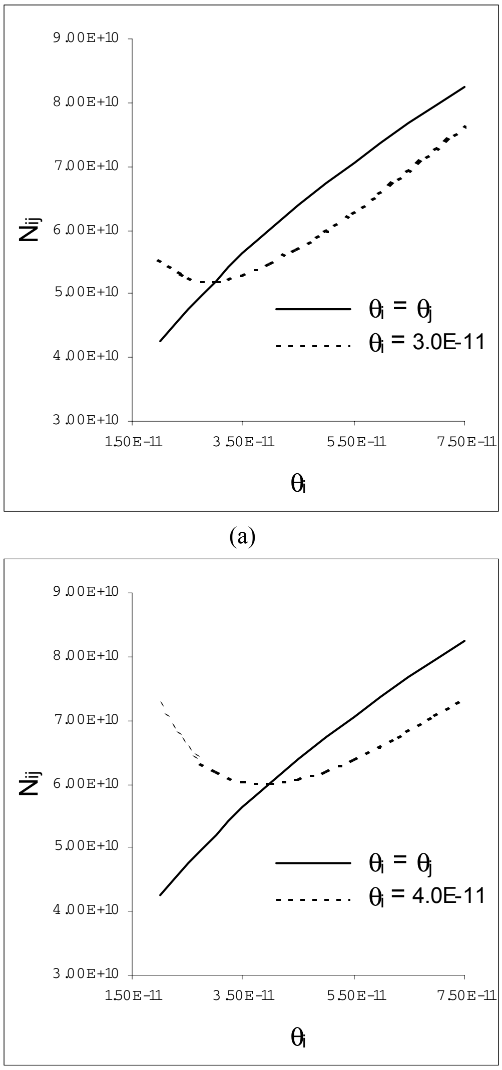
$$N_{ij} = \frac{\sqrt{\pi}}{4} d_{ij}^2 n_i n_j g_o \left(\frac{m_i m_j}{\theta_i \theta_j}\right)^{3/2} \times S_1 \quad (12)$$

$$S_1 = \frac{1}{A^{3/2} D^2}\left[1 + \frac{3\pi B^2}{2AD} + \frac{45\pi B^4}{8(AD)^2} \cdots\cdots\right]$$

If $\theta_i = \theta_j$, and m_i=m_j, then equation (8) boils down to the one used for particles of equal granular temperature and equal mass as showed by Gidaspow (1994).

Particulate Viscosity

In dilute kinetic theory of gases, the intuitive concept of the mean free path plays a very important role. Through the use of this concept, transport coefficients such as viscosity are obtained that are surprisingly close to those obtained from the exact theory (Gidaspow, 1994). The mean time between successive collisions of phase i, called the



(a)

(b)

Figure 1. Computed Number of collisions with granular temperature of species j.

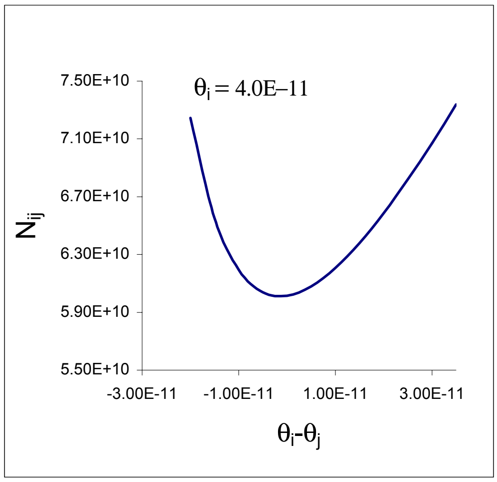


Figure 2 Computed Number of collisions with granular temperature difference.

