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Experimental study of high Reynolds number turbulent boundary layers - mean flow scaling

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Abstract

Preliminary experimental results are presented for high Reynolds number turbulent boundary layers. A flat plate zero pressure gradient layer has been studied in a new high Reynolds number boundary layer tunnel. Measurements were made of the mean flow up to Reynolds numbers of 6.0×10^4 based on momentum thickness. The data supports the existence of a logarithmic law of the wall in the overlap region and constants $\kappa = 0.41$ and $A = 5.0$ are found to best fit the data.

Introduction

One of the most important aspects in the study of wall bounded turbulent flow is a knowledge of the correct scaling law for the mean velocity profile. The well established similarity laws that describe the mean velocity profile are inner-flow similarity

$$\frac{U}{U_\tau} = f\left[\frac{zU_\tau}{\nu}\right] \tag{1}$$

and outer-flow similarity

$$\frac{U_1 - U}{U_\tau} = g\left[\frac{z}{\delta_c}\right] \tag{2}$$

where U is the mean streamwise velocity, U_τ is the friction velocity (ie. $\tau_0 = \rho U_\tau^2$ where τ_0 is the wall shear stress and ρ is the fluid density), U_1 is the local freestream velocity, z is the distance normal to the wall, ν is the kinematic viscosity and δ_c is the boundary layer thickness. Equation (1), first proposed by Prandtl, is known as the law of the wall and equation (2), first proposed by von Kármán, is known as the velocity defect law. Using the Millikan [7] argument that there exists a region of overlap between equation (1) and equation (2) gives the classical results

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\ln\left[\frac{zU_\tau}{\nu}\right] + A \qquad \text{and} \tag{3}$$

$$\frac{U_1 - U}{U_\tau} = -\frac{1}{\kappa}\ln\left[\frac{z}{\delta_c}\right] + B \tag{4}$$

where κ is the Karman constant, A is the universal smooth wall constant and B is a constant that depends on the large scale flow geometry and, in general, varies with streamwise distance. However, for zero pressure gradient layers B is often assumed to attain a constant value. Traditional values are $\kappa = 0.41$ and $A = 5.0$. The overlap region will be referred to as the fully turbulent wall region (TWR) and it is often assumed to begin at $zU_\tau/\nu = 100$ and extend to $z/\delta_c = 0.15$, although it should be noted the upper limit will be effected by the the large scale flow geometry, eg. pressure gradients.

Recent studies have renewed the debate on the validity of equation (3) for example see Barenblatt et al. [2] and Osterlund et al. [10] and several researchers suggest a Reynolds number dependent power law exists. Perhaps some of the most important experiments with regards to this are the superpipe experiments of Zagarola and Smits [11]. They measured mean velocity profiles (using a Pitot tube) in fully developed pipe flow using pressurised air as the working fluid, allowing them to obtain results up to a Reynolds number that was an order of magnitude higher than previous studies (maximum Reynolds number was 3.5×10^7, based on average velocity and pipe diameter). From their results they suggest that equation (3) is valid at sufficiently high Reynolds numbers. However they find that the constants are $\kappa = 0.436$ and $A = 6.15$. Conversely recent experimental results in zero pressure boundary layers reported by Osterlund [9] give numerical constants to be $\kappa = 0.38$ and $A = 4.1$ independent of Reynolds number and with no evidence of a power law. It has always been assumed in the past that equation (3) is valid for both pipes and boundary layers with the same constants.

The purpose of the current work is to provide a large overlap region in the mean velocity profile in order to evaluate the most appropriate functional form and to determine the constants which give "best fit". The high Reynolds number required are achieved through a long development length and a total of 39 profiles covering the Reynolds number range $3.5 \times 10^3 < R_\theta < 6.0 \times 10^4$ are analysed.

Experimental Method

Experiments were performed in an open return blower wind tunnel, as shown in figure 1. The important feature of the tunnel is the working section length of 27 m. This allows high Reynolds numbers to be obtained through the long development length, thus avoiding many of the experimental difficulties associated with using the alternative methods of achieving high Reynolds numbers, such as the use of compressed air or high velocities. The tunnel was run at three reference Reynolds numbers; $U_\infty/\nu = 6.48 \times 10^5$, 1.33×10^6 and 1.94×10^6 /m, corresponding to nominal reference freestream velocities of $U_\infty = 10\,\mathrm{m/s}$, $20\,\mathrm{m/s}$ and $30\,\mathrm{m/s}$. With further work it is anticipated that measurements at a nominal freestream velocity of $40\,\mathrm{m/s}$ will be possible, extending the Reynolds number range to $R_\theta = 8 \times 10^4$.

