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Theoretical and Experimental Modelling of Thermal Erosion by Laminar Lava Flows

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Abstract

Thermal erosion by hot lavas during their laminar flow over cold ground is investigated both theoretically and experimentally. An analysis of the steady forced convective heat transfer by a laminar channel flow at large Péclet numbers is presented, which is used to determine the final steady-state erosion velocity as well as the thicknesses and timescales of the associated thermal boundary layers in both the lava and ground. The initial transient period involving the growth and remelting of a basal chill layer is also quantified. Laboratory experiments are described in which hot, molten wax flowed over and thermally eroded an underlying bed of solidified wax. The experimental observations of chill formation and thermal erosion are found to be in excellent agreement with the theoretical predictions. The theory also predicts rates of thermal erosion consistent with those observed in recent flows of basaltic lava in Hawaii.

Introduction

Over the past thirty years, there has been a continuing interest in understanding the circumstances under which lavas can thermally erode the ground that they flow over. This work has been motivated both by observations of subaerial and submarine lava flows on Earth, and by the desire to explain the extraterrestrial development of lunar sinuous rilles, Martian channels, Venusian canali, and lava flows on Io.

The first quantitative study of thermal erosion was made by Hulme [3], who determined the rate of thermal erosion by a turbulent lava flow using an empirical formulation for heat transfer in a pipe. This work was applied to lunar sinuous rilles, and it has since been extended to describe the submarine flow of komatiite lavas, and the formation of major nickel deposits, in the Archaean [5].

Numerous field observations have shown that thermal erosion also occurs in laminar lava flows (e.g. [7]; [11]). To date, there have been two attempts to model thermal erosion by such laminar flows. The first attempt was a numerical model by Carr [2]. However, this model assumed the interfacial boundary condition that, when any ground material reaches the melting temperature, it is replaced by the interior lava temperature. This boundary condition therefore ignores the thermal boundary layer at the base of the flowing lava, and hence it cannot correctly predict the heat flux from the lava to the ground. Indeed, the boundary condition assumed by Carr effectively imposes an infinitely fast heat transfer in the lava, and it should result in infinitely fast thermal erosion. The finite erosion rates obtained by Carr must merely be an artifact of the arbitrary grid spacings and timesteps used in his numerical scheme.

The second attempt to quantify thermal erosion by laminar lava flows was by Hulme [4]. In his theoretical model, he assumed that the lava flowed at a uniform velocity u, and then estimated the thickness of the thermal boundary layer as $h_l \approx (4\kappa x/u)^{1/2}$, where κ is the thermal diffusivity of the lava and x is the distance downstream. However, this model ignores the hydrodynamic boundary condition of no-slip (i.e. that the lava velocity is zero next to the ground). The model therefore underestimates the thickness of the thermal boundary layer, and overestimates the heat flux to the ground and the erosion rate.

In this paper, I summarize a recent theoretical and experimental study that quantifies thermal erosion in laminar lava flows [8]. I first outline the fundamental fluid dynamics that determines the final steady-state forced convective heat transfer due the laminar flow of a hot, viscous fluid over a cold solid. This theory yields expressions for the rate of thermal erosion of the solid, and for the thicknesses of the associated thermal boundary layers in both the fluid and the solid. I also examine the initial transient behavior in which a basal chill layer is formed and then remelted at the base of the flow. The theoretical results are then compared with two sets of laboratory experiments that explore thermal erosion in wax flows, and excellent agreement is obtained. The theory is then applied to provide a quantitative understanding of the thermal erosion seen in basaltic lava channels.

Theory

Consider the steady lava flow shown in figure 1. The lava has a density ρ, viscosity μ, kinematic viscosity ν, thermal diffusivity κ and a uniform initial temperature T_l at the origin ($x = 0$). For simplicity, the flow is taken to be two dimensional, which is reasonable if the lava channel width w is much greater than the flow depth d. Laminar flow down a slope α has a parabolic velocity profile $u(z) = Uz(2d - z)/d^2$, where the surface velocity $U = g\rho \sin\alpha\, d^2/2\mu$, and the flow rate per unit width Q is

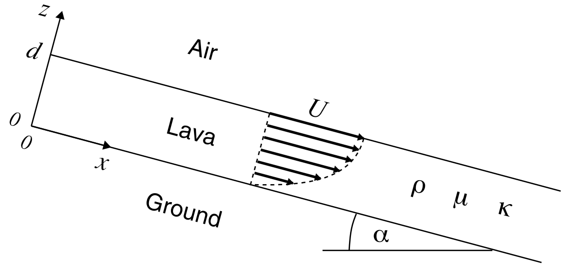

Figure 1: Schematic diagram of a laminar lava flow.

