14th Australasian Fluid Mechanics Conference
Adelaide University, Adelaide, Australia
10-14 December 2001

Effect of Column Spacing on Wave Force

C.E. Quadros

**Graduate School of Engineering, Department of Naval Architecture & Ocean Engineering
Osaka University, 2-1, Yamadaoka, Suita, Osaka 565 0871, JAPAN.**

Abstract

The study is of an offshore structure consisting of slender columns near the free surface and large buoyancy pontoons near the base that is moored to the sea bottom like a Tension Leg Platform. The effect of the spacing between the columns of the structure on the horizontal component of the total wave force due to an irregular wave has been investigated. Since the columns are slender, diffraction of the incident wave field has been neglected and the wave force has been computed using a method proposed by Borgman for an irregular wave. This research has been done to get an insight into the design of an offshore structure that will experience low horizontal force in the west coast of India where the use of this structure is contemplated.

Introduction

An offshore structure consisting of slender columns near the free surface and large buoyancy pontoons near the base is considered to be advantageous since the wave force and hence wave induced responses are low when compared to a structure consisting of large diameter columns near the free surface. Since the columns are slender, diffraction of the incident wave field can be considered to be negligible and the wave force can be computed using Morison's equation. For a given sea state, the total force on the structure will depend on the relative disposition between cylinders. The effect of this spacing between columns on the total force on the structure has been researched upon.

Figure 1 shows wave statistics data for the west coast of India from Global Wave Statistics Handbook of BMT [3]. It can be observed that the most probable zero crossing period is between *5s* and *6s* and has a probability of *349* in *1000*. The length of the wave in relation to the spacing of columns in the structure has a significant role on the total force on the structure. Since the wavelength is dependent on the zero crossing period, the total force on the structure will hence be considerably influenced by the zero crossing period. Hence, the most probable zero crossing period has been selected. The zero crossing period in this range has a mean of *5.5s*. This corresponds to a mean wave period T_0 of *6.0s*. The probability of exceedance of a significant wave height $H_{1/3}$ of *6m* is *20* in *1000* which is very low. Hence the wave of $H_{1/3} = 6m$ and $T_0 = 6s$ has been selected as the design wave

Significant Wave Height (m) \ Zero Crossing Period (s)	<4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10	>11	Total
Total	46	229	349	245	102	30	7	1	1000
> 9	-	-	-	-	-	-	-	-	2
8 - 9	-	-	-	1	1	-	-	-	2
7 - 8	-	-	1	2	1	1	-	-	5
6 - 7	-	1	2	4	3	1	-	-	11
5 - 6	-	1	5	8	6	2	1	-	23
4 - 5	-	3	12	16	10	4	1	-	47
3 - 4	1	9	30	33	18	6	2	-	97
2 - 3	2	26	68	61	27	8	2	-	194
1 - 2	8	74	133	85	28	6	1	-	336
0 - 1	36	115	97	35	7	1	-	-	291

Figure 1. Wave Statistics Data

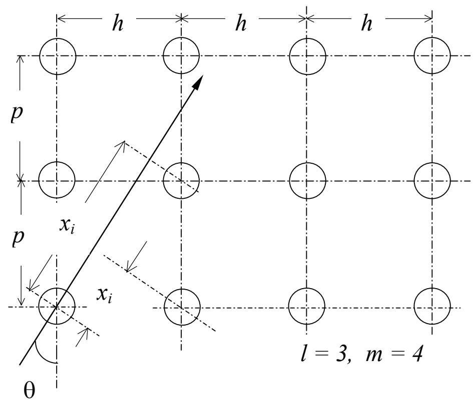

Figure 2. Regular Array

Theoretical Background

For slender columns, the horizontal force can be computed by Morison's [4] equation given by

$$dF(t) = c_1 A(t) + c_2 u(t)\left|u(t)\right| \tag{1}$$

where *u(t)*, *A(t)* and *dF(t)* are the water particle velocity, acceleration and total force respectively as functions of time at a section of the column. c_1 and c_2 are coefficients occurring in Morison's equation which are related to cylinder diameter, the inertia and drag coefficients.