collision time τ_i, can be obtained as shown below for dilute flow where g_o is one:

$$\tau_i = \frac{n_i}{N_{ij}} \tag{13}$$

and the Mean free path can be expressed as

$$\lambda_i = < c_i > \tau_i \tag{14}$$

where

$$< c_i > = \sqrt{\frac{8\theta_i}{\pi m_i}} \tag{15}$$

Using the mathematical techniques as shown in Chapman & Cowling (1972) and Dimitri Gidaspow (1994) the viscosity of phase *i* can be written as

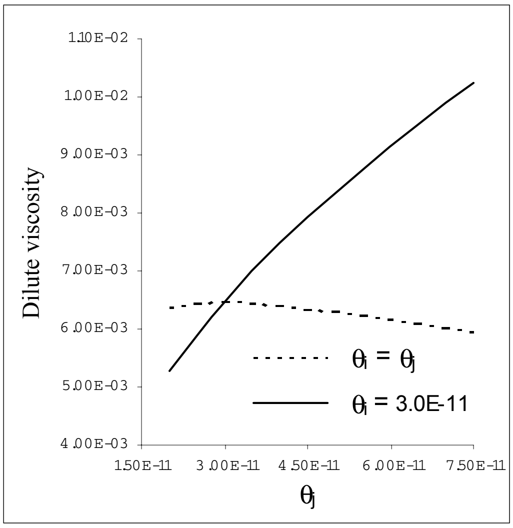
$$\mu_i = \frac{24\varepsilon_{si}\theta_i\rho_i}{\pi^2 d_i^3 m_i \sum_j N_{ij}} \tag{16}$$

If $\theta_i = \theta_j$, and m_i=m_j, equation (12) reduces to the one used by Dimitri Gidaspow (1994) and Chapman & Cowling (1970) for particles of equal granular temperature and equal mass.

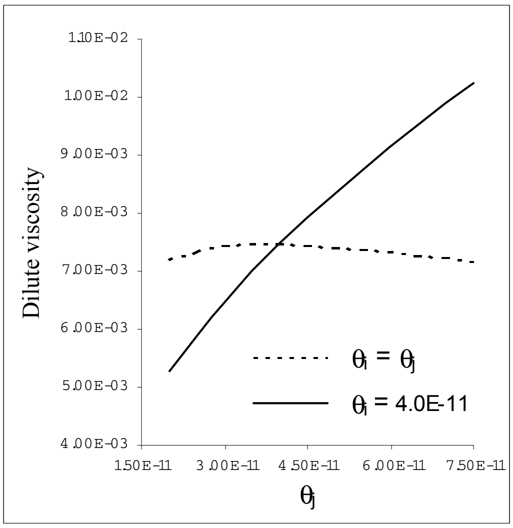
Results and Discussion

We considered two types of particle with m_i = 1.6 E-07 kg, m_j = 2.21 E-07 kg and d_i = 5.0E-04 m, d_j= 1.2E-03 m. Fig. 1(a) shows the collisional number of the multi-particle granular mixture as a function of granular temperature of species j, keeping the granular temperature of species i constant at 3.0E-11kg-m^2/s^2. Results obtained for $\theta_i = \theta_j$ are also presented in the figure. Figure 1 clearly indicates that for unequal granular temperature the rate of no. of collision per unit volume decreases with the increase of granular temperature of species j, in the region $\theta_i > \theta_j$. The trend is opposite in the region $\theta_i < \theta_j$. This is expected because in the region $\theta_i > \theta_j$, with the increase of granular temperature of species j the volume of collisional cylinder decreases as relative velocity ($c_{ij} = c_i - c_j$) decreases, leading to lower no. of collisions. Similarly in the region $\theta_i < \theta_j$, with the increase of granular temperature of species j, the volume of collisional cylinder increases as relative velocity ($c_{ij} = c_i - c_j$) increases, leading to higher no. of collisions. The curve with unequal granular temperature intersects with the curve with equal granular temperature at the point $\theta_i = \theta_j$. Because for $\theta_i = \theta_j$, the equation (10) for N_{ij} reduces to the one that is used for particles with equal granular temperature [Gidaspow 1994, Chapman & Cowling 1970]. This validates the newly derived function for rate of number of collision per unit volume. Figure 2 shows the number of collision per unit time per unit volume as a function of the difference between the granular temperatures ($\theta_j - \theta_i$). Which clearly indicates that the nature of the curve for rate of collision per unit volume depends on the nature of $\theta_j - \theta_i$. Based on the nature of $\theta_j - \theta_i$ (-ve or +ve), N_{ij} may decrease or increase.

