Effect of Double Diffusion on Mixing of Discharges from Coastal Outfalls

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Abstract

The discharge of an axisymmetric buoyant jet of less salty and colder wastewater into an unstratified stagnant sea leads to the formation of a cloud of pollutant at the sea surface. We consider the transport of pollutants arising from double diffusion at the lower interface of this cloud; this phenomenon has not been considered in conventional models hitherto [1]. Conservation equations are solved numerically to obtain the temperature, salt and pollutant concentration profiles in the buoyant jet, cloud and surrounding sea water. Two outfall scenarios corresponding to low and high interface stability are considered. The interaction between double diffusion and buoyant jet entrainment results in the recirculation and accumulation of pollutant. When the interface stability is low, the pollutant concentration doubles near the buoyant jet, in the timescale required for the polluted cloud to reach the coast. The effect of double diffusion on pollutant transport in the high stability case is found to be small.

Introduction

In many coastal towns, sewage and industrial effluents are usually discharged via coastal outfalls [2]. Wastewater is usually less dense than sea water and hence rises as a buoyant jet above the outfall. In an unstratified sea, it then spreads out at the sea surface as a less salty and colder cloud of pollutant above the warmer and saltier sea water. Dilution of the polluted cloud thereafter is mainly thought to be controlled by horizontal turbulent and molecular diffusion [2]. Vertical transport is usually considered to be small as diffusion effects are orders of magnitude smaller than advective horizontal transport.

However, the temperature and salt concentration of the polluted cloud have opposing effects on the vertical density distribution and consequently molecular diffusion can drive vigorous buoyant convection. This phenomenon is known as double diffusion [12]. With the colder and less salty cloud overlaying the warmer and saltier sea water, the diffusive type interface is formed. Laboratory experiments [11] have shown that the double diffusive fluxes across such an interface can be much larger than the vertical transport in a single component fluid.

In this article, we model the evolution of the temperature, salt and pollutant concentrations of such a cloud. We shall assume the sea has negligible currents. This is an important worst case scenario as currents usually increase dilution [10]. Two outfall scenarios will be presented to assess the impact of double diffusion on discharges from coastal outfalls.

Theory

Modelling of Buoyant Jets

We consider a turbulent axisymmetric buoyant jet formed from a discharge of colder and less salty wastewater into the sea of depth H_{total} (Figure 1). We shall assume that the ambient fluid is of infinite lateral extent and motionless. The ambient is not stratified initially and is vertically well-mixed by eddies from the diffusive interface after the onset of double diffusion.
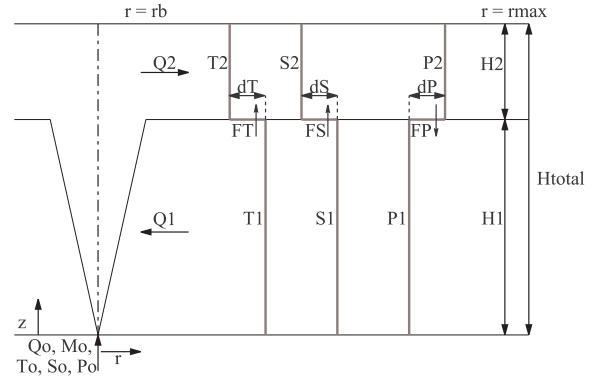

Figure 1: Schematic diagram of discharge of wastewater buoyant jet to calm unstratified sea

We invoke the Boussinesq approximation, i.e. the density variations are small and only important in buoyancy terms. We approximate the density as a linear function of salt concentration and temperature [9]:

$$\rho = 0.99708(1 + 0.70181S - 2.546 \times 10^{-4}(T - 25))$$

Here ρ is density (g/cm^3), S is the mass fraction of salt in solution and T is the temperature(^oC).

Equations for conservation of volume, momentum and buoyancy of plumes were first derived by Morton et al. [7]. We extend these conservation statements to a top-hat buoyant jet where density is a function of both temperature and salt concentration:

$$\frac{dQ}{dz} = 2\alpha_e M^{1/2}\pi^{1/2} \tag{1}$$

$$\frac{dM}{dz} = Q(B_S + B_T)/M \tag{2}$$

$$\frac{dB_S}{dz} = 0 \tag{3}$$

$$\frac{dB_T}{dz} = 0 \tag{4}$$

Here Q is the buoyant jet volumetric flowrate; M is the buoyant jet momentum; B_S is the buoyancy flux due to salt; B_T is the buoyancy flux due to temperature and z denotes the vertical coordinate, increasing upwards, with its origin at the source. The entrainment constant, α_e, is taken as 0.076 for a strong jet behaviour $(l_M = M_o^{3/4}/B_o^{1/2} > H_{total})$ and 0.117 for a strong plume behaviour $(l_M \ll H_{total})$; here M_o and B_o are the momentum flux and buoyancy flux of the buoyant jet at the source, respectively.

