14th Australasian Fluid Mechanics Conference
Adelaide University, Adelaide, Australia
10-14 December 2001

Bifurcation of Neutrally Buoyant Jets in Stratified Environments

P. L. Morgan and S. W. Armfield

School of Aerospace, Mechanical and Mechatronic Engineering
Sydney University, New South Wales, 2006 AUSTRALIA

Abstract

In homogeneous planar jet's the existence of any bifurcation structure depends only on the jet Reynolds number Re. In the case of a planar jet entering a density stratified medium when the jet density is that of the ambient density at the jet entrance, the occurrence of a bifurcation will depend on both the jet Reynolds number and the degree of ambient stratification, which is parameterised by a Richardson number Ri. Direct numerical simulations have been carried out using a non-staggered fractional step scheme to investigate the behaviour of laminar incompressible neutrally buoyant horizontal jets entering a stratified environment. It was observed that at jet initialisation a jet intrusion front is formed with associated trailing vortex pairs. The intrusion front transits the domain and for low Reynolds number/low Richardson number the unsteadiness associated with the initialisation eventually decays and a steady state flow results. At a critical combination of Reynolds and Richardson numbers the jet becomes unstable and bifurcates to an unsteady flow. These results will be presented together with critical (Re, Ri) combinations showing the transition behaviour of the flow.

Introduction

Neutrally buoyant jets discharging into stratified environments can be readily seen in industry and in nature. In industrial and domestic environments a common application of neutrally buoyant jets is found in low speed flow devices such as heating and cooling systems. In nature cephalopods or squids employ this type of jet as a means of propulsion, where fluid is drawn in from its immediate surroundings and discharged resulting in the jet having essentially the same density as the ambient.

All jets exhibit evolving coherent structures, and their development stems from an initial perturbation, that of the fundamental frequency f_o [6, 4, 8, 12]. The energy content of this wave increases until completely saturated, at which point an energy transfer takes place and the first sub-harmonic begins to grow and ultimately becomes the new fundamental or dominant frequency [8]. In non-stratified environments the planar jets initial fundamental frequency is parameterised only by the Reynolds number. At a critical Reynolds number the jet may exhibit bifurcation and a characteristic mode of instability.

There has been extensive research compiled on flow characteristics and preferred modes of stability of two dimensional jets [10, 11, 1]. Sato [10] found experimentally that two kinds of instability occur when the laminar flow field becomes unstable, and that these instabilities were predominantly dependent on the jets exit velocity profile. A uniform jet exit velocity profile was found to be associated with symmetric/varicose mode instability, whilst a fully developed parabolic jet exit profile was found to be associated with asymmetric/sinuous mode instability. However observations of the near field planar homogeneous jet by Beavers and Wilson [3] showed that the flow consisted of periodic bifurcations which are predominantly symmetric for low Re with the instability sensitive to external perturbations. Rockwell and Niccolls [9] concluded that the growth rate of the instability was directly related to the flow initial conditions, and at high Re the flow cycles between varicose and sinuous instability modes. Hsiao and Ho [7] solved, in their investigations of plane jets in air, the quasi parallel Rayleigh equation. This semi analytic solution was compared with experimental results and showed that before the initial merging of the shear layers the varicose mode instability was dominant over the sinuous mode instability. However as the flow developed, the sinuous mode growth rate exceeded the varicose growth rate and dominated the flow downstream.

The two dimensional neutrally buoyant jet has flow characteristics that are similar to the homogeneous planar jet, and it is proposed that the existence of any bifurcation structure depends on the jet Reynolds number and the degree of ambient stratification, which is parameterised by the Richardson number. In the analysis that follows numerical simulations have been carried out for two aspect ratios, $AR = 21$ and 41 and results are presented showing the transition behaviour of the flow.