In order to maintain a zero pressure gradient the ceiling incorporates adjustable spanwise slots which allow for the bleeding of air. In addition, the height of the ceiling can be varied. Through these mechanisms it was possible to maintain the C_p distribution to within $\pm 0.5\%$, where C_p is the pressure coefficient given by

$$C_p = 1 - (U_1/U_\infty)^2 \, .$$

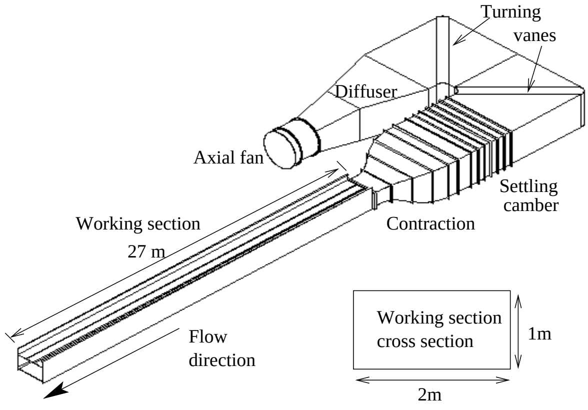

Figure 1: Isometric view of wind tunnel.

Two dimensionality in the mean was checked by measuring U_1 across the span (ie. in the y direction) at the most downstream station ($x = 26m$). U_1 was found to be within $\pm 0.1\%$ of the center line value once outside the influence of the sidewall boundary layers. Maximum sidewall boundary layer thickness was found to be $0.2w$ were w is the tunnel width.

Mean velocity profiles were measured using a Pitot-static probe in conjunction with a MKS Baratron 698 pressure transducer. The Pitot tube readings were corrected for the effect of shear using the MacMillan [6] correction, which gives $\delta_1/d = 0.15$, where δ_1 is the effective location of the Pitot tube above its centre line and d is the Pitot tube diameter, in this case 1 mm.

For each flow case measurements were made at 13 streamwise stations, with a spacing between stations of 2 m and starting at $x = 1$ m, where x is the streamwise distance measured from the beginning of the working section. Transition to turbulence was initiated by a trip wire of diameter 0.4 mm placed at $x = 0$. The boundary layer studied developed on the smooth aluminum floor of the working section. The freestream turbulence level was found to be less than 0.4%.

For the results presented here the Clauser chart was used to determine the values of U_τ. It must be noted that the Clauser chart relies on the validity of equation (3) with specified constants A and κ. If the constants used are correct it is a necessary condition that the values of U_τ returned collapses the data well onto equation (3). However good collapse onto equation (3) does not guarantee that the constants assumed are correct. Many *incorrect* combinations of κ and A will return *incorrect* U_τ values but still give equally good collapse, resulting in U_τ values which are all incorrect by a common factor. The approach taken here is to analyse the data using Clauser charts for different combinations of constants. If the data shows poor collapse onto equation (3) the constants can be discounted as being the correct ones. Three cases were considered: traditional constants of $\kappa = 0.41$ and $A = 5.0$; the Osterlund [9] constants of $\kappa = 0.38$ and $A = 4.1$; and the Zagarola and Smits [11] constants of $\kappa = 0.436$ and $A = 6.15$. Note that since Osterlund [9] proposes that the TWR begins at $z^+ = 200$ when assuming his constants, the beginning of the Clauser chart fit region was taken as $z^+ = 200$ for his case. For the other cases it was taken as $z^+ = 100$, where $z^+ = zU_\tau/\nu$.

To make any further comment on the validity of the constants requires an independent method of determining U_τ. Therefore at certain Reynolds numbers U_τ was also measured using a laser interferometer skin friction meter. This technique provides a direct measurement of the wall shear stress and does not rely on the assumption of a universal law of the wall. Instead it is based on the relationship between the wall shear stress and the gradient of a film of oil placed on the wall. Further details of the meter and skin friction results are given in Nishizawa et al. [8].

