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On the relationship between large-scale forcing and small-scale statistics in decaying grid turbulence

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Abstract

For moderate Reynolds numbers, the isotropic relations between second-order and third-order moments for temperature (Yaglom's equation) or velocity increments (Kolmogorov's equation) are not respected, reflecting a non-negligible correlation between the scales responsible for the injection, the transfer and the dissipation rate of turbulent energy. For grid turbulence, the dominant large-scale phenomenon is the non-stationarity (or, in an experimental context, the streamwise non-homogeneity) of statistical moments resulting from the decay of energy downstream of the grid. The objective of our paper is to quantify the influence of this non-homogeneity on various properties associated with the inertial and dissipative ranges of scales. In particular, we will show that a new term must be added to Yaglom's and to Kolmogorov's equations to account for the decay of second-order moments and thus explain the observed departure of the inertial range from the isotropic '4/3rds' and '4/5ths' laws. Similar contributions must also be retained in the isotropic forms of the budget equations for the mean dissipation rates $\langle\varepsilon_\theta\rangle$ and $\langle\varepsilon\rangle$. One of the inferences from our work is that the reported non-universal inertial-range properties are most often a result of the large-scale influence rather than an indication of strong departure from isotropy.

Introduction

One of the basic assumptions in statistical theories of fully developed turbulence (in particular far from walls or boundaries) is that, at large enough Reynolds numbers, three-dimensional effects involved in the transfer of energy result in the existence of a range of scales for which statistical properties become independent from the large-scale production process. These properties are then universal (i.e. they do not depend on the large-scale flow specific features nor on the Reynolds number value), and tend to satisfy isotropy over a range of scales (for which the expression *local isotropy* is commonly used).

In this context, a relatively simple relation was derived by Kolmogorov [5, see also 6] between the second- and third-order moments of the longitudinal velocity increment $\Delta u = u(x+r) - u(x)$ (the angular brackets denote time averaging, the separation r is along the longitudinal direction, $\langle\varepsilon\rangle$ is the mean dissipation rate of the turbulent kinetic energy and ν is the kinematic viscosity of the fluid),

$$-\langle(\Delta u)^3\rangle + 6\nu\frac{d}{dr}\langle(\Delta u)^2\rangle = \frac{4}{5}\langle\epsilon\rangle r. \qquad (1)$$

The counterpart of Eq. (1) for a passive scalar such as temperature, which then relates the second-order moment of the temperature increment $\Delta\theta = \theta(x+r) - \theta(x)$ and the third-order mixed moment $\langle\Delta u(\Delta\theta)^2\rangle$, was obtained by Yaglom [9, see also 6] (where k_0 is the thermal diffusivity),

$$-\langle\Delta u(\Delta\theta)^2\rangle + 2k_0\frac{d}{dr}\langle(\Delta\theta)^2\rangle = \frac{4}{3}\langle\epsilon_\theta\rangle r, \qquad (2)$$

or $A + B = C$, where $\langle\varepsilon_\theta\rangle$ is the mean dissipation (or destruction) rate of half the temperature variance.

If one considers decaying grid flow, where the assumption of isotropy is reasonably well satisfied (and the values of $\langle\varepsilon_\theta\rangle$ and $\langle\varepsilon\rangle$ are determined with very good accuracy from the streamwise decay rate of half the temperature variance and the turbulent kinetic energy respectively, [4]), neither Eq. (1) nor Eq. (2) is verified even at moderate Reynolds numbers. This is illustrated in figure (1), where A and B, as well as their sum, are plotted as a function of the normalized separation r^* (r/η, with $\eta = (\nu^3/\langle\varepsilon\rangle)^{1/4}$; note that, throughout the text, quantities normalized by their associated Kolmogorov velocity or temperature scale will be designated with a superscript *) after they have been divided by $\langle\varepsilon_\theta\rangle r$ in order to highlight the presence (or absence) of a plateau at the level of 4/3 for a particular range of scales.