Since the pollutants are passive in the entrainment process, their concentrations can be calculated by

$$P(z) = \frac{(Q(z) - Q_o)}{Q(z)} \times P_1 + \frac{Q_o}{Q(z)} \times P_o \qquad (5)$$

where Q_o is the volumetric flowrate from the port and P, P_o and P_1 are the pollutant concentration in the buoyant jet at height z, at the port and in the sea water near the margin of the jet, respectively.

Modelling the Cloud of Pollutants

The wastewater buoyant jet rises to the sea surface and spreads out radially producing a cloud of pollutants. Advection, molecular diffusion and convection due to double diffusion determine the temperature, salt and pollutant concentrations. We shall assume that the depth of the sea (H_{total}) is uniform in the region considered to simplify the problem. The established cloud of pollutant typically has a thickness, H_2, of approximately 0.15 times H_{total} [8]. Conservative pollutant (i.e. no decay with time due to predation, sedimentation etc.) is considered here for simplicity.

Mass Balances
Laboratory experiments, e.g. [6], have shown that eddies generated at the edge of the diffusive interface convectively stir the layers making them well-mixed. In this article, we assume that the stirring action by the eddies is strong enough so that the sea water and the cloud of pollutants are well-mixed vertically. Hence temperature, salt and pollutant concentrations in both the cloud and the sea are functions of the radial coordinate, r, only. This assumption will give the largest concentration changes and hence will lead to an upper bound for the changes in dilution due to the diffusive interface. This will be useful to assess the impact of the diffusive type interface on wastewater discharges via coastal outfalls.

The depth integrated transport equations for temperature, salt and pollutants are

$$H_2 \frac{\partial \chi_2}{\partial t} = \left(-\frac{Q_2}{2\pi} + H_2 \kappa_\chi\right) \frac{1}{r} \frac{\partial \chi_2}{\partial r} + H_2 \kappa_\chi \frac{\partial^2 \chi_2}{\partial r^2} + F_\chi \qquad (6)$$

$$H_1 \frac{\partial \chi_1}{\partial t} = \left(\frac{Q_1}{2\pi} + H_1 \kappa_\chi\right) \frac{1}{r} \frac{\partial \chi_1}{\partial r} + H_1 \kappa_\chi \frac{\partial^2 \chi_1}{\partial r^2} - F_\chi \qquad (7)$$

where r is the radial coordinate; t is time; H_1 denotes the depth of the sea; Q_1 is the volumetric flowrate of the sea water being entrained into the jet; Q_2 is the volumetric flowrate of the polluted cloud; χ represents T, S and P, i.e. temperature, salt and pollutant concentrations, respectively. κ_χ and F_χ represent the diffusivities and the double diffusive fluxes of the components, T, S and P, respectively. F_χ is taken as positive for a net flux into the surface current. Subscripts 1 and 2 denote the sea water layer and the surface current, respectively.

Flux Laws for Salt and Temperature
The fluxes of heat and salt across the diffusive interface have been investigated in many experiments. Turner [11] proposed correlations of the form

$$\alpha F_T = C \left(\frac{g\kappa_T^2}{\nu}\right)^{1/3} (\alpha \Delta T)^{4/3} \qquad (8)$$

$$\beta F_S = \alpha F_T R_F \qquad (9)$$

where g is the gravitational constant, ν is the kinematic viscosity, ΔT is the temperature difference between the layers and R_F is called the flux ratio. Kelley [3] constructed best fit curves for the parameters C and R_F using previous experimental data. We

shall adopt his expressions for C and R_F.

$$C = 0.0032 \exp(4.8/R_\rho^{0.72}) \qquad (10)$$

$$R_F = \frac{R_\rho + 1.4(R_\rho - 1)^{3/2}}{1 + 14(R_\rho - 1)^{3/2}} \qquad (11)$$

where R_ρ is stability ratio for the diffusive interface defined as $R_\rho = \beta \Delta S / \alpha \Delta T$.