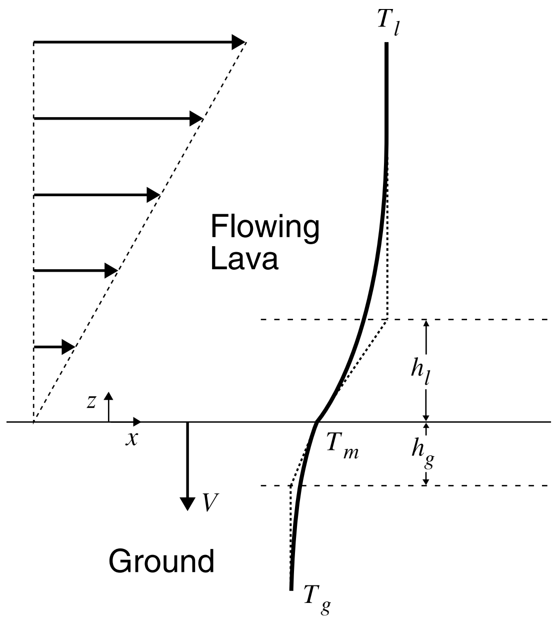

Figure 2: Schematic diagram of the temperature profile near the base of the lava, where the flow has a linear velocity gradient.

given by

$$Q = \frac{g\rho \sin\alpha\, d^3}{3\mu} = \frac{2Ud}{3}. \qquad (1)$$

The assumption of laminar flow is valid provided that the Reynolds number $Re = Q/\nu$ is less than about 500, which is typically the case for terrestrial lava flows.

As the hot lava flows, heat is transferred to the cold underlying ground, raising its temperature from T_g initially until it melts at a temperature T_m (figure 2). Ahead of the melting front, the steady-state temperature in the solid has an exponential profile

$$T(z) = T_g + (T_m - T_g)\mathrm{e}^{z/h_g}, \qquad (2)$$

where the lengthscale $h_g = \kappa_g/V$, κ_g is the thermal diffusivity of the ground and V is the melting velocity.

Turning now to the more complicated problem of quantifying the heat transfer in the flowing lava, I begin by making the initial assumption that the slow movement of the interface between the lava and the ground can be neglected. Also, it is important to realize that the thermal diffusivity κ of a lava is much smaller than its kinematic viscosity, which results in the Péclet number $Pe = Q/\kappa$ being very large. As a consequence, the thermal boundary layer is typically much thinner than the thickness of the flow, and it therefore lies within the region at the base of the flow where the velocity gradient can be taken to be linear. The advection-diffusion equation for the steady temperature field in the lava is then

$$\kappa \frac{\partial^2 T}{\partial z^2} = u(z)\frac{\partial T}{\partial x} = \frac{2U}{d} z \frac{\partial T}{\partial x}, \qquad (3)$$

when the alongstream diffusion of heat is neglected. This equation can be integrated, subject to the boundary condition of $T = T_m$ at $z = 0$, to show that the temperature profile in the fluid is given by

$$\frac{T - T_l}{T_l - T_m} = \frac{1}{\Gamma(\frac{4}{3})} \int_\zeta^\infty \mathrm{e}^{-\eta^3}\mathrm{d}\eta, \qquad (4)$$

[9], where $\zeta = z\,(2U/9\kappa d x)^{1/3}$ and $\Gamma(\frac{4}{3}) = 0.8929795...$. The heat flux to the ground is then given by

$$H = k\frac{\partial T}{\partial z}\bigg|_{z=0^+} = \frac{k(T_l - T_m)}{\Gamma(\frac{4}{3})}\left(\frac{2U}{9\kappa d\,x}\right)^{1/3} \qquad (5)$$

where k is the thermal conductivity of the lava. From equation (5), the thickness of the thermal boundary layer can be defined as

$$h_l = \frac{k(T_l - T_m)}{H} = \Gamma(\frac{4}{3})\left(\frac{9\kappa d\,x}{2U}\right)^{1/3}. \qquad (6)$$