Results and Discussion

Mean velocity profiles

Mean velocity profiles are shown in figure 2 for the three flow cases considered. Here the data is normalised assuming $\kappa = 0.41,\ A = 5.0$ and it can be seen good collapse in the TWR is achieved across the full Reynolds number range. The quality of the collapse is more clearly seen in figure 3 where equation (3) has been subtracted from the data (ie. $\Delta(U/U_\tau)$ is plotted), note only data in the TWR is plotted. Also shown in figure 3 are the results of normalising the data assuming a priori that the correct constants are $\kappa = 0.38,\ A = 4.1$ and $\kappa = 0.436,\ A = 6.15$. It can be seen that the traditional constants (ie. $\kappa = 0.41,\ A = 5.0$) best collapse the data onto equation (3) with specified constants. When the other constants are used the data shows more scatter and this is most pronounced in the case of the [11] constants. This scatter is a consequence of fitting the wrong log-law over a large Reynolds number range. Further, for the [9] and [11] constants the data does not appear to fit the required gradient $1/\kappa$ as a consequence the values of U_τ obtained from best fit (ie. Clauser chart) will be sensitive to the limits defining the TWR.

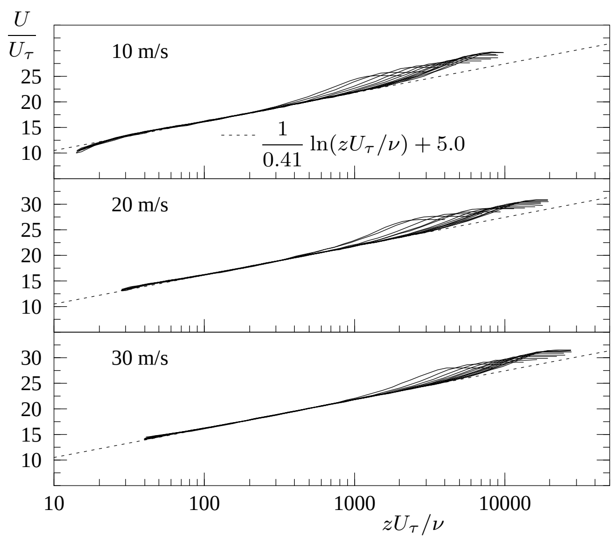

Figure 2: Mean velocity profiles for the complete range of streamwise stations.

The suitablity of a logarithmic law in the overlap region can be investigated by plotting the non-dimensional velocity gradient pre-multiplied by z^+, ie.

$$D_1 = z^+ \frac{dU^+}{dz^+}\ , \qquad (5)$$

where $U^+ = U/U_\tau$. If a log-law exists, equation (5) should equal a constant. Further, if the profiles have been scaled with the correct values of U_τ the constant

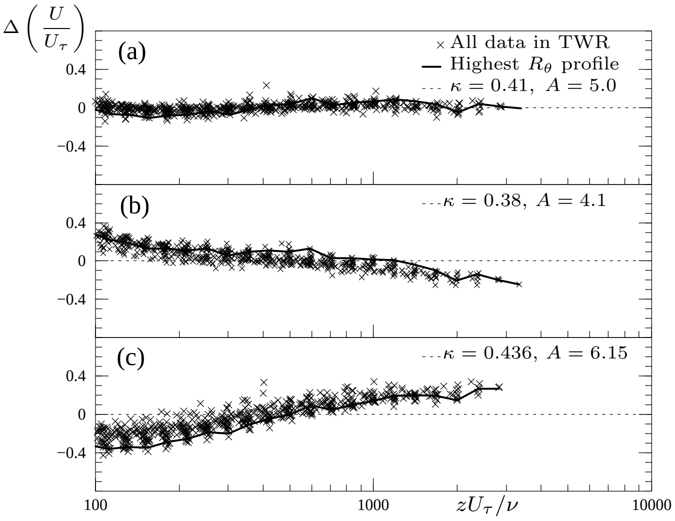

Figure 3: Deviation from log-law equation (3) when data is normalised to best fit; (a) $\kappa = 0.41$, $A = 5.0$, (b) $\kappa = 0.38$, $A = 4.1$ and (c) $\kappa = 0.436$, $A = 6.15$. Only data in the TWR ($100 < z^+ < 0.15K_\tau$) is shown.