Borgman [1] has shown that for an array of N cylinders, the spectral density of force for the group after linearizing the drag term is given by

$$S_{QQ}^{(N)}(\omega) \approx S_{QQ}^{(1)}(\omega) \times T_{MP}^{(N)}(\omega) \tag{2}$$

where $S_{QQ}^{(1)}(\omega)$ is the spectral density of force for a single cylinder and $T_{MP}^{(N)}(\omega)$ is a transfer function called multiple-pile transfer function which depends on the arrangement of cylinders given by

$$S_{QQ}^{(1)}(\omega) = S_{\eta\eta}(\omega) \times RAO^2 \tag{3}$$

$$T_{MP}^{(N)}(\omega) = \sum_{m=1}^{N}\sum_{n=1}^{N} \cos\left\{k(\omega)(x_n - x_m)\right\} \tag{4}$$

where RAO is the response amplitude operator for force for a single cylinder given by

$$RAO^2 = \frac{8}{\pi}\left[\frac{\omega c_1}{\sinh kd}\int_0^d \sigma(s)\cosh ks\, ds\right]^2 + \left[\frac{\omega^2 c_2}{\sinh kd}\int_0^d \cosh ks\, ds\right]^2 \tag{5}$$

For a rectangular array of equally spaced cylinders, the multiple pile transfer function has the following closed form solution.

$$T_{MP}^{(N)}(\omega) = \frac{1-\cos(kpl\cos\theta)}{1-\cos(kp\cos\theta)} \times \frac{1-\cos(khm\sin\theta)}{1-\cos(kh\sin\theta)} \tag{6}$$

where p and h are the spacing column-wise and row-wise respectively and l and m are the number of columns in each column and row respectively as shown in figure 2. θ is the incident wave direction.

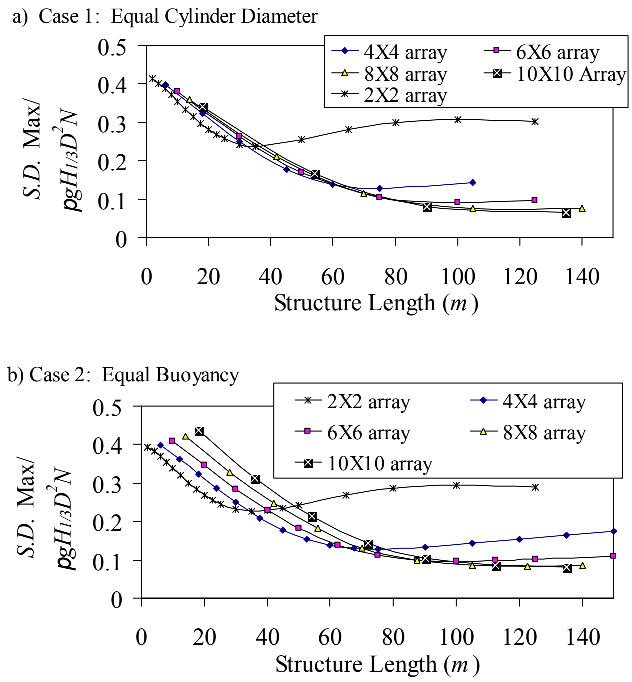

Figure 3. S.D. Maxima for Regular Arrays

Optimum Spacing

The spacing between columns for which the horizontal wave force is minimum is considered to be the optimum spacing. Since the incident wave is an irregular wave, the variance of total wave force has been used as the deciding criterion. Hence, the optimum spacing is that spacing for which the variance of total horizontal force given by the following expression is minimum

$$Variance = \int_0^\infty S_{QQ}^{(N)}(\omega)\, d\omega \tag{7}$$

The variance given by equation (7) has been computed using a computer program developed by the author for various spacing, number of columns in the array and incident wave direction. For a given array, the variance has been computed for various angles of incidence and the maxima has been chosen since the maxima is of concern to us. The results of this computation for a 10×10, 8×8, 6×6, 4×4 and 2×2 array have been plotted in figure 3 for two different cases in terms of the non-dimensional maxima of standard deviation (*S.D.*) of total array force divided by the number of cylinders (*N*) in the array. In the figure, *D* is the cylinder diameter, $H_{1/3}$ is the significant wave height, *g* is the acceleration due to gravity and *p* is the density of water . In Case 1, the cylinder diameter *D* is kept constant and equal to *1.0m* for each array. In Case 2, the buoyancy of all the cylinders put together is kept constant for the various arrays. It can be observed that Case 1 exhibits the same trend as Case 2 and the minimum occurs at the same value of structure length for each array. The numerical values are however different. For each of the array configurations, the minimum occurs corresponding to an overall length of the structure of *35m* and above for the sea state considered. In the above case, as far as the arrangement of cylinders is concerned we had only one control variable *p*.