Numerical Method

Domain and Boundary Conditions

The computational domain is a rectangular box with a non-dimensional length $x = 20$ and height depending on the aspect ratio $y = 21, 41$, where all the lengths are non-dimensionalised by the jet inlet width H. The aspect ratio is defined to be the ratio of domain height to the jet inlet width and thus the domain height is equal to the aspect ratio, AR. The jet is located at $y = AR/2$ on the left hand boundary and has a parabolic velocity profile with a temperature distribution equal to that of the ambient fluid at that height, which has a stable linear stratification. The governing equations are therefore solved in the domain $0 \leq x \leq 20$, $0 \leq y \leq AR$, with the jet inlet at $x = 0$ and $(AR - H)/2 \leq y \leq (AR + H)/2$. The domain and boundary conditions for the x-velocity u, and the y-velocity v, and the temperature T are shown in Figure 1. Velocities are non-dimensionalised by the maximum jet inlet velocity, and temperature by ΔT, the variation of the initial ambient temperature stratification over the jet height H. At initiation the fluid is quiescent with $u = v = 0$ and $T(x, y) = y$. In the calculation the temperature is represented in terms of background and fluctuating components, that is $T = \overline{T} + \phi$, where $\overline{T}$ is the background temperature, which is a function of y only with $d\overline{T}/dy = 1.0$. The jet inlet temperature is

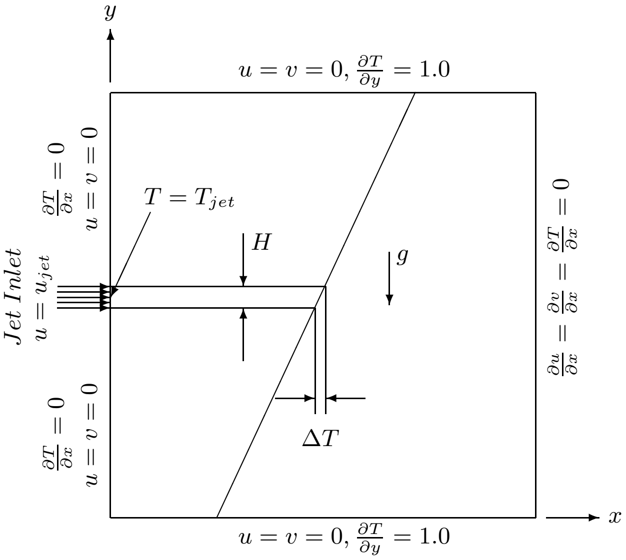

Figure 1: Geometry Configuration and Boundary Conditions

set equal to the background temperature at that height, and therefore the jet is stratified and neutrally buoyant with respect to $\overline{T}$ with $\phi = 0$ at $x = 0$ in the jet inlet region. On the rest of the $x = 0$ boundary and the other boundaries the normal gradient of ϕ is set to zero.

Governing Equations

The governing equations are the Navier-Stokes equations, which are written in incompressible and non-dimensional form, using the Boussinesq approximation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \quad (2)$$

$$\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + Ri\phi, \quad (3)$$

$$\frac{\partial \phi}{\partial \tau} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + v = \frac{1}{RePr}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right), \quad (4)$$

The Reynolds number $Re = \frac{V_0 H}{\nu}$, Richardson number $Ri = \frac{g\alpha H \Delta T}{V_0^2}$, and Prandtl number $Pr = \frac{\nu}{\kappa}$, with V_0, the dimensional maximum jet velocity, ν the kinematic viscosity, κ the thermal diffusivity and α the coefficient of thermal expansion.

Method of Solution

The governing equations are solved using a time accurate second-order fractional step Navier-Stokes solver, with the domain discretised on a non-staggered grid using finite volumes. When values of dependent variables are required at locations other than the nodes, linear interpolation is used. The viscous, pressure gradient and divergence terms are approximated using standard second-order centre differencing and the advection terms are discretised using the QUICK third-order up-wind scheme. Time integration is achieved using an explicit Adams-Bashforth scheme for the advection terms and an implicit Crank-Nicolson scheme for the diffusive terms.

