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Three-dimensional modelling of dam-break induced flows using Smoothed Particle Hydrodynamics

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Abstract
This paper presents simulations of dam-breakage using real topography obtained from the US Geological Survey (USGS). Three-dimensional simulations were performed using the full Navier-Stokes equations with the particle-based Lagrangian Smoothed Particle Hydrodynamics (SPH) method. Since SPH is mesh free it is well suited to simulating free-surface flows of this kind, involving splashing and fluid layer break up. This paper demonstrates the ability of the SPH method to handle these large geophysical phenomena having length scales of the order of kilometres. Due to computational limitations, the highest resolution for which simulations were carried out used a particle separation of around 6.0 m for a topographic area of approximately 15 sq km. An example of a potential dam breakage flow, and its interaction with the surrounding topography, is presented here.

Introduction
The failure of a dam often results in dramatic flood conditions further downstream. The estimation and assessment of these conditions is of critical importance to engineers and land-planners. For example, forecasting the timing of the flood wave in the event of a dam failure allows one to be better prepared with an emergency plan. Also, being able to predict the characteristics of the flow will permit the identification of man-made structures likely to be destroyed.

Simulations of the dam-break process are usually carried out using shallow water equations, which basically represent conservation of mass and momentum of a layer of water. Typical simulations of this kind are reported in [1, 2, 3]. All such simulations have been carried out in one or two dimensions using grid based finite difference or finite element methods. With these approximations, it becomes difficult to resolve three-dimensional features of the flow, which can be important in many dam-break situations, particularly when the topography is complex. Measurements of real dam-break events are very few and far between in the literature. The only such measurements, which the authors have come across is of the Malpasset dam-break that occurred in the late 1950's [1]. The data indicated approximately the propagation time of the flood wave and the maximum heights to which the water reached on the banks.

Smoothed Particle Hydrodynamics (SPH) was originally developed in the 1970's to solve compressible astrophysical problems [4]. Many applications involving free surface flows have been solved using the SPH method since then. Some examples include bursting of a dam and generation of a wave in two dimensions [5] and high pressure and gravity die-casting [6,7]. Since the two-dimensional dam-breaking phenomenon is handled extremely well by the SPH method [5], the full three-dimensional dam-breaking simulation was thought to be a natural extension using this method. The mesh free nature of the SPH method is well suited for simulating free-surface flows of this kind involving splashing and fluid break up.

The SPH methodology
A brief summary of the SPH method is presented here. For more comprehensive details one can refer to [8, 9]. The interpolated value of a function A at any position $\mathbf{r}$ can be expressed using SPH smoothing as:

$$A(\mathbf{r}) = \sum_b m_b \frac{A_b}{\rho_b} W(\mathbf{r} - \mathbf{r}_b, h) \tag{1}$$

where m_b and ρ_b are the mass and density of particle b and the sum is over all particles b within a radius 2h of $\mathbf{r}$. Here $W(\mathbf{r},h)$ is a C^2 spline based interpolation or smoothing kernel with radius 2h, that approximates the shape of a Gaussian function but has compact support. The gradient of the function A is given by differentiating the interpolation equation (1) to give:

$$\nabla A(\mathbf{r}) = \sum_b m_b \frac{A_b}{\rho_b} \nabla W(\mathbf{r} - \mathbf{r}_b, h) \tag{2}$$

Using these interpolation formulae and suitable finite difference approximations for second order derivatives, one is able to convert parabolic partial differential equations into ordinary differential equations for the motion of the particles and the rates of change of their properties.

Continuity equation:

From Monaghan [8], our preferred form of the SPH continuity equation is:

$$\frac{d\rho_a}{dt} = \sum_b m_b (\mathbf{v}_a - \mathbf{v}_b) \nabla W_{ab} \tag{3}$$

where ρ_a is the density of particle a with velocity $\mathbf{v}_a$ and m_b is the mass of particle b. We denote the position vector from particle b to particle a by $\mathbf{r}_{ab} = \mathbf{r}_a - \mathbf{r}_b$ and let $W_{ab} = W(\mathbf{r}_{ab}, h)$ be the interpolation kernel with smoothing length h evaluated for the distance $|\mathbf{r}_{ab}|$.