The melting velocity V satisfies the interfacial heat flux condition

$$H = V(\rho_g L + \rho_g c_g (T_m - T_g)), \qquad (7)$$

where ρ_g, L and c_g are respectively the density, latent heat and heat capacity of the ground. From equation (5) and equation (7), we obtain

$$V = \frac{k(T_l - T_m)}{\Gamma(\frac{4}{3})(\rho_g L + \rho_g c_g(T_m - T_g))}\left(\frac{2U}{9\kappa d\,x}\right)^{1/3}. \qquad (8)$$

In the above analysis, the assumption was made that the slow melting of the ground does not significantly decrease the vertical heat transfer by increasing the effective thickness of the thermal boundary layer in the lava. To examine this assumption, I observe that the timescale for heat to diffuse across the thermal boundary layer is $t_l \sim h_l^2/\kappa$. On this timescale, the thickness of ground melted is $h_m = V t_l \sim V\,h_l^2/\kappa$, and the ratio of h_m to h_l is

$$\frac{h_m}{h_l} \sim \frac{\rho c(T_l - T_m)}{\rho_g L + \rho_g c_g (T_m - T_g)} = \frac{1}{\mathcal{S}}, \qquad (9)$$

where $\mathcal{S} = (\rho_g L + \rho_g c_g(T_m - T_g))/\rho c(T_l - T_m)$ is the Stefan number and c is the heat capacity of the lava. Hence the analysis will be valid provided the Stefan number is large, which is typically the case for thermal erosion by lava flows. A more detailed asymptotic analysis of this moving boundary problem [10] shows that, to first order in powers of $\mathcal{S}^{-1}$, the melting velocity in equation (8) is multiplied by the term $1 - 3\Gamma(\frac{2}{3})\Gamma(\frac{1}{3})^{-2}\mathcal{S}^{-1} \approx 1 - 0.566\,\mathcal{S}^{-1}$ i.e.

$$V = \frac{1}{\Gamma(\frac{4}{3})\mathcal{S}}\left(\frac{2U\kappa^2}{9d\,x}\right)^{1/3}(1 - 0.566\,\mathcal{S}^{-1}). \qquad (10)$$

It is also important to realize that, before steady-state erosion is finally established in a lava channel, there is an initial transient period in which the lava on the floor freezes to form a chill layer that is then melted back (e.g. [1]; [6]; [10]). This phenomenon reflects the fact that, while the heat flux into the ground is initially greater than the initial heat flux from the lava to the floor, the conductive heat flux into the ground then steadily decreases with time until it becomes less than the steady-state convective heat flux H from the lava to the floor. From the work of Huppert [6], it can be estimated that the basal chill layer has a maximum thickness $h_f \approx C_1\, k_g(T_m - T_g)/H$, is grown by the time $t_f \approx C_2\, k_g^2(T_m - T_g)^2/\kappa_g H^2$ and is remelted by the time $t_r \approx C_3\, k_g^2(T_m - T_g)^2/\kappa_g H^2$, where k_g is the thermal conductivity of the ground and the constants C_1, C_2, C_3 are known functions of the ground Stefan number $\mathcal{S}_g = L/c_g(T_m - T_g)$. I also note that a timescale $t_g \sim \kappa_g/V^2 \sim \mathcal{S}^2 t_l$ is required to establish the final temperature profile given by equation (2).

Laboratory Experiments

For comparison with the theoretical results above, two carefully chosen sets of laboratory experiments were performed [8]. The experiments were conducted in a channel that was 1.2 m long and 5.3 cm wide (figure 3). The channel had side walls and removable end walls made of Perspex, and an aluminium base that sat on two heat exchangers. The experimental material was a polyethylene glycol wax (PEG 600) that had a melting temperature of about 20.0°C (and a freezing temperature of about 16.0°C) on the timescale of the experiments. The molten wax was first poured into the channel, to a depth of 2 to 3 cm, and then solidified by pumping cold coolant through the underlying heat exchangers.

The first set of experiments (Set A) was aimed at investigating the rate of steady-state thermal erosion. The experiments were started by first removing the end walls of the channel and tilting the channel on an angle of 7.3°. Hot molten wax (at a temperature $T_l = 37.5$°C) was then allowed to flow under gravity from an overlying reservoir through a tube onto polystyrene foam insulation at the upper end of the channel, from where it flowed downstream over the cold solidified bed of wax, whose thickness was initially equal to that of the polystyrene (figure 3). Throughout the experiment, additional hot wax was continuously added to the overlying reservoir to maintain a constant depth and hence a constant gravitational head and a constant flow rate.