Grid and Time Dependency Tests

The mesh used for this investigation consists of 195×186 nodes in the x and y directions respectively, with 33 nodes spanning the inlet. The smallest cell size in the x direction is set to $\Delta x = 3.8 \times 10^{-2}$, and smallest cell size in the y direction set to $\Delta y = 3.0 \times 10^{-2}$. Grid dependency tests were carried out on fine and coarse mesh configurations and their solutions compared with the solution obtained for the mesh outlined above. The fine mesh has a grid size of 257×276 nodes in the x and y directions respectively, and 36 nodes spanning the inlet. The smallest cell size in the x direction being $\Delta x = 2.7 \times 10^{-2}$, and the smallest cell size in the y direction being $\Delta y = 2.7 \times 10^{-2}$. The coarse mesh has a grid size of 44×115 nodes in the x and y directions respectively, and 13 nodes spanning the inlet. The smallest cell size in the x direction being $\Delta x = 3.0 \times 10^{-1}$, and the smallest cell size in the y direction being $\Delta y = 1.0 \times 10^{-1}$. Three time steps were tested and compared for all mesh configurations, $\Delta\tau = 5.0 \times 10^{-2}$, $\Delta\tau = 1.0 \times 10^{-2}$ and $\Delta\tau = 1.0 \times 10^{-3}$ with $\Delta\tau = 1.0 \times 10^{-2}$ used for this investigation.

Results and Discussion

At jet initialisation, $t = 0$ the fluid impulsively enters a density stratified quiescent ambient, with the density of the jet equal to that of the ambient at the jet entrance. An intrusion front is formed and transits the domain. The perturbations generated by the jets initialisation also progress in the streamwise direction and eventually decay resulting in a steady state flow. At a critical Re,Ri combination the unsteadiness associated with these perturbations amplifies and the jet becomes unstable and begins to bifurcate. The detailed jet flow temporal behaviour near the critical Re for $Ri = 2.5 \times 10^{-5}$, and $AR = 41$, showing the transformation from symmetric to asymmetric flow, is presented in Figure 2, where ϕ contours are plotted. The jet has entered the domain at $t = 50$, with the intrusion front entraining fluid from the ambient. The next four plots for $t = 100, 150, 200, 250$, show the jet intrusion front transiting and ultimately leaving the domain. At $t = 500$, the flow begins an unsteady sinusoidal motion. The oscillation steadily amplifies at $t = 600, 700, 800, 900$, resulting in a large amplitude sinuous mode bifurcation, a characteristic also observed in experimental investigations of two dimensional planar jets by Sato [10, 11], Batchelor [1] and others. To obtain critical Re,Ri combinations for the neutrally buoyant jet a series of numerical simulations have been conducted. In each set of simulations a critical Re was found for a particular Ri, which indicated the point of initial unsteady transition, by initially obtaining a low Re steady solution and then obtaining solutions with Re increased by 10 each time until an unsteady fully developed flow was observed. The results for $AR = 21$ with $60 \leq Re \leq 140$ and $1 \leq Ri \leq 1.0 \times 10^{-6}$ are shown in Figure 3. Two sets of results are presented here. Those shown as open circles represent solutions in which an unsteady non-periodic solution was obtained. In these cases the flow would exhibit an amplifying sinusoidal motion for a period of time, but then the jet would attach to the upper or lower boundary for a short time, detach and exhibit another period of amplifying sinusoidal motion until again attaching to the upper or lower boundary. For these cases it was not possible to obtain a Reynolds number for which a small amplitude periodic sinusoidal flow existed without periodic boundary attachment. The