The next step was to study an irregular array shown in figure 4 which consists of *4* groups of cylinders. In each group, the cylinders are equally spaced. For this arrangement, we have two control variables i.e. the smaller spacing *p* and the larger spacing *P*. This arrangement has been studied to explore whether the presence of two control variables enables us to get a greater reduction in the horizontal wave force when compared to the regular array.

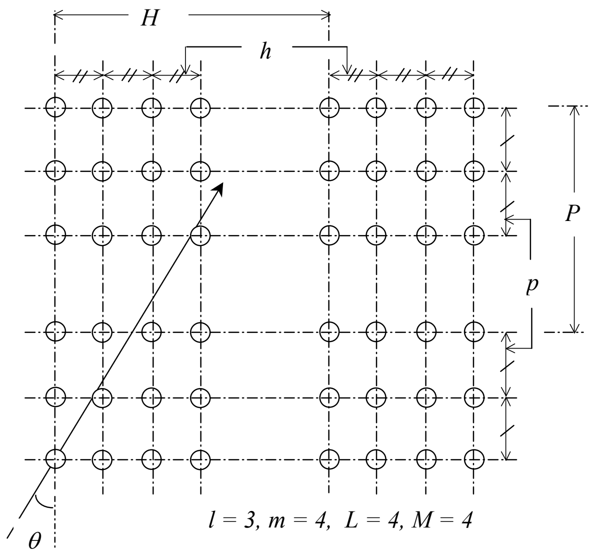

Figure 4. Irregular Array

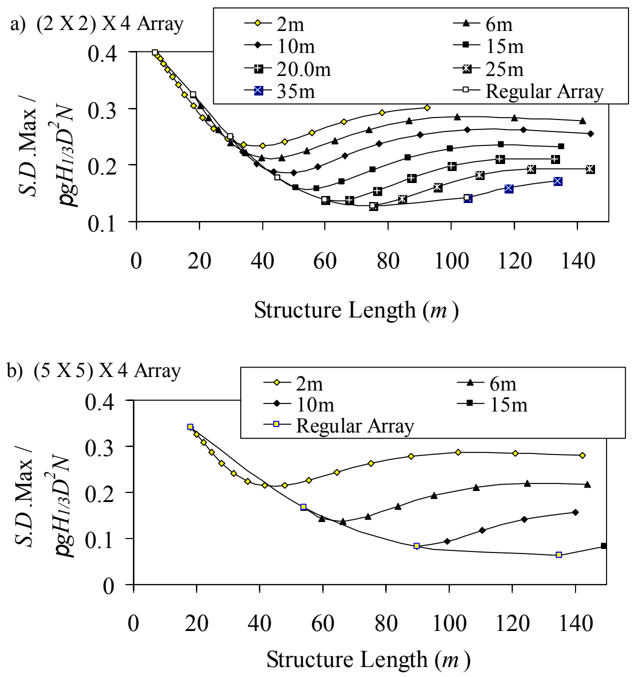

Figure 5. S.D. Maxima for Irregular Arrays – Case 1

For such an arrangement, using equation (4), a closed form expression has been obtained by the author which is given below.

$$T_{MP}^{(N)}(\omega) = \frac{1-\cos(kpl\cos\theta)}{1-\cos(kp\cos\theta)}\,\frac{1-\cos(khm\sin\theta)}{1-\cos(kh\sin\theta)} \times \frac{1-\cos(kPL\cos\theta)}{1-\cos(kP\cos\theta)}\,\frac{1-\cos(kHM\sin\theta)}{1-\cos(kH\sin\theta)} \tag{8}$$

Using equation (8), the variance given by equation (7) has been computed for various irregular arrays. The results for a $(2 \times 2) \times 4$ and $(5 \times 5) \times 4$ array configuration are shown in figure 5 for Case 1 in terms of the non-dimensional maxima of *S.D.* of total array force divided by the number of cylinders in the array. In this figure, curves have been drawn through points corresponding to the same smaller spacing *p* shown in the legend. A single curve has been drawn through the left-end point of each

