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Optimization of Pipelines Containing Fluid against Divergence

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Abstract

An exact method is presented for determining the optimal design of fluid-containing pipelines to enhance their resistance against divergence instability. The mathematical procedure uses the transmission matrix technique along with the method of Newton – Raphson to solve the associated eigenvalue problem. Calculations are carried out for thin-walled tubes consisting of uniform modules having different length and cross-sectional properties. Design variables include the mean diameter, wall thickness and length of each module. The model accounts for an elastically supported pipeline in order to cover a wide range of boundary conditions. Numerical examples demonstrate the efficiency and effectiveness of the model in arriving at global optimal solutions.

Introduction

A large number of publications dealing with the eigenvalue optimization problems can be found in the literature where several computational approaches have been developed and applied. Related topics cover both frequency and buckling optimization [4,6,8] using either calculus of variation methods for continuous models or mathematical programming techniques for discrete finite-element models. Such problems are usually formulated by finding the minimum weight that satisfies prescribed eigenvalues, or alternatively by maximizing the fundamental eigenvalue for a given structural weight. Limited research may be found that deals with maximization of the critical flow velocity in a pipeline. Borglund [1] formulated the minimal mass design problem of a cantilevered pipeline for a fixed critical flow speed. Analysis was performed using the finite element method to solve the associated equations of motion. No attempt was made to maximize the critical flow speed for a given structural mass. Sällström [5] maximized the imaginary part of the fundamental frequency of bending vibration of a cantilevered uniform beam conveying fluid. The fluid velocity was kept constant and design variables included the location and values of lumped masses, springs, or dampers connected to the beam. Tanaka et al. [7] employed variational principles combined with finite elements to maximize the critical flow velocity through a cantilevered pipeline with given structural mass. The pipe inner diameter was kept constant, while the wall thickness distribution was determined through the optimization process. The present paper deals with the maximization of the critical flow velocity through an elastically supported non-uniform pipeline for a prescribed total mass. The effect of the pipe inner diameter on the overall stability, which has not received any attention in previous publications, is dealt with herein. To avoid the highly nonlinear shapes of continuous models, which can be difficult to fabricate and produce economically, a multi-module pipe model is optimized with the effective design variables chosen to be the mean diameter, wall thickness and length of each uniform module. The exact divergence speed is determined using the transfer matrix technique [3] and solving the associated eigenvalue problem for known boundary conditions. Appropriate non-dimensionalization of the various parameters and variables has led to a naturally scaled optimization model. As a case study, the developed model is applied to a simply supported pipeline consisting of two, three and more modules. Extensive computer experimentation has shown that the critical flow speed is well behaved and continuous in the selected design space. Global optimality has been attained showing significant improvement in the overall fluid-structure stability as compared with a baseline design.

Mathematical Formulation

Basic Assumptions

1- Fluid is incompressible and flow is steady and fully developed laminar. Variation in the velocity across the pipe cross section is ignored.
2- Effects of structural damping, damping of surroundings and gravity are not considered.
3- Thin-walled rounded tubular slender pipes are only considered so that the classical beam theory can be applicable.

Governing Differential Equation

Païdoussis and Issid [2] introduced the basic governing differential equations of a fluid-flowing pipe. For the case of static instability, the governing equation takes the form

$$(EIw'')'' + m_f\, u(u'w' + uw'') = 0 \qquad (1)$$

where E = the modulus of elasticity, I = area second moment of inertia, w = bending displacement, m_f = fluid mass per unit length and u = flow velocity. The notation $(\,)'$ means derivative with respect to the axial coordinate x. It must be mentioned here that the problem of determining the critical flow velocity in a pipeline cannot be and never be fully similar to that of the column's buckling problem. Some investigators in the field believe in full similarity, which cannot be true. The distribution of the shearing force is not the same in both problems. Furthermore, the axial flow velocity in a non-uniform pipe is not constant lengthwise, whereas in the case of column's buckling the axial force is constant along the entire length.