The flow rate per unit width Q of the wax in the channel was about 4.43 cm^2/s. This flow rate is consistent with the measured surface velocity U of 16.4 cm/s and the observed flow depth d of about 4.0 mm, given the measured density (1112 kg/m^3) and viscosity (0.0693 Pa s) of the hot flowing wax. The wax flow in the channel was observed to be laminar, which is consistent with its Reynolds number of 7.1. The Péclet number of the flow was very large (about 5500). Using typical values for the thermophysical properties of the PEG 600 wax and the measured temperature of the solidified wax block (about 12.0°C), the Stefan number $\mathcal{S}$ was about 3.8.

In the experiments, the flow of the hot wax caused the underlying wax block to be thermally eroded, at a rate that decreased with distance downstream from the polystyrene foam insulation (figure 4). The melting velocity V was quantified by measuring the depth eroded during an experiment, which was observed to be uniform across the channel. At each position x, the measurements showed that the melting rate did not vary with time, to within the experimental error of about 5% (figure 5). This rapid approach to steady-state thermal erosion in these experiments is due to (i) the absence of any initial basal chill layer (since $T_l - T_m > T_m - T_g$) and (ii) the large value of $\mathcal{S}_g$, which ensures that thermal diffusion in the ground has a small effect.

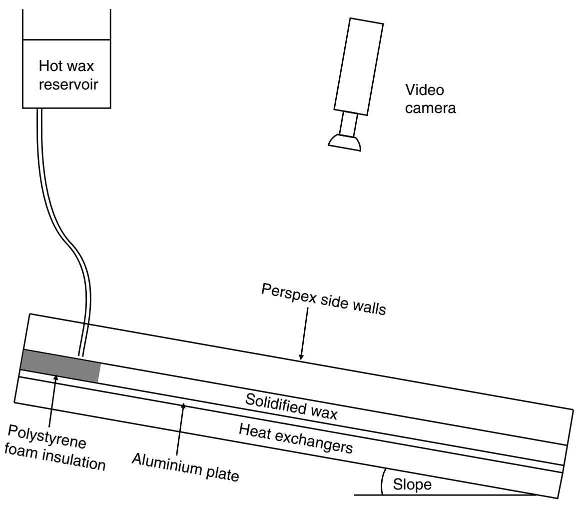

Figure 3: Sketch of the experimental apparatus.

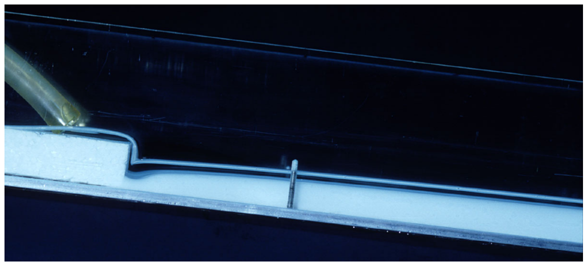

Figure 4: Photograph after 8 minutes in a thermal erosion experiment from set A, showing the decrease in thermal erosion with distance downslope.

As shown in figure 5, the experimental melting velocities vary with position x, and are consistent within experimental error with the line $V = 2.57\ x^{-1/3}$ where V is measured in mm/min and x in cm. This result is in excellent agreement with equation (8), which predicts that $V = 2.47\ x^{-1/3}$, with an error in V of about 9%.

The second set of laboratory experiments (Set B) was aimed at investigating the growth and subsequent remelting of an initial chill layer at the base of the hot wax flow. To facilitate the formation of such a chill layer, these experiments had a smaller channel slope (4.1°), a smaller flow rate (1.11 cm^2/s), a lower wax temperature (26.3°C) and a lower ground temperature (about -6°C). The experiments had a Reynolds number of 1.0, a Péclet number of 1400 and a Stefan number of 13.4. The wax ground layer was dyed a blue colour (using a powdered dye) so that it could be clearly distinguished from an overlying chill layer of white wax.