It is only for very small scales ($< 5\eta$) that the value of 4/3 is attained, this value being asymptotically imposed by the fact that $3k_0[d\langle(\Delta\theta)^2\rangle]/dr$ must tend to twice the isotropic value of $\langle\varepsilon_\theta\rangle$ when r approaches η. In particular, it is obvious that using relation (2) (or relation (1)) to evaluate $\langle\varepsilon_\theta\rangle$ (or $\langle\varepsilon\rangle$) by adjusting a linear behavior of $-\langle\Delta u(\Delta\theta)^2\rangle$ (or $-\langle\Delta u^3\rangle$) (which would be characteristic of an inertial range behavior) will provide very poor results at small Reynolds numbers. This would lead to $\langle\varepsilon_\theta\rangle$ being overestimated . The objective of the present paper is therefore to derive new relations which provide turbulent energy scale budgets that are more accurate than Eqs. (1) and (2), and compare them with experimental data. For economy of space, the analytical development will be reported for the temperature field. The reader can refer to Danaila et al. [2] for the velocity field analysis.

Experimental details

Measurements were made on the centreline of the working section (350 mm X 350 mm, 2.4 m long) of a non-return blower-type wind tunnel, downstream of a biplane grid, in the range 20 < x/M < 80 (M is the mesh size of the grid). The mean longitudinal velocity of the flow, U, was 7 m/s. A square mesh (M = 24.76 mm, with 4.76 mm

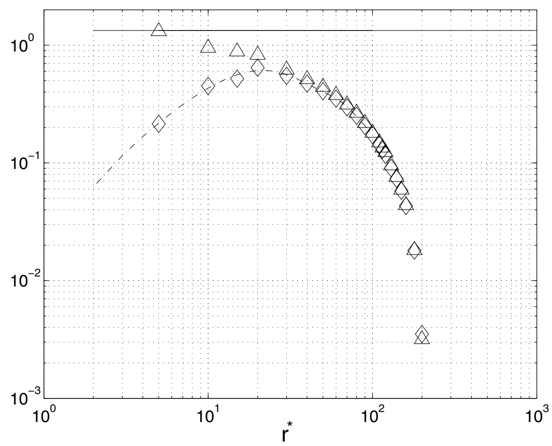

Figure 1: Comparison between normalized terms in Eq. (2), at x/M=70: $\diamond$, A/C, $\triangle$, (A+B)/C. The horizontal line is 4/3.

× 4.76 mm square rods) grid, with a solidity of 0.35, was used. The Taylor micro-scale Reynolds number R_λ ($= u'\lambda/\nu$, where u' is the rms value of the longitudinal velocity fluctuations and λ is the Taylor micro-scale) is approximately constant in x, with a value of about 70.

A mandoline was used to heat the flow in a way similar to [8]. For all the measurements, the mandoline was fixed at 1.5M downstream of the grid. It was constructed from fine Chromel-A wires of 0.5 mm diameter. The mandoline comprised two parts separated by 15 mm in the streamwise direction: the wires were horizontal in one and vertical in the other. Each part had a resistance of about 22 Ω and was heated by a power supply. The total power consumption was about 2 kW. The mean temperature ΔT relative to ambient was about 3 K in the tunnel. The wire separation in each part was 24.76 mm, i.e. the same as M. To prevent sagging due to thermal expansion, small springs were used to keep each wire under tension.

Simultaneous measurements of the three components of velocity and temperature were conducted, using a probe comprising 2 X-wires and a cold wire. One X-wire was in the (x ; y)-plane, and the other one was in the (x ; z)-plane. To avoid contamination of the cold wire measurements from the hot wires, the cold wire was located 1 mm below the centre of the two X-wires and shifted 0.5 mm upstream. The wires were etched from Wollaston Pt-10% Rh.

The active length of the cold wire was about $800d_w$ ($d_w = 0.63\,\mu m$ is the wire diameter). For the hot wires, the diameter was d = 2.5 μm, the length of the active part being 200d. The hot wires were operated with in-house constant-temperature anemometers with an overheat ratio of 1.5. The cold wire was operated with a constant-current (0.1 mA) circuit, also built in-house. The output signals from the constant-current and constant-temperature anemometers were passed through buck and gain circuits, and low-pass filtered at a cut-off frequency f_c close to f_k, the Kolmogorov frequency (estimated via $f_k = U/2\pi\eta$). The cut-off frequency is therefore a function of the position behind the grid: it varies from 5 kHz at x/M = 20 to 1.6 kHz at x/M = 80. The signals were then digitized into a personal computer using a 12 bit A/D converter at a sampling frequency of $2f_c$. The record duration was 52 s.