This form of the continuity equation is Galilean invariant (since the positions and velocities appear only as differences), has good numerical conservation properties and is not affected by free surfaces or density discontinuities. The use of this form of the continuity equation is very important for predicting free surface flows of the present kind.

As two particles approach each other, their relative velocity is negative so that there is a positive contribution to $d\rho_a/dt$ causing ρ_a to rise, leading to a positive pressure that pushes the particles apart again. As two particles move apart their densities decrease creating a negative pressure that pulls the particles back towards each other. This interplay of velocity and density/pressure ensures that the particles remain 'on average' equally spaced and that the density is close to uniform so that the fluid is close to incompressible.

Momentum equation:

The SPH momentum equation used here is:

$$\frac{d\mathbf{v}_a}{dt} = \mathbf{g} - \sum_b m_b \left[\left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} \right) - \frac{\xi}{\rho_a \rho_b} \frac{4\mu_a \mu_b}{(\mu_a + \mu_b)} \frac{\mathbf{v}_{ab}\mathbf{r}_{ab}}{\mathbf{r}_{ab}^2 + \eta^2} \right] \nabla_a W_{ab} \quad (4)$$

where P_a and μ_a are pressure and viscosity of particle a and $\mathbf{v}_{ab} = \mathbf{v}_a - \mathbf{v}_b$. Here ξ is a factor associated with the viscous term [10], η is a small parameter used to smooth out the singularity at $\mathbf{r}_{ab} = 0$ and $\mathbf{g}$ is the gravity vector.

The first two terms involving the pressure correspond to the pressure gradient term of the Navier-Stokes equation. The next term involving viscosities is the Newtonian viscous stress term. This form ensures that stress is automatically continuous across material interfaces and allows the viscosity to be variable or discontinuous.

Equation of state:

Since the SPH method used here is quasi-compressible one needs to use an equation of state, giving the relationship between particle density and fluid pressure. This relationship is given by the expression:

$$P = P_0 \left[\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right] \quad (5)$$

where P_0 is the magnitude of the pressure and ρ_0 is the reference density. For water we use $\gamma = 7$. This pressure is then used in the SPH momentum equation (4) to give the particle motion. The pressure scale factor P_0 is given by:

$$\frac{\gamma P_0}{\rho_0} = 100 V^2 = c_s^2 \quad (6)$$

where V is the characteristic or maximum fluid velocity. This ensures that the density variation is less than 1% and the flow can be regarded as incompressible.

Setting up the simulations

Simulations of dam-breakage were carried out using real topography, obtained from the US Geological Survey (USGS) website [11], shown in figure 1.

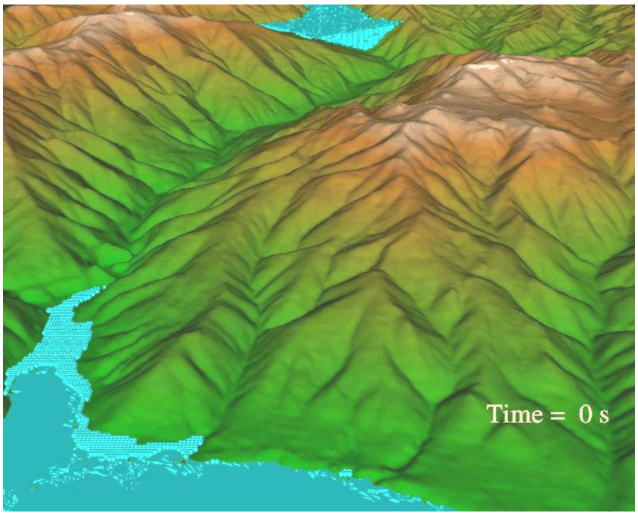
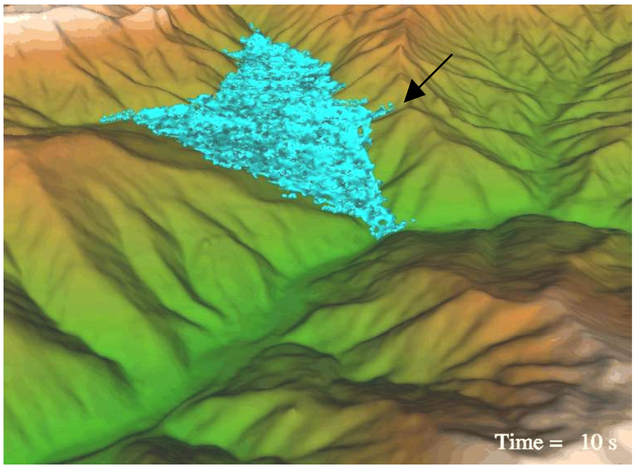