Figure 3(a) shows the particulate viscosity as a function of granular temperature of species j, keeping the granular temperature of species i constant at 3.5E-12 kg-m^2/s^2.



(a)



(b)

Figure 3 Computed dilute viscosity with granular temperature of species j.

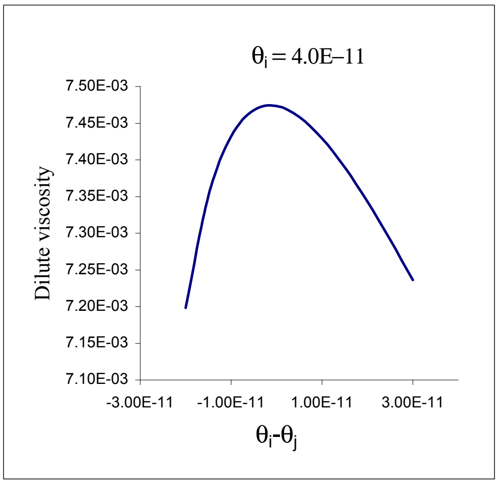


Figure 4 Computed dilute viscosity with granular temperature difference.

Figure 3 clearly indicates that for unequal granular temperature the particulate viscosity increases with the increase of granular temperature of species j in the region $\theta_i > \theta_j$. The trend is opposite in the region $\theta_i < \theta_j$. This is expected because in the region $\theta_i > \theta_j$, with the increase of granular temperature of species j the relative movement of the particles decreases, leading to high particulate viscosity. Similarly in the region $\theta_i < \theta_j$, with the increase of granular temperature of species j the relative movement of the particles increases, leading to lower particulate viscosity. The curve with unequal granular temperature intersects with the curve with equal granular temperature at the point $\theta_i = \theta_j = 3.0E-12$ kg-m^2/s^2. This is because for $\theta_i = \theta_j$, the particulate viscosity equation (16) reduces to the one that is used for particles with equal granular temperature [Gidaspow 1994]. This validates the newly derived function for particulate viscosity. Figure 4 shows the particulate viscosity of species i as a function of the difference between the granular temperatures. Which clearly indicates that the nature of the curve for particulate viscosity depends on the nature of $\theta_j - \theta_i$. Based on the nature of $\theta_j - \theta_i$ (-ve or +ve) μ_i may increase or decrease.

Conclusion

Reliable values of particulate viscosities are needed for numerical simulation and design improvements. No handbook values are available for such viscosities. The available theories developed to obtain such viscosities are applicable for particles of same mass and same granular temperature. In this paper, a new method of obtaining particulate viscosities for a multi particle system has been proposed. The computed results indicate that the number of collision and the dilute viscosities are sensitive to the granular temperature difference. For particles of equal mass and equal granular temperature the proposed equations boils down to the available one that is used for particles with equal mass and equal granular temperature. Work is in progress to complete the kinetic theory for particles with unequal granular temperature.

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Notations

λ = Mean free path.
ρ = Density
θ = Granular temperature
μ = Particulate viscosity
ρ = density
τ = stress tensor
ε = volume fraction
$\varepsilon_{s,\,max}$ = maximum solid packing
g_o= Radial distributive function.
A, B, D = auxiliary constants
c = velocity vector
d = Particle diameter
m = mass of the particle
n = No. of particles.
r = position vector

Subscripts
i = ith phase
j = jth phase
s = solid
k = kinetic
c = collisional

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