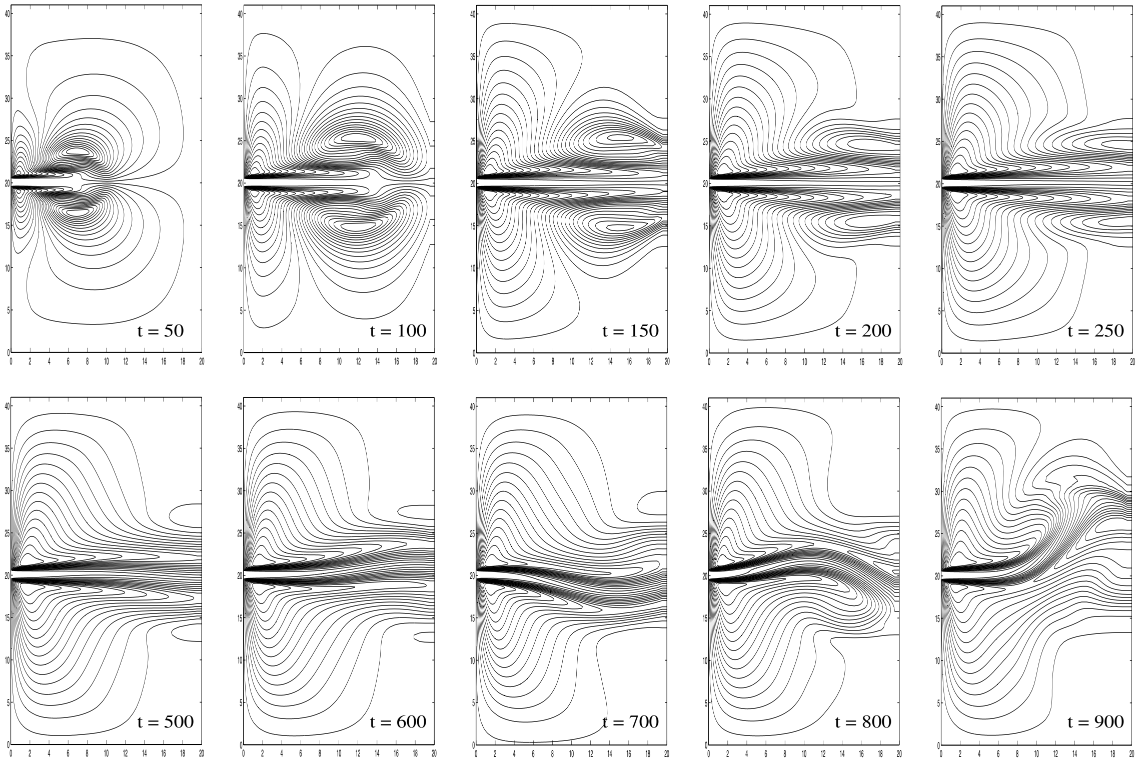

Figure 2: ϕ contours for $Re = 30$, $Ri = 2.5 \times 10^{-5}$ and $AR = 41$. The contour step is $\Delta\phi = 0.75$.

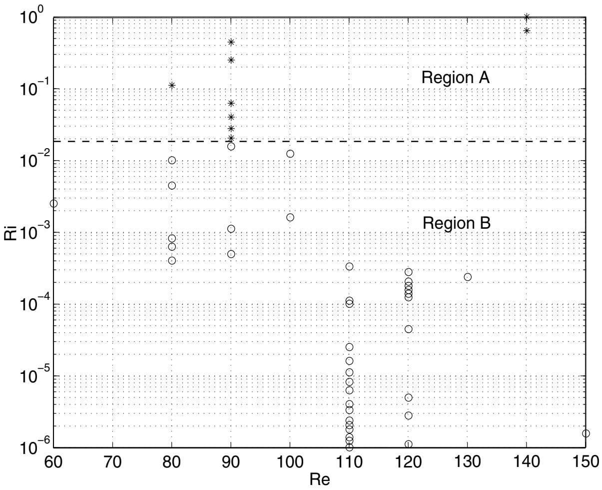

Figure 3: Critical solutions for aspect ratio, $AR = 21$ for various Re,Ri combinations.