Figure 1. Digital elevation Model of Triunfo Pass supplied by the US Geological Survey. Sections coloured blue (or the lighter shade in black & white), situated in a mountain trough are fluid particles representing the water behind a dam wall. The sea (lower-left) was also incorporated into the simulation using additional fluid particles.

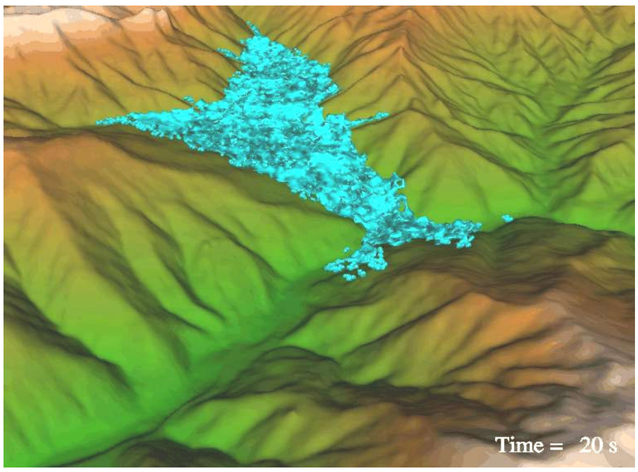
The topography used was that of the Triunfo Pass located in the state of California in the USA. The topographic data was in the form of a Digital Elevation Model (DEM). A DEM is a digital file consisting of terrain elevations for ground positions at regularly spaced horizontal intervals. This data was used to generate boundary particles with an interpolation length in the range of 10.0 m for the coarse resolution simulations. Fluid particles (the section coloured in blue or the lighter shade in black and white and situated in a mountain trough) were set up on a portion of the topography as shown in figure 1 to create the dam. The adjoining sea was also incorporated into the simulation using layers of additional fluid particles. The interpolation length of the fluid particles was fixed at 13.0 m. This ensured that the fluid particles would not penetrate the boundary in regions where the topography is too steep and the boundary particle interpolation length is significantly greater than 10.0 m. For the fine resolution simulations the interpolation lengths for the fluid and boundary particles were halved. The total number of fluid particles constituting the dam for the coarse and fine resolution simulations were around 20,000 and 160,000 respectively. The configurations generated were used as the starting conditions for the SPH simulations.

Results and Discussion

Figure 2 shows a series of frames of the water flowing through the valley after the dam-break event, starting from the initial stage shown in figure 1.



(a)