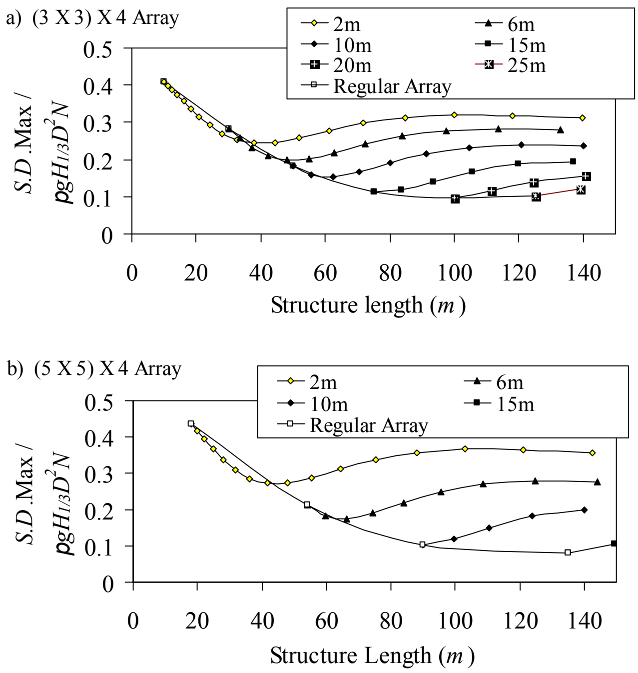

Figure 6. S.D. Maxima for Irregular Arrays – Case 2

curve which indicates the case of a regular array of figure 2. From the curves, it is observed that for values of smaller spacing p less than *8m*, the irregular array experiences lower total force when compared to the regular array. However, this difference is not significant. For values of smaller spacing p greater than *8m*, the irregular array experiences greater force. This difference is quite significant as seen in figure 5. Arrays with greater number of cylinders experiences lesser force when compared to arrays with lesser number of cylinders. Hence the presence of a greater number of cylinders makes the force time histories of each cylinder work against each other.

Figure 6 shows the non-dimensional maxima of S.D. of array force for Case 2 for a $(3\times3)\times4$ array and a $(5\times5)\times4$ array. The $(2\times2)\times4$ array of Case 2 is the same as that of Case 1 as this is the base on which other array dimensions have been determined. Case 2 indicates the same trend as Case 1. The numerical values are however different. Case 2 when compared to Case 1 indicates Case 2 experiencing higher force for each of the arrays. The structure length at which the minima occurs is however the same for Case 1 and Case 2 for each array.