Pollutant Flux Law
Unlike for salt and temperature, flux laws for passive components have not been correlated. Therefore, a pollutant flux law will be developed here.

Experimental results [11] show that there are two transport regimes for the heat-salt system at different stability ratios. The "constant" regime occurs at high stability when the flux ratio is approximately constant. The "variable" regime describes the rapid change of flux ratio at low stability. Linden [5] argued the increased transport at low stability is due to entrainment across the interface. He modelled the transport of double diffusing components across the diffusive interface as composed of two parts, namely "diffusive flux" and "entrainment flux". The diffusive flux results directly from double diffusive instability. The entrainment flux results from the mechanical mixing across the interface by the convective motions of eddies in the layers. He argued that the two fluxes are independent of one another, hence are additive. We shall extend Linden's [5] model to obtain a flux law for pollutants. The flux of pollutants is composed of two parts, diffusive and entrainment.

$$F_P = u_e \Delta P + F_P^d \qquad (12)$$

Here F_P is the flux of pollutants, u_e is eddy entrainment velocity and F_P^d is the "diffusive" flux of pollutants.

The ratio of flux of pollutants to flux of temperature is:

$$\frac{\text{flux of P}}{\text{flux of T}} = \frac{F_P}{F_T} = \frac{u_e \Delta P + F_P^d}{u_e \Delta T + F_T^d} \qquad (13)$$

Adopting Linden's [5] approximations and curve fit results of Kelley [3], the flux of pollutants can be manipulated to be

$$F_P = F_T \times \frac{\Delta P}{\Delta T} \times \frac{1 + 14\sqrt{\kappa_P/\kappa_T}(R_\rho - 1)^{3/2}}{1 + 14(R_\rho - 1)^{3/2}} \qquad (14)$$

Boundary and Initial Conditions
Boundary conditions have to be determined to solve the set of 6 PDE's above. At $r = rb$, the buoyant jet spreads out to form the polluted layer; hence the pollutant cloud has properties equal to those of the buoyant jet. Equation (16) corresponds to the open sea boundary condition at $r = rmax$ [4]: an inflow of mass without back return so that the pollutant cloud at large distance from the jet has a uniform diffusive flux in the radial direction.

$$\chi_2(r = rb, t) = \chi_{jet}(z = H_{total}, t) \qquad (15)$$

$$\left. \frac{\partial}{\partial r}\left(r \frac{\partial \chi_2}{\partial r}\right)\right|_{r=rmax,t} = 0 \qquad (16)$$

For the sea water, the temperature, salt and pollutant concentrations far from the outfall are not affected by the wastewater discharge. Thus the temperature, salt and pollutant concentrations at the shoreline ($r = rmax$) are at their initial background levels and their rate of change with radial position is zero.

$$\chi_1(r = rmax, t) = \chi_1(r = rmax, t = 0) \qquad (17)$$

$$\left. \frac{\partial \chi_1}{\partial r}\right|_{r=rmax,t} = 0 \qquad (18)$$

The moment the jet reaches the sea surface is taken as $t = 0$. Hence the temperature, salt and pollutant concentrations in the upper layer at $t = 0$ are the same as those for the sea water.

Equations 6 and 7 were solved using Mathematica^{TM}.

Results and Discussion

In this section, we apply our model to two scenarios of coastal outfalls to assess the importance of double diffusion to pollutant transport. In both scenarios, the outfalls are located 5km offshore and discharge at a depth of 30m. The volumetric flowrate is 1m^3/s and the momentum flux is 4.5m^4/s^2. Cadmium is chosen as the conservative pollutant. A discharge level of 1μg/l [13] is chosen and a zero background level is adopted for simplicity. The properties of the buoyant jet and sea water are as in Table 1.

		Low R_ρ	High R_ρ
wastewater	T_o	15°C	15°C
buoyant jet	S_o	0.032g/g	0.020g/g
	P_o	10$^{-9}g/cm^3$	10$^{-9}g/cm^3$
	α_e	0.076	0.117
Sea water	T_1	20°C	20°C
	S_1	0.034g/g	0.035g/g
	P_1	0g/cm^3	0g/cm^3
R_ρ at $t = 0, r = rb$	R_ρ	1.10	8.27

Table 1: Properties of system investigated

Scenario 1: Low R_ρ Case

Some industries discharge effluents with a high salt content to the sea. Examples include desalination plants extracting fresh water from sea water, oil and gas production as a by-product and manufacture or treatment of caustics. In this case, the interface formed between the polluted layer and the sea water has a small ΔS and hence a low interface stability. We model this low interface stability case next.