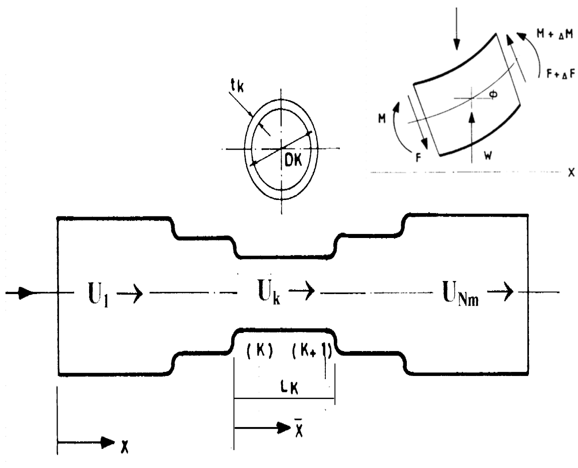

Figure 1. Multi-module pipeline model and free body diagram of an element dx.

Figure 1 shows a general discretized pipeline model composed of N_m-uniform modules, each of which may have different length and cross-sectional properties. Setting $u' = 0$, and substituting for $m_f = \rho_f A_k$ in equation 1, the governing differential equation for the Kth uniform module reduces to

$$E I_k w'''' + \rho_f A_k u_k^2 w'' = 0 \qquad (2)$$

It is convenient to deal with dimensionless quantities so that the analysis can be valid for any arbitrary pipeline configuration. The various parameters are non-dimensionalized by their corresponding values of a reference uniform pipe having the same total length, material and fluid properties. Referring to Table 1, it is noted that the same symbols that define the actual parameters are reused to define their corresponding dimensionless quantities in order to avoid having many subscripts and symbols in the derived equations. For example, the notation $w \leftarrow w/L$ means that the dimensionless deflection is equal to its dimensional value divided by the total pipe length. Therefore, dividing by EI_k/L^3, Equation 2 takes the following dimensionless form

$$w'''' + \lambda_k^2 w'' = 0, \quad \lambda_k = u_k \sqrt{\frac{A_k}{I_k}} = \frac{u A_{max}}{\sqrt{A_k I_k}}, \; k = 1,2,..,N_m \qquad (3)$$

where u is the critical flow velocity and A_{max} the maximum cross-sectional area of the modules.

Quantity	Notation	Non-dimensionalization
Axial coordinate	x	$x \leftarrow x/L$
Module length	L_k	$L_k \leftarrow L_k/L$
Wall thickness	t_k	$t_k \leftarrow t_k/t$
Mean diameter	D_k	$D_k \leftarrow D_k/D$
Cross-sectional area	A_k	$A_k \leftarrow A_k/A$ $(=D_k^2)$
2nd moment of Inertia	I_k	$I_k \leftarrow I_k/I$
Bending deflection	w	$w \leftarrow w/L$
Bending moment	M	$M \leftarrow M^*(L/EI)$
Shearing force	F	$F \leftarrow F^*(L^2/EI)$
Rotational spring	K_φ	$K_\varphi \leftarrow K_\varphi^*(L/EI)$
Transversal spring	K_w	$K_w \leftarrow K_w^*(L^3/EI)$
Flow velocity	u_k	$u_k \leftarrow u_k^*(\rho_f A L^2/EI)^{1/2}$
Structural mass	M_s	$M_s \leftarrow M_s/M_r$ $(=\sum D_k t_k L_k)$

Table 1. Definition of dimensionless quantities. Reference pipe has the following uniform properties: area $A=\pi D^2/4$, inertia $I \cong \pi D^3 t/8$, mass $M_r=\rho_p \pi D t L$, where ρ_p is the pipe mass density and L total length.

Equation 3 has the exact solution

$$w(\bar{x}) = B_1 + B_2 \bar{x} + B_3 \sin \lambda_k \bar{x} + B_4 \cos \lambda_k \bar{x} \qquad (4)$$

The constants B_i are determined by applying the appropriate boundary conditions.