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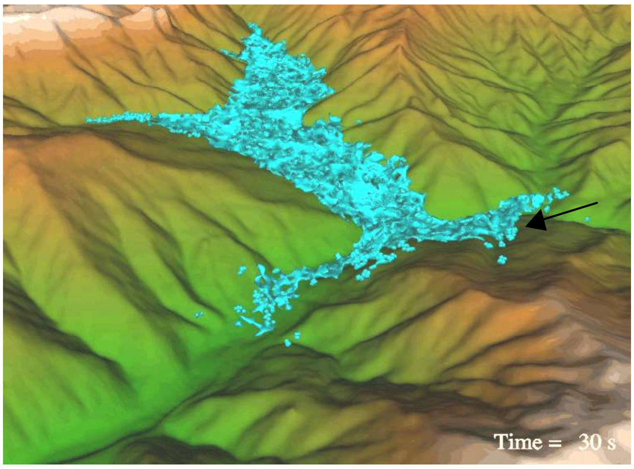
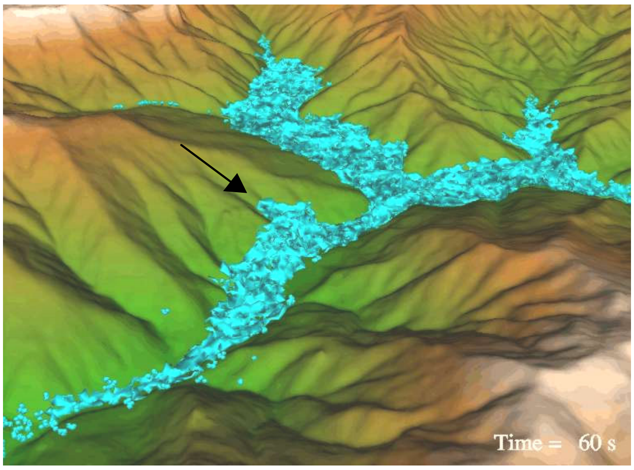


(c)

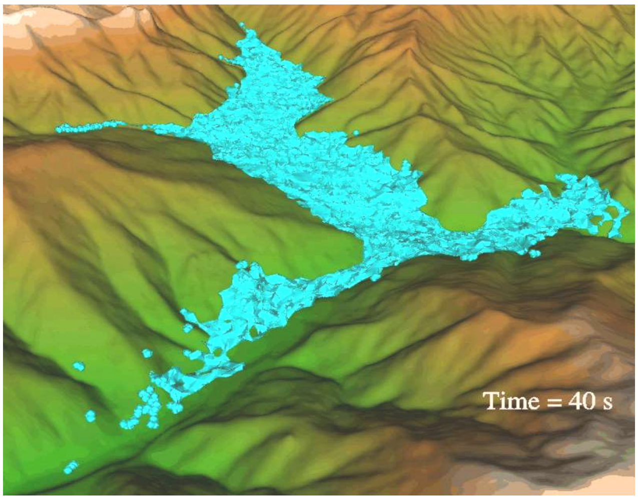


(d)

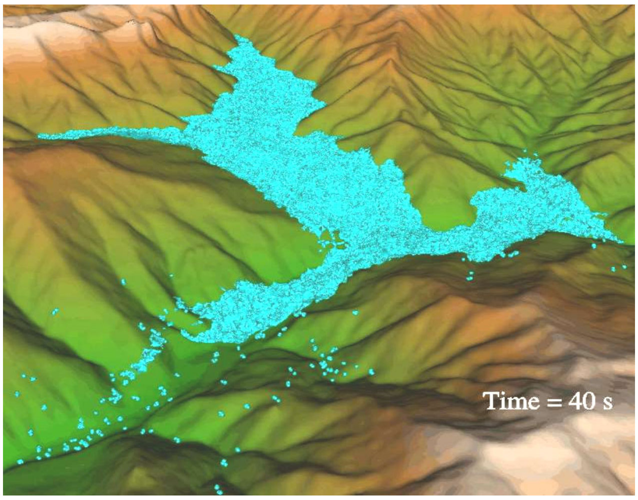
Figure 2. Water flowing through the valley after the dam-break event. Course resolution simulations are presented here. Arrows in (a), (c) and (d) indicate the positions of the water surging over ridges.