As the ocean depth (30m) is less than l_M(38m), we consider an entrainment constant α_e of 0.076 for a jet. The stability ratio, R_ρ, just after the start of the wastewater discharge is 1.10.

Salt and heat are transported from the sea water below to the overlaying polluted cloud. Thus the salt concentration and temperature in the sea decrease with time (Figure 2). Pollutants are transported from the polluted cloud to the sea water leading to an increase in pollutant concentration in the sea with time (Figure 3). When the front of the polluted layer reaches the coast at approximately 240 days, the salt concentration and temperature of the sea close to the jet has decreased by 0.31% and 4.9%, respectively.

In the polluted layer, we expect the temperature and the salt concentration to increase and the pollutant concentration to decrease, comparing with the no double diffusion case, due to the transport by double diffusion. However close to the jet, the opposite is observed (Figures 2 and 3). This apparent up-gradient transport is due to recirculation. Part of the pollutant excess, temperature deficit and salt deficit are transported from the polluted layer to the sea water due to double diffusion. These are then entrained back into the jet which carry them to the polluted layer. The jet entrainment and double diffusion leads to recirculation. At 240 days, the decreases of salt concentration and temperature close to the jet are 0.30% and 4.7%, respectively. The pollutant concentration close to the jet increases by
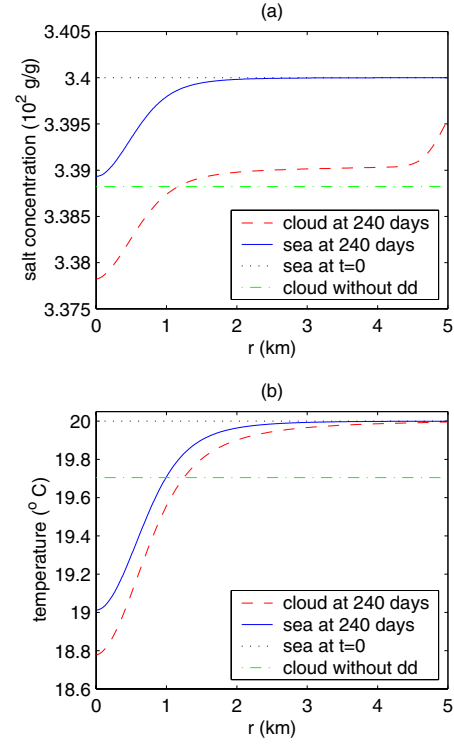

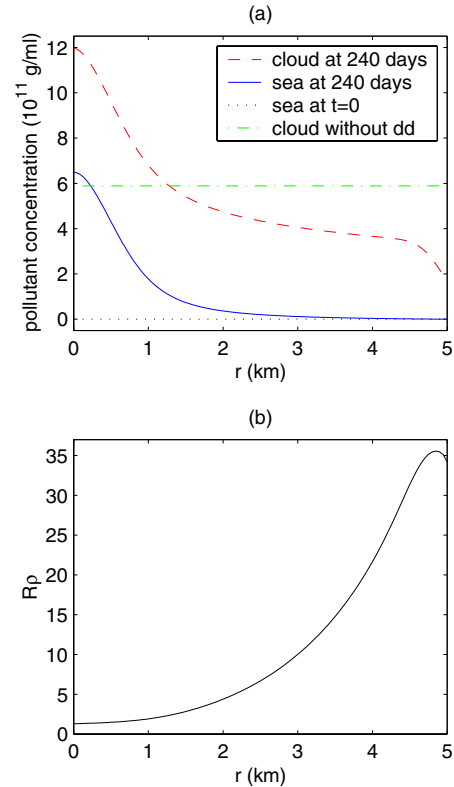Figure 2: (a) Salt concentration and (b) temperature profiles at 240 days

Figure 3: (a) Pollutant concentration and (b) R_ρ profiles at 240 days.

103% compared to the value in the absence of double diffusion at 240 days. Beyond 1.3km from the jet, the temperature and salt concentration are higher, and the pollutant concentration is lower, than the corresponding values in the absence of double diffusion.