Analysis by Transmission Matrix Method

The exact critical flow velocity of a multi-module pipeline model can be best obtained by applying the transmission matrix technique [3] and solving the associated eigenvalue problem. The state vector, $\underline{Z}_k$, at any joint (k) within the pipeline is defined as follows

$$\underline{Z}_k^T = [\, w \;\; \varphi \;\; M \;\; F \,]_k = [\, w \;\; -w' \;\; -Iw'' \;\; -Iw''' \,]_k \qquad (5)$$

At two successive joints (k) and (k+1) the state vectors are related to each other by the matrix equation

$$\underline{Z}_{k+1} = [T_r]_k \, \underline{Z}_k \qquad (6)$$

Where $[T_r]_k$ is a square matrix of order 4x4 known as the transmission or transfer matrix of the kth pipe module. Its individual elements can be obtained by first expressing the coefficients B_i in terms of the state variables at joint (k), and then expressing the state variables at joint (k+1) in terms of those at joint (k). Defining $C_k = \cos \lambda_k L_k$ and $S_k = \sin \lambda_k L_k$, the final derived form of the transmission matrix is:

$$[T_r]_k = \begin{bmatrix} 1 & -L_k & (C_k - 1)/I_k \lambda_k^2 & (\frac{S_k}{\lambda_k} - L_k)/I_k \lambda_k^2 \\ 0 & 1 & S_k / I_k \lambda_k & (1 - C_k)/I_k \lambda_k^2 \\ 0 & 0 & C_k & S_k / \lambda_k \\ 0 & 0 & -\lambda_k S_k & C_k \end{bmatrix} \qquad (7)$$

For a pipeline made of N_m-modules, Equation (6) can be applied at successive joints to obtain

$$\underline{Z}_{Nm+1} = [T] \underline{Z}_1 \qquad (8)$$

Where [T] is called the overall transmission matrix found by taking the products of all the intermediate matrices of the individual modules. Therefore, applying the boundary conditions and considering only the non-trivial solution, the resulting characteristic equation can be solved for the critical flow velocity.

Boundary Conditions

In order to make the analysis valid for variety of boundary conditions, the pipeline is considered to be elastically supported at both ends. Therefore, considering shear and moment balances, one gets:

at x=0: $w = -(I/K_w)\, w'''$, $w' = (I/K_\varphi) w''$ (9.1)

at x=1: $w = (I/K_w)\, w'''$, $w' = -(I/K_\varphi) w''$ (9.2)

For the common types of boundary conditions, Table 2 gives the final form of the characteristic equation for determining the critical divergence speed.

Type of Boundary Conditions	Characteristic Equation	Reference Value of u (One Module)
Pinned - Pinned	$T_{12}T_{34} - T_{14}T_{32} = 0$	3.14159
Clamped - Pinned	$T_{13}T_{34} - T_{14}T_{33} = 0$	4.49336
Clamped - Clamped	$T_{13}T_{24} - T_{14}T_{23} = 0$	6.28319

Table 2. Divergence-characteristic equation for common types of boundary conditions.

Note that divergence instability is not possible for a cantilevered pipeline, where the non-trivial solution of the associated characteristic equation results in a vanishing bending displacement over the entire span of the pipeline. For such a configuration, only dynamic instability (flutter) can be considered. This is now under study by the authors, and will be hopefully published in the near future.

Application and Computational Results

As a basic case of study, we consider first a simply supported uniform pipeline consisting of one module. Referring to Table 2, the associated characteristic equation takes the form $\sin\lambda_1 = 0$, which results in the non-trivial solution for the divergence speed:

$$u = \pi\sqrt{D_1 t_1} \qquad (10)$$

It is obvious that there is no way to increase u above its reference value π without the penalty of increasing the structural mass ($M_s = D_1 t_1$). We consider next, pinned-pinned configurations consisting of several modules to see how the critical velocity can be changed with the selected design variables.

Pipelines made of Two Modules

For a pinned-pinned pipeline composed of two modules, the corresponding transcendental equation for calculating divergence speed reduces to the following compact form (see Table 2):

$$C_1 S_2 \lambda_1 + S_1 C_2 \lambda_2 = 0 \qquad (11)$$

Extensive computer solutions for the above equation have shown that the computed values of the divergence speed, u, can be repeated in spite of the wide variation in the chosen design variables $(D, t, L)_K$, k=1,2. This proves the existence of the velocity level curves in the selected design space. Figure 2 shows the developed star-like level curves in the (D_1-D_2) design space for a two module model having uniform thickness of unity with the span divided into two equal portions. Contours of the structural mass are also indicated.