Water from the dam flows through the valley and into the sea under gravity. Immediately after the dam break at 10 s (figure 2a) the flow splits into two sections, one going into the valley upstream and the other downstream. Some water also tends to surge over shallow sections of the mountain as indicated by the arrow in figure 2a but is unable to move over the ridge. In figure 2b at 20 s the splitting of the flow into two sections is seen more clearly with distinctive but small surges of water occurring over ridges. In figure 2c at 30 s the flow is seen to track the valley between the mountains with more fluid flowing into the valley downstream. There is again a surge of fluid over a ridge indicated by the arrow in figure 2c. In figure 2d at 60 s the fluid has spread into a substantial section of the terrain. The fluid flowing upstream into the valley has also split into two sections. The flow upstream has substantially decelerated. The flow downstream into the sea has, however, accelerated due to the steepness of the terrain, with water surging over ridges (shown by the arrow in figure 2d) as it moves through the valley and into the sea. The splashing of the water and fluid break up are seen quite distinctively from these figures, indicating that SPH can effectively handle the free surface flow.

In order to check on the independence of the solution with resolution, the simulations were run at two different resolutions as mentioned before. As can be seen from figure 3a (coarse resolution) and figure 3b (fine resolution), the main features of the flow remain unchanged with a change in the resolution. However, with the finer resolution the details of the flow are resolved more clearly.



(a)



(b)

Figure 3. (a) Coarse, and (b) Fine resolution simulation results at t = 40 s to indicate the effect of change in resolution on the dam-break flow patterns.

Figure 4 compares the horizontal distance travelled by the fluid front with time for the two simulations. It demonstrates quantitatively that there is good agreement between the simulations. The variations from a linear behaviour are caused by changes in the slope of the valley floor.

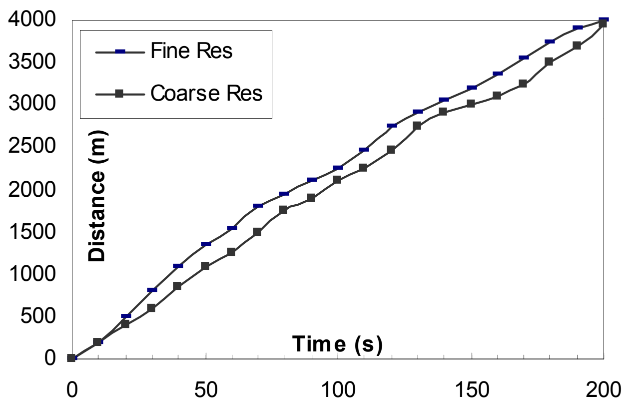
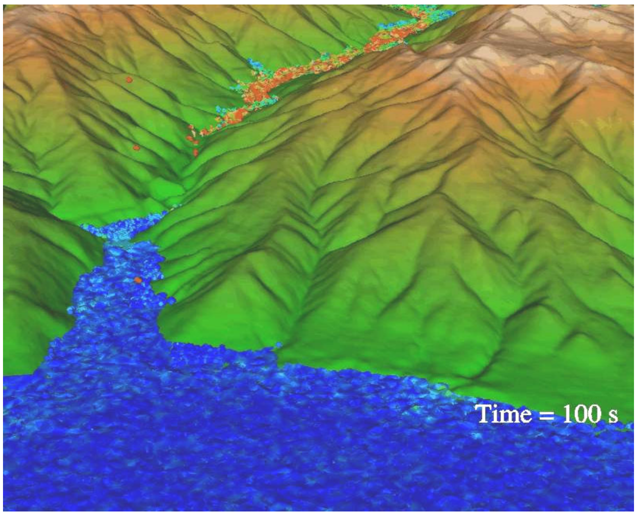
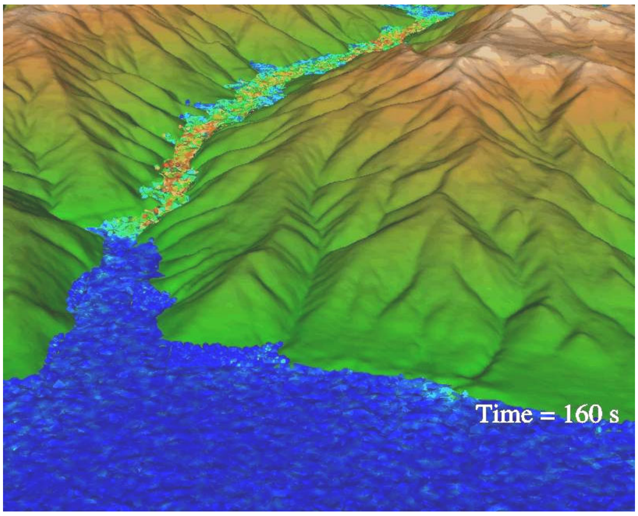