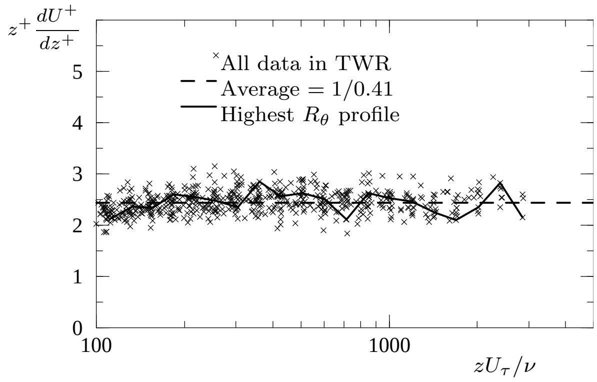

Figure 4: Logarithmic law diagnostic function equation (5) for all profiles, data from TWR.

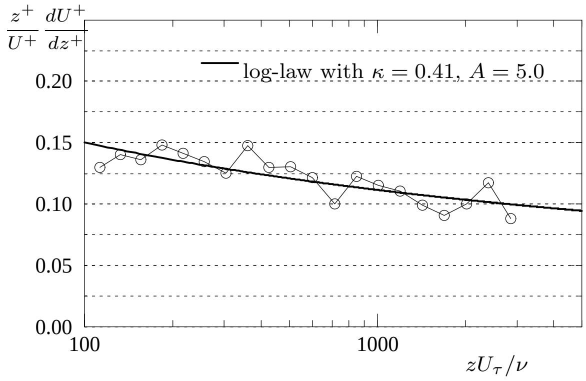

Figure 5: Power law diagnostic function equation (7) for highest R_θ profile for data from TWR.

should equal $1/\kappa$. Figure 4 shows the result of plotting equation (5) for all data in the range $100 < z^+ < 0.15K_\tau$. There is no smoothing or averaging of data, so figure 4 contains a degree of scatter. Nevertheless, the data does show agreement with the logarithmic law. The agreement is more clearly seen by considering the individual profile shown in figure 4, which corresponds to the highest R_θ profile. The data used in figure 4 was scaled assuming traditional constants, therefore it is not surprising that the average value of all the data is $D_1 = 1/0.41$. If the average value of D_1 is calculated for an individual profile there is no evidence of Reynolds number dependence. When D_1 is calculated using the data scaled assuming [9] constants and assuming [11] constants the change to figure 4 is very small. The scatter is slightly more while the averages of D_1 obtained hardly change. For both the [9] constants and the [11] constants the average is $D_1 = 1/0.40$. Again this indicates the traditional constants are more likely to be the correct ones and that it is not possible to collapse the data onto log-laws with either [9] constants or the [11] constants.

Several researches have proposed the correct scaling is not logarithmic but takes the form of a power law. For example, Barenblatt [1] proposes

$$\frac{U}{U_\tau} = A\left(\frac{zU_\tau}{\nu}\right)^\alpha \tag{6}$$

where A and α depend on Reynolds number. Figure 5 shows the function

$$D_2 = \frac{z^+}{U^+}\frac{dU^+}{dz^+} \tag{7}$$

plotted in the range $100 < z^+ < 0.15K_\tau$, for the highest Reynolds number profile. If a power law is the correct form equation (7) should plot as a constant value equal to α. Since this constant will in general vary with Reynolds number only the highest R_θ profile is shown. It does show a preferred slope (consistent with a log law) which suggests the power law is not the correct functional form. A similar trend is observed for the other profiles.

Skin friction

From the preceding section it is apparent that neither the [9] constants or the [11] constants can be made to fit the data and that the traditional constants best collapse the data. It is required that the constants also be compatible with the *true* values of wall shear velocity. Measurements have been made with an oil-film skin friction meter for selected values of R_θ across the Reynolds number range, see [8]. Results so far indicate the values of U_τ determined from Clauser chart with traditional constants are within $\pm 3\%$ of the oil-film results. Figure 6 shows the comparison of C_f' values as a function of R_θ. Also shown in figure 6 are semi-empirical formulae correlating C_f' to the Reynolds number. These formulae are based on the assumption that the layer has reached an equilibrium state, that is the defect plots are self-similar. When considered in the context of the law of the wall law of the wake, this implies a constant wake strength factor Π. Here we are referring to a modified version of the Coles [4] law of the wall/wake given by