results shown as a star represent solutions for which a periodic sinusoidal motion was obtained without a boundary attachment. In Figure 3 the two data sets are seen to lie in separate regions. The periodic bifurcation results exist for $1 \leq Ri \leq 2.3668 \times 10^{-2}$, which is labelled region A, while the results where attachment occurs exist for $2.0408 \times 10^{-2} \leq Ri \leq 1.0 \times 10^{-6}$, which is labelled region B. The region A results show critical Reynolds numbers of 80 and 90 for all except the two largest Richardson numbers. It would appear that the critical Reynolds number is relatively insensitive to Ri for a range of Ri, then shifts to almost twice the value as Ri increases. This may indicate a resonant effect, although more results would be required to verify this. It is difficult to see any structure in the region B results, although generally higher Reynolds numbers are obtained for smaller Ri. The $Ri = 0$ result, for which there is no stratification, has a critical $Re \sim 20$ [11], and thus the trend of increasing Re with decreasing Ri must reverse at some stage, however no such effect is seen here. The streamwise velocity time series of two Re,Ri combinations affected by wall attachment, and one Re,Ri combination not affected are shown in Figure 4 for $AR = 21$. The unsteady development of the $Re = 110$, $Ri = 4.0 \times 10^{-6}$ and $Re = 110$, $Ri = 1.234 \times 10^{-6}$ combinations show the wall influence on the jet, with the large velocity peaks and troughs indicating the time at which the jet attaches and detaches from the wall. It can also be seen in the figure that the $Re = 90$, $Ri = 4.0 \times 10^{-2}$ combination show only periodic flow. Results have also been obtained for $AR = 41$ where again a periodic sinusoidal flow is observed only for $1 \leq Ri \leq 2.3668 \times 10^{-2}$ with similar Wall attachment effects observed for lower Ri values. Critical results for $AR = 21$ and 41 for the periodic sinusoidal flow are are given in Figure 5, and show that for a given Ri higher critical Re values are obtained for $AR = 41$. Linear regression of each data set have been used to obtain the trend lines with the following best fit equations,

$$Ri = 0.0131Re - 1.0193, \quad (AR = 21)\,, \tag{5}$$

$$Ri = 6.8450 \times 10^{-4}Re - 0.0180, \quad (AR = 41)\,. \tag{6}$$

Analysis of the residuals show no pattern or trends and indicate that the above equations are appropriate fits to the data.

Conclusions

The impulsively started jet has an intrusion front which crosses the domain. At low Re and low Ri, unsteadiness associated with the initial conditions decays and the flow

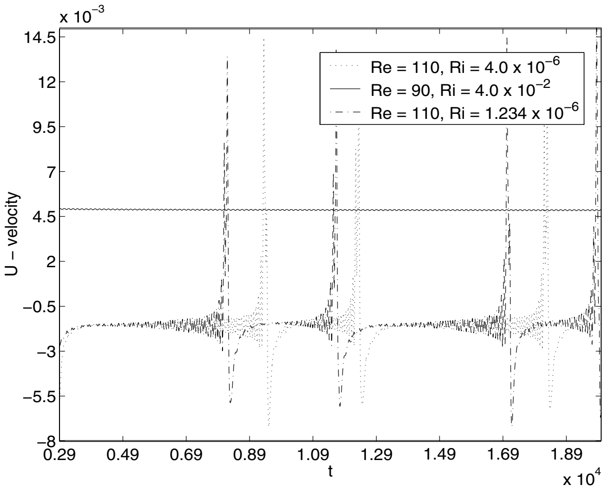

Figure 4: U-velocity time series at $x = 20, y = AR/2$ with and without wall attachment effects for aspect ratio, $AR = 21$.

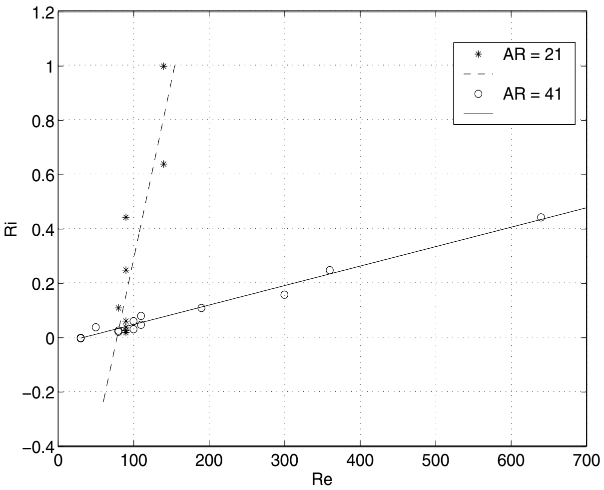

Figure 5: Critical values for periodic sinusoidal flow for two different aspect ratios, $AR = 21, 41$ for $1 \leq Ri \leq 2.3668 \times 10^{-2}$, Re combinations