Figure 4. Comparison of horizontal distance travelled by the fluid front as it approaches the sea for the fine and coarse resolution simulations.



(a)



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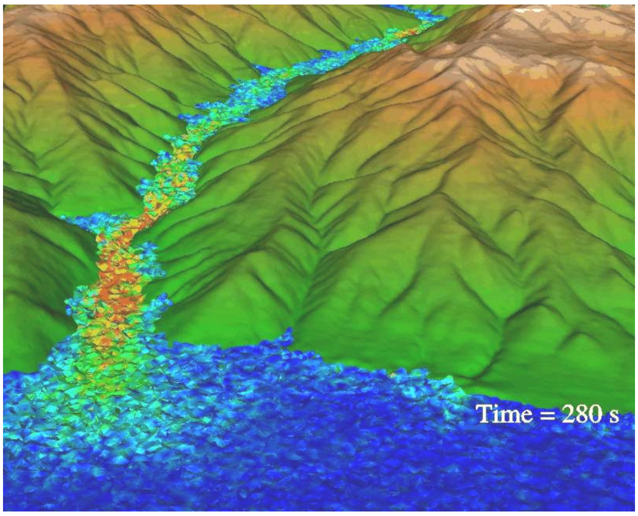

(c)

Figure 5. Flow pattern of the water from the dam at various times as it approaches and finally enters the sea. The fluid is coloured according to the speed of the flow. In black and white the lighter shade indicates that the fluid is flowing at a greater speed. Course resolution simulations are presented here.

Figure 5 shows the flow of dam water as it approaches and enters the sea. After the dam-break the water travels through the valley between the mountains as seen from all frames in figure 5. The speed of the flow changes depending on the steepness of the topography. In figure 5a at 100 s the fluid is travelling through the valley at a relatively higher velocity with the front moving at approximately 60 m/s and is yet to reach the sea. At 160 s in figure 5b the water has just entered the sea but has still not interacted much with the sea water. The speed of the water has decreased considerably to around 30 m/s except in regions close to where it meets the sea where the speeds are still around 60 m/s because of the steepness of the topography. Figure 5c at 280 s shows the mixing of the dam water with the sea water as it enters the sea (in black and white this is seen as the entry of the lighter coloured fluid into the almost black sea).

The coarse resolution simulation here takes approximately 80 hrs of CPU time for 300 s of simulation time, using a single 500 MHz processor on a COMPAQ ES40 and contains approximately 60,000 fluid particles including the dam and sea. Although we have not yet carried out any comparisons with measured data for such types of simulations, the above exercise gives reasonable levels of confidence that SPH can be used to solve real scale geophysical problems with relative ease. The water from the dam has a flow path through the valley and into the sea that seems to be intuitively correct. Thus, it might be expected that SPH simulations could be used to identify man-made structures that are likely to be destroyed in the event of a dam-break. Our next step is to use real data such as that from the Malpasset dam-break event [1] and compare the simulations with the measured data. Once an acceptable level of confidence is achieved, SPH can be used as a valuable tool to assess the effect of dam breakage on the surrounding topography.

Conclusions

The paper presents the use of SPH for simulating dam-breakage on a real three-dimensional topography. The results obtained from the simulations suggest that SPH can simulate the dam-breakage with interactions with the surrounding topography that seem intuitively correct and that these results are not particularly sensitive to simulation resolution. The next step in our study will use real data for comparison with SPH simulations. This will give additional confidence in establishing SPH as a useful tool in assessing the effect of dam breakage.

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