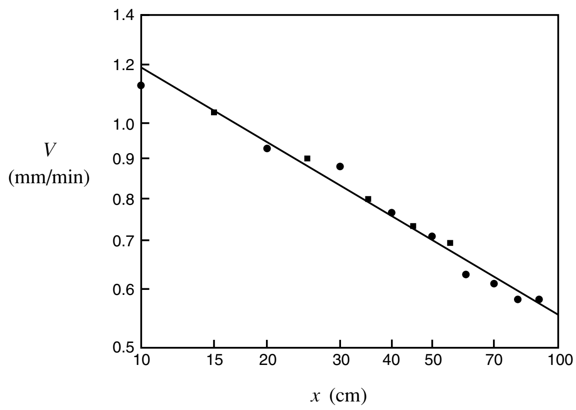

Figure 5: Melting velocity V versus distance x, for two experiments from set A with run times of 13 min and 18 min. The data are consistent with the line $V = 2.57\,x^{-1/3}$.

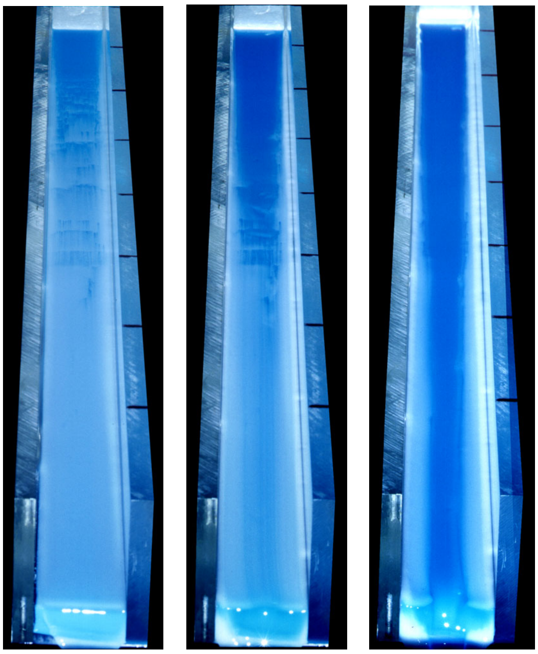

Figure 6: Views along the channel towards the source during a chill formation experiment from Set B: after 2 minutes (left photo); after 14 minutes (center photo); after 36 minutes (right photo).

A sequence of photographs showing the evolution of an experiment is shown in figure 6. In the first 20 seconds that it took for the wax flow to reach the end of the channel, a chill layer of white wax formed at the base of the flow that extended to within 3 cm of the foam insulation. The chill layer then began to be remelted upstream, while it continued to grow further downstream (figure 5a). Eventually, the region of chill remelting extended along the whole length of the channel (figure 5b). After about 36 minutes, the chill layer had been completely melted in the centre of the channel, although it still remained in thin boundary layers along the sides of the channel (figure 5c). The observed time for remelting along the channel is in very good agreement with the predicted value of $t_r = 32$ min at $x = 90$ cm. After 54 minutes, the flow of hot wax from the reservoir was stopped. While most of the hot wax then drained from the channel, some of the hot wax froze to form a thin new chill layer of white wax that coated the base of the thermal erosion channel.

Application to Basaltic Lava Flows

When the theory is applied to basaltic lava channels on Hawaii [8], it is necessary to take into account the change in the lava's crystal content with temperature, the substantial volatile content of the lava and of the ground, and the presence of lava falls and tube breakouts. The theory is found to explain field observations of constant erosion rates of several cm/day on timescales of several months [7]. The theory also predicts that, over flow distances of 10–1000 m: (i) the thermal boundary layer at the base of the lava is only 1–5 cm thick and is established after only 0.26–5.6 hours; (ii) an initial chill layer reaches a maximum thickness of 7.3–34 cm after a time of 0.21–4.6 days, before thermal erosion and chill meltback begins. This thermal erosion of the ground is responsible for the observation by Kauahikaua *et al.* [7] that "except where inflation is active, lava streams only partially fill their tubes". Also, the predicted decrease in thermal erosion with decreasing ground slope is consistent with the field observation that the amount of thermal erosion decreases with decreasing preflow slope. Finally, the decrease in thermal erosion with x provides a mechanism for the continued increase in the height of lava falls with time, since thermal erosion will be very rapid below a lava fall but relatively slow upsteam of the lava fall. Indeed, recent observations indicate that up to 30% of the elevation drop in Hawaiian lava tubes can occur in lava falls.

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