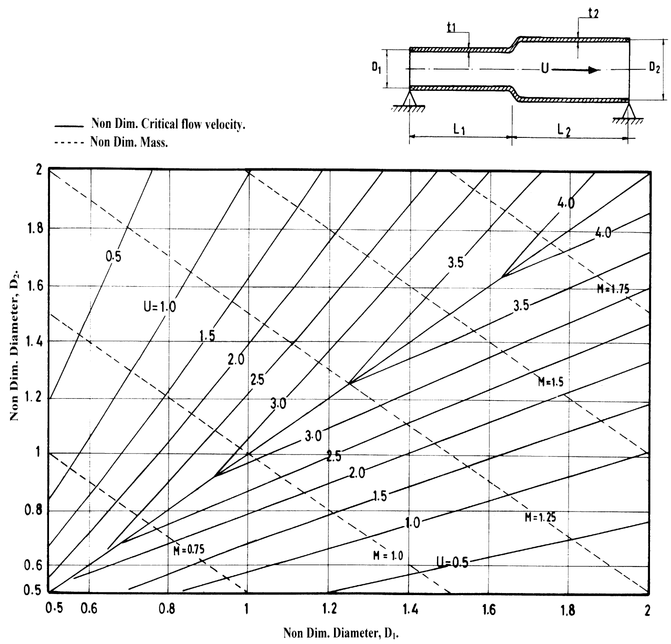

Figure 2. Divergence speed and mass level curves for a two module pinned-pinned pipeline ($t_1 = t_2 = 1$ & $L_1 = L_2 = 0.5$).

It is seen that the diameters of the individual modules affect significantly the behavior of the overall stability of the system, a factor that has not been considered in previous publications. Several practical applications in industry utilize pipelines with different module diameters. Since cost is directly proportional to structural mass, the level curves of the divergence speed have been generated for the case of unit dimensionless mass (i.e. the optimized pipe has the same mass as that of the reference design). Figure 3 depicts the optimum zone for a constant-diameter two-module pipeline model. The absolute maximum value of the divergence speed is seen to be very close to 3.238, which corresponds to the design point $(t,L)_K$=(.39, .135), (1.095, .865).

Pipelines Built of More Than Two Modules

Several other case of studies, including the optimization of three, four, and more modules, have been implemented and investigated in detail. For pinned-pinned and clamped-clamped

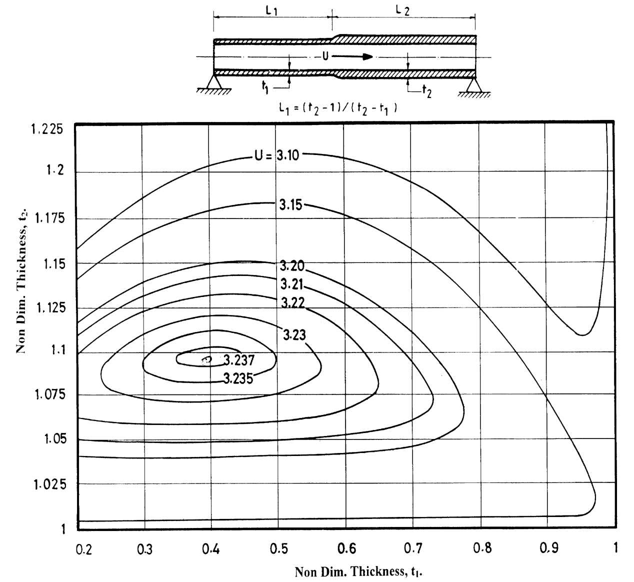

Figure 3. Behavior of divergence speed for simply supported two-module pipeline with constant diameter.

pipelines, the obtained results have indicated that optimum patterns must be symmetrical about the mid-span point. When considering starting designs with even number of modules, it was found that the optimization process discarded one of the modules by letting its length sink to zero, or sometimes, by making two consecutive modules have the same diameter and wall thickness (i.e. reduced to one module). Therefore, it may be easier to cope with symmetrical configurations, which reduce the computational efforts significantly by only considering half of the design variables. For example, in the case of pinned-pinned pipeline the boundary conditions become:

$$\text{at } x=0 \qquad w = w'' = 0 \qquad (12.1)$$

$$\text{at } x=1/2 \qquad w' = w'' = 0 \qquad (12.2)$$

The associated characteristic equation takes the form:

$$T_{22}T_{44} - T_{24}T_{42} = 0 \qquad (13)$$

For a symmetrical three-module pipeline, The compacted form of Equation 13 is

$$\lambda_1^2 C_1^2 S_2 + 2\lambda_1\lambda_2 S_1 C_1 C_2 - \lambda_2^2 S_1^2 S_2 = 0 \qquad (14)$$