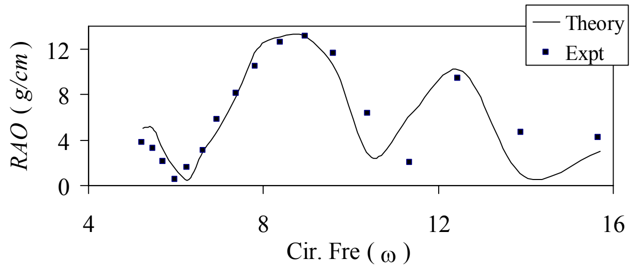

Figure 8. Model Wave Force *RAO*

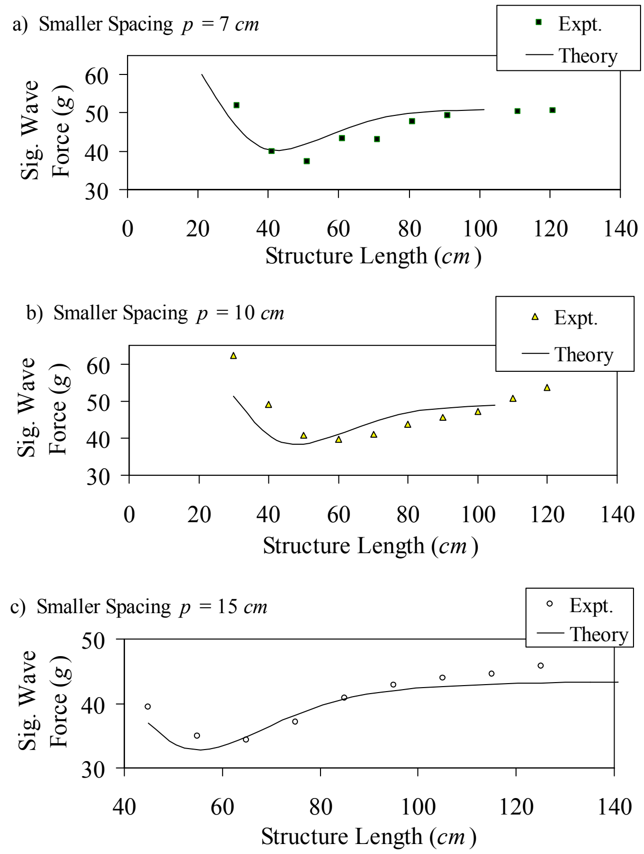

Figure 9. Experimental Results for Irregular Wave

Validation by Experiment

The numerical simulation has been checked by performing an experiment. Figure 7 shows the experimental set-up. Tests have been carried out in a wave tank *14m* in length, *30cm* in width having a water depth of *50cm*. The response amplitude operator (*RAO*) for force for the group has been determined by generating regular waves. Figure 8 shows the *RAO* measured experimentally along with the theoretical *RAO* for smaller spacing p = *10cm* and overall length = *90cm*.

An irregular wave has been generated with $T_0 = 0.705s$ and $H_{1/3} = 5.44cm$ in the wave tank and the group force has been measured for various array configurations. The results are shown in figure 9 along with prediction from numerical

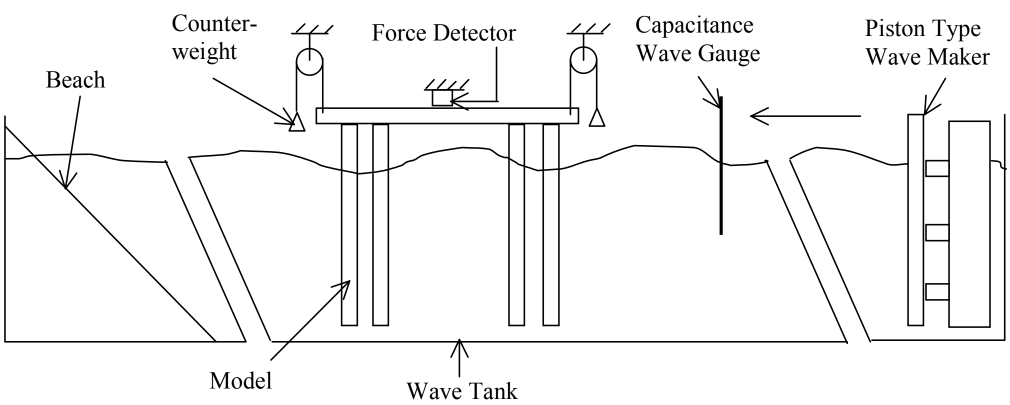

Figure 7. Experimental Set-up

simulation for three different values of smaller spacing *p*. It is observed that there is good agreement between experiment and theory.

Choice of Structure

In the case of regular arrays, it is observed that for both cases, among the 10×10, 8×8, 6×6, 4×4 and the 2×2 arrays, the lesser the number of columns the lower the total array force. The optimum spacing for the 2×2 array corresponds to an overall length of *35m*. On the above basis, the choice of a 2×2 array of overall length of *35m* would mean a column to column spacing of *35m*. This spacing, it appears, will be too large to achieve an economical structure of adequate strength. The higher arrays with greater number of columns such as the 4×4, 6×6, 8×8 and 10×10 arrays also face a similar problem. The column to column spacing decreases as we go for higher arrays. The total force, however, increases due to the increase in the number of columns.

The irregular array, unlike the regular array, leaves considerable amount of space in the centre of the structure. This space will be useful for routing the risers required for exploitation of oil. In the case of the irregular arrays also, the lower the number of columns the lower the total array force. The $(2 \times 2) \times 4$ array experiences minimum force corresponding to an overall length of *75m*. This occurs when the irregular array converges to the regular array as seen in figure 5. Considering structural aspects, it is advisable to place the columns in each of the groups not too far apart. At large overall lengths, for small values of smaller spacing *p*, the irregular array experiences the same force as that for shorter overall lengths. Since a smaller overall length will be more economical, it is advisable to select a shorter overall length. On the above basis, a $(2 \times 2) \times 4$ array of smaller spacing of *6m* and overall length of *40m* has been selected. The buoyancy pontoons have been placed at a depth of *65m* since wave activity diminishes considerably at this depth. This has resulted in a conceptual design of the structure that is shown in figure 10.