becomes steady. At a critical combination of Re,Ri in the range $1 \leq Ri \leq 2.3668 \times 10^{-2}$, the jet becomes unstable and bifurcates to a periodic sinusoidal flow, for the two aspect ratios tested. For Re,Ri in the range $2.0408 \times 10^{-2} \leq Ri \leq 1.0 \times 10^{-6}$ periodic sinusoidal flow is not obtained for any Re. Rather the flow transits directly from steady state to one in which the jet periodically attaches to the upper and lower boundary. It is clear that for the lower AR the flow is influenced by the presence of the upper and lower boundaries for the full range of Ri considered. For the larger Ri the jet is seen to periodically attach to the upper or lower boundary. For the smaller Ri the jet does not come into contact with the horizontal boundaries, however shifting to a larger AR changed the critical Reynolds numbers, demonstrating that the aspect ratio is still influencing the flow. At the larger AR the wall attachment effect is still present for large Ri. Additional results at a larger AR still are required to determine if the flow remains dependent on the aspect ratio.

The wall attachment is a complex process. The presence of the wall acts to produce an asymmetric pressure field with a lower pressure adjacent to the jet on the side nearest the wall. This apparently acts to amplify the jet oscillation, while occasionally capturing the jet momentarily when the oscillations become large enough. Further investigation is required to parameterise the wall effect and to determine large enough aspect ratios to make the effect negligible.

Acknowledgements

This project was supported by the Australian Research Council through grant No. A00104152.

References

[1] Batchelor G. K. and Gill A. E. Analysis of the stability of axisymmetric jets. *J. Fluid Mech.*(1962) **14**, 529–551.

[2] Battaglia F., Tavener S. J., Kulkarni A. K. and Merkle C. L. Bifurcation of low Reynolds number flows in symmetric channels. *AIAA Journal*(1997) **35**, 99–105.

[3] Beavers G. S. and Wilson T. A. Vortex growth in jets *J. Fluid Mech.*(1970) **44**, 97–112.

[4] Cohen J.and Wygnanski I. The evolution of instabilities in the axisymmetric jet. Part 1. The linear growth of disturbances near the nozzle *J. Fluid Mech.*(1987) **176**, 191–219.

[5] Fearn R. M., Mullin T. and Cliffe K. A. non-linear flow phenomena in a symmetric sudden expansion. *J. Fluid Mech.*(1990) **211**, 595–608.

[6] Ho C. and Huerre P. Perturbed Free Shear Layers. *Ann. Rev. Fluid Mech.*(1984) **16**, 365–424.

[7] Hsiao F. and Ho C. The sinuous mode and the varicose mode in a 2-D jet *Bull. Am. Phys. Soc.*(1984) **29**, 1519.

[8] Hsiao F. and Huang J. On the evolution of instabilities in the near field of a plane jet *Phys. Fluids A*(1989) **2**, 400–412.

[9] Rockwell D. O. and Niccolls W. O. Natural Breakdown of planar jets *Trans. ASME J. Basic Eng*(1972) **94**, 720–728.

[10] Sato H. The stability and transition of a two-dimentional jet. *J. Fluid Mech.*(1960) **1**, 53–80.

[11] Sato H.and Sakao F. An experimental investigation of the instability of a two-dimensional jet at low Reynolds numbers. *J. Fluid Mech.*(1964) **20**, 337–352.

[12] Thomas F. O. and Prakash K. M. K. An experimental investigation of the natural transition of an untuned planar jet. *Phys. Fluids A*(1991) **3**, 90–105.