$$\frac{U}{U_\tau} = \frac{1}{\kappa}\log\left[\frac{zU_\tau}{\nu}\right] + A - \frac{1}{3\kappa}\eta^3 + \frac{\Pi}{\kappa}W_c[\eta] \tag{8}$$

where $\eta = z/\delta_c$ and W_c is the Coles wake function. Note the $-\eta^3/(3\kappa)$ term is included to give the correct behaviour at the edge of the layer. Using equation (8) leads to the non-explicit formula

$$R_\theta = \exp(\kappa S)E[\Pi]\left(C_1[\Pi] - \frac{C_2[\Pi]}{S}\right) . \tag{9}$$

where $S = (2/C_f')^{1/2}$ and $E[\Pi]$, $C_1[\Pi]$ and $C_2[\Pi]$ are known analytical functions of Π. Equation (9) is plotted

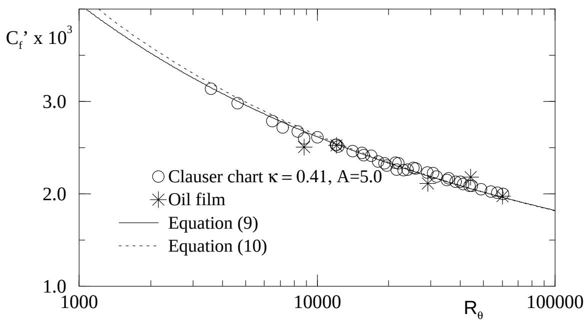

Figure 6: Skin friction coefficient.

in figure 6, assuming Π to be equal to a constant corresponding to the average experimental value. Often it is also assumed in equation (9) that $C_2[\Pi]/S$ is constant, leading to the well known skin friction law

$$S = \frac{1}{\kappa}\ln(R_\theta) + C \qquad (10)$$

where C is an additive constant. The difference between equation (9) and equation (10) is shown in figure 6, where the value of C was determined from best fit. Osterlund [9] curve fitted equation (10) to his C_f' data and then calculated U_τ based on equation (10). This may have had a subtle effect on the final constants he arrived at as it assumes self-similarity in terms of velocity defect.

The Π values are shown as a function of R_θ in figure 7. The 20 m/s and 30 m/s results show reasonable agreement and suggest Π decreases from its initial value to a constant value of $\Pi \approx 0.65$ beyond $R_\theta \approx 25000$. This value differs from the Coles [5] quoted value of $\Pi = 0.55$ due to the inclusion of the $-\eta^3/(3\kappa)$ term in equation (8), if this term is omitted from equation (8) the asymptotic value of Π is found to be closer to 0.51. The data for the 10 m/s flow does not tend to an asymptotic value and, in fact, shows two distinct peaks. A possible explanation can be found by considering the Clauser [3] pressure gradient parameter

$$\beta = \frac{\delta^*}{\tau_0}\frac{dp}{dx}.$$

Due to the higher displacement thickness of the 10 m/s flow case it can be shown that the parameter β is more sensitive to the presence of slight pressure gradients.

Conclusions

The mean velocity profiles are found to be well described by a logarithmic law of the wall in the fully turbulent wall region. The traditional values of $\kappa = 0.41$ and $A = 5.0$ are found to best collapse the data and comparison with oil-film skin friction data suggests the constants are accurate to approximately $\pm 3\%$.

The Coles wake factor (Π) tends to a constant for the higher Reynolds number cases, indicating the mean flow has obtained a state of equilibrium. However, for the 10 m/s case, Π appears to be sensitive to the presence of slight pressure gradients.

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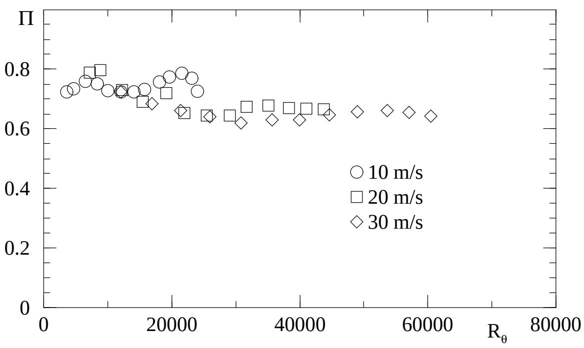

Figure 7: Wake strength factor Π from fit to equation (8).

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