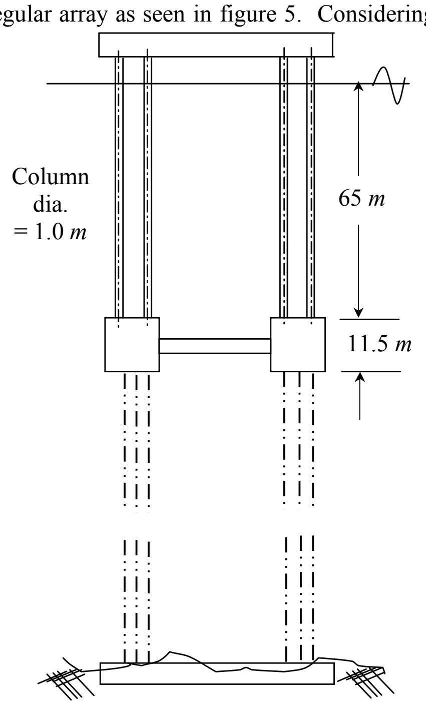

Figure 10. Conceptual Design

Change in sea state:

The force the above structure will experience under sea states having the same significant wave height of *6m* but different mean wave periods has been investigated. Figure 11 shows the maxima of *S.D.* of group force in non-dimensional form. It is observed that for sea states with lower mean wave period, the force is lower and is higher for sea states with higher mean wave period. The force is however not grossly affected by change in mean wave period. From figure 1, it can be observed that for higher mean wave periods, at lower values of significant wave height, the probability of exceedance of wave height is low. Hence, it is expected that the chosen structure will not experience a force greater than that experienced for the design sea state which has a significant wave height of *6m* and a mean wave period of *6s*. Hence the force experienced under the sea state of $T_0 = 6.0s$ and $H_{1/3} = 6.0m$ can be considered to be the design force for other considerations and the chosen structure can be considered to have the optimum arrangement of cylinders.

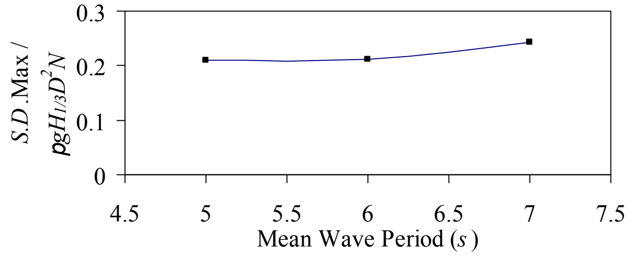

Figure 11. S.D. for different sea states

Conclusion:

In the case of regular arrays, the variance of total force is minimum for structure lengths of *35m* and above. These lengths mean a large column to column spacing which may make an economical design with adequate structural integrity not feasible.

In the case of irregular arrays, for the case with smaller spacing less than *8m*, the group force is lesser than that for an evenly spaced array of the same length. However this difference is not significant. Hence the choice of an irregular array does not give a considerable benefit as far as the wave force is concerned. For smaller spacing above *8m*, the unevenly spaced array experiences higher force. The irregular array at lower overall lengths experiences the same force as that for larger overall lengths for smaller values of smaller spacing *p*. Hence it is advisable to select a structure of length around *40m* and a smaller spacing of *6m*. The above structure when subjected to sea states other than the design sea state is not expected to experience greater force.

Acknowledgement:

I thank Professor S. Naito for his guidance and assistance in doing this research.

References:

[1] Borgman, L.E., The Spectral Density for Ocean Wave Forces, *Coastal Engineering*, Santa Barbara Speciality Conference, October, 1965

[2] Borgman, L.E., A Stastical Theory for Hydrodynamic Forces on Objects, Wave Res. Rep., HEL 9-6, 1965, Hydraulic Eng. Lab., Univ. of Calif., Berkeley, Calif.

[3] British Maritime Technology Ltd., Global Wave Statistics, 1986, Area 50, Annual, All directions

[4] Morison, J.R., O'Brien, M.P., Johnson, J.W. & Schaaf, S.A., The Force Exerted by Surface Waves on Piles, *Petroleum Trans*., AIME, 189, 1950, 149-154.