The stability ratio of the diffusive interface, R_ρ, increases with time close to the buoyant jet, from 1.10 at $t = 0$ to 1.30 at 240 days. R_ρ at the front of the polluted layer current also increases with time. At 240 days, R_ρ is 34, implying the interface at the current front is very stable to double diffusion. Thus, we expect a small transport of salt and pollutant across the interface. This is consistent with the temperature profiles obtained, which show that the temperature difference between the polluted layer and the sea water has almost all diffused away close to the shore. This also explains the almost horizontal region in the salt and pollutant concentration profiles for the polluted layer beyond 1.3km from the jet.

Scenario 2: High R_ρ Case

The second scenario corresponds to a buoyant jet with a low salt content. Examples include stormwater and some wastewater from sewage treatment works. l_M is found to be 10.3m and hence we consider an entrainment constant of 0.117, appropriate for a plume.

With a small salt concentration difference between the wastewater and the sea, the resulting diffusive interface is stable and has a high R_ρ. The stability parameter just after the start of the wastewater discharge is 8.3. The changes of the temperature, salt and pollutant concentrations in both layers are qualitatively similar to that in the low R_ρ case, but with much smaller magnitudes. The front of the polluted layer reaches the shore at approximately 135 days. In the sea, the salt concentration and temperature close to the plume decrease by 0.01% and 0.39%, respectively. In the cloud, the pollutant concentration increases by 5.6% close to the plume compared to the value in the absence of double diffusion. The temperature and salt concentration close to the plume decrease by 0.37% and 0.01%, respectively.

Conclusions

The significance of the recirculation due to double diffusion and the buoyant jet entrainment cannot be captured by mixing zone models adopted by many countries for coastal outfall discharges, since the "near-field" (the area where the buoyant jet rise and interacts with the ocean surface, bottom and any stratification) and the "far-field" (the area where ambient environment controls the advection and mixing of the polluted layer) are modeled separately in mixing-zone models.

For a low stability interface between the polluted layer and the sea water, convection due to double diffusion significantly changes the temperature, salt and, especially pollutant concentrations in both layers, as observed in our first scenario. The pollutant concentration profiles are also very different from what are expected in the absence of double diffusion. When the diffusive interface is stable, double diffusion does not significantly alter the salt and pollutant concentrations in both layers.

References

[1] Doneker, R.L., private communication on Cormix, 2001.

[2] Grace, R.A., *Marine Outfall Systems: Planning, Design and Construction*, Prentice–Hall, 1978.

[3] Kelley, D.E., Fluxes through Diffusive Staircase: a New Formulation, *J. Geophys. Res.*, **95 C3**, 1990, 3365–3371.

[4] Koutitas, C.G., *Mathematical Models in Coastal Engineering*, Pentech Press, 1988.

[5] Linden, P.F., A note on the Transport across a Diffusive Interface, *Deep–Sea Res.*, **21**, 1974, 283–287.

[6] Linden, P.F. and Shirtcliffe, T.G.L., The Diffusive Interface in Double–diffusive Convection, *J. Fluid Mech.*, **87**, 1978, 417–432.

[7] Morton, B.R., Taylor, G. and Turner, J.S., Turbulent Gravitational convection from Maintained and Instantaneous Sources, *Proceedings of the Royal Society London*, **234A**, 1956, 1–23.

[8] Roberts, P.J.W., Sea Outfalls, in *Environmental Hydraulics*, editors V.P. Singh and W.H. Hager, Kluwer Academic Publishers, 1996, 63–110.

[9] Ruddick, B.R. and Shirtcliffe, T.G.L., Data for Double Diffusers: Physical Properties of Aqueous Salt-sugar solutions, *Deep–Sea Res.*, **26A**, 1979, 775–787.

[10] Tsanis, I.K. and Valeo, C., *Mixing Zone Models for Submerged Discharges*, Computational Mechanics Publishers, 1994.

[11] Turner, J.S., The Coupled Turbulent Transports of Salt and Heat across a Sharp Density Interface, *Int. J. Heat Mass Trans.*, **8**, 1965, 759–767.

[12] Turner, J.S., Multicomponent Convection, *Ann. Rev. Fluid Mech.*, **17**, 1985, 11–44.

[13] Whyte, M.W., Marine Disposal of Sewage and Sludge – Australian Practice, in *Marine Treatment of Sewage and Sludge*, I. Civ. E., 1988, 79–83.