The developed isomert curves for patterns with constant diameters and unit dimensionless mass are depicted in Figure 4. The final optimum results for different number of modules are summarized in Table 3. It is important to mention here that the attained optimum configurations depend, to some extent, on the prescribed lower limits imposed on the pipe wall thickness. Such limits are usually related to considerations of local instability that might be caused by buckling.

N_m	Optimum [$(t, L)_K$]	u_{max}	Gain%
3	$[(0.45, 0.15625), (1.25, 0.34375)]_s$	3.3590	6.9
5	$[(0.2500, 0.075), (0.75, 0.15) (1.3409, 0.275)]_s$	3.4121	8.6
7	$[(0.15, 0.050), (0.50, 0.075), (0.90, 0.125), (1.37, 0.250)]_s$	3.4332	9.3

Table 3. Optimum patterns of simply supported pipelines with different number of modules. The subscript (s) denotes symmetry about the mid-span point.

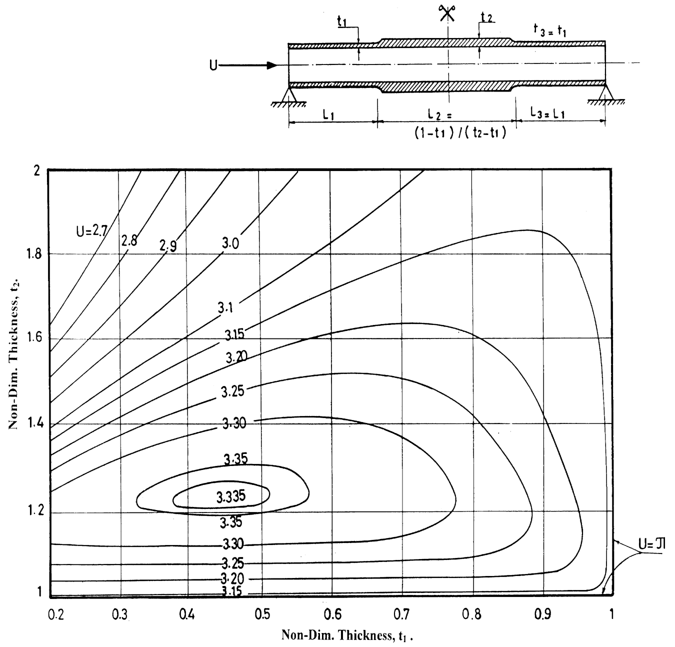

Figure 4. Optimum zone for a symmetrical three-module pipeline.

Conclusions

As a major stability criterion for the design of flexible pipes conveying fluid, the divergence speed is maximized for given total length and structural mass. To avoid the highly complicated nonlinear shapes of continuous structural models, which can be difficult to fabricate and produce economically, a multi-module pipeline model is optimized with the design variables selected to be the mean diameter, wall thickness and length of each module. Based on the fact that an exact solution for a uniform pipe is available and well established, the exact critical flow velocity is determined using the transmission matrix technique and solving the associated eigenvalue problem for known boundary conditions. The number of modules does not affect the accuracy of the resulting solutions. The present analysis leads to the exact divergence speed no matter the number of modules is. This can ensure the exact determination of the static stability boundary. Non-dimensionalization of the various parameters has eliminated the need for scaling design variables as usually suggested by similar optimization procedures. Extensive computer analysis of a pinned-pinned pipeline model has proved that the divergence speed, even though implicit function in the design variables, is well behaved, monotonic and defined everywhere in the selected design space. The study has also shown that the critical velocity is very sensitive to variation in the module's length. Investigators who use the finite element method and consider only cross-sectional properties as main design variables always miss this variable. Finally, the proposed model has succeeded in arriving at global optimality, showing significant improvements in the overall fluid-structure stability as compared with a baseline design. Future study shall consider the effect of support flexibility on the attained optimum designs as well as flutter optimization of similar pipeline configurations.

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