Analytical considerations

In order to understand the physical significance of the difference between the two terms of Eq. (2), we carefully reconsider its derivation, as it is presented in [6]. We start with the heat transport equation, which we first write at point $\mathbf{x}$,

$$\frac{\partial\theta}{\partial t} + \vec{u}\nabla\theta = k_0\nabla^2\theta,$$

and then at point $\mathbf{x}+\mathbf{r}$ (with the variables at this location being designated with a superscript +),

$$\frac{\partial\theta^+}{\partial t} + \vec{u}^+\nabla^+\theta^+ = k_0\nabla^{2+}\theta^+.$$

After multiplying the first of these equations by θ^+ and the second by θ, and adding them, one obtains

$$\frac{\partial}{\partial t}\langle\theta\theta^+\rangle - 2\nabla_{\vec{r}}\langle\vec{u}\theta\theta^+\rangle = 2k_0\nabla^2_{\vec{r}}\langle\theta\theta^+\rangle. \qquad (3)$$

Using the usual forms of the gradient and Laplacian operators [e.g. 6] for homogeneous and isotropic turbulence, only the derivatives with respect to r, the modulus of $\mathbf{r}$ will be used. If we also consider that the temperature variance θ'^2 decays behind the grid, and therefore retain the corresponding term in Eq. (3), and use the fact that (mainly because R_λ is constant) $\langle(\Delta\theta)^2\rangle/\theta'^2$ remains unchanged with x (or time t) for any value of r, then, after multiplying by r^2, integrating with respect to r and dividing by r^2, we obtain the generalized form of Yaglom's equation in decaying grid turbulence:

$$\begin{aligned} &-\langle\Delta u(\Delta\theta)^2\rangle + 2k_0\frac{d}{dr}\langle(\Delta\theta)^2\rangle \\ &-\frac{1}{r^2}\int_0^r s^2\frac{\partial}{\partial t}\langle(\Delta\theta)^2\rangle ds = \frac{4}{3}\langle\epsilon_\theta\rangle r, \end{aligned} \qquad (4)$$

where s is a dummy variable identifiable with the separation.

The new term on the left hand side of Eq. (4) can be interpreted as a "source term" since it accounts for the large-scale flow properties, and we therefore call it S.

Results and discussion

Figure 2 shows that the influence of this term is quite important, since the balance between $(4/3)r^*$ and the sum of the three normalized terms $A^* + B^* + S^*$ is quite good for all the scale range, underlining the fact that the decay is crucial in this kind of flow. It also validates the three-dimensional isotropy of the passive scalar field in grid turbulence, even at moderate Reynolds numbers, while it was sometimes argued that the observed departure from the "four-thirds law" was mainly a consequence of a lack of isotropy at moderate Reynolds numbers. The same kind of analysis can be performed for the velocity field [2], the additional term in Kolmogorov's equation being

$$S_u = -3\frac{U}{r^4}\int_0^r s^4\frac{\partial}{\partial x}\langle(\Delta u)^2\rangle ds.$$

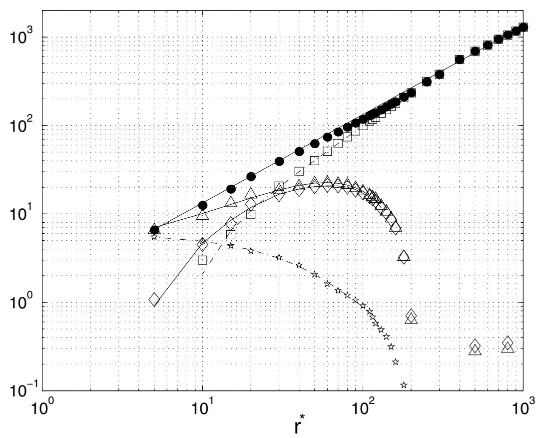

Figure 2: Terms in the generalized Yaglom equation. $\diamond$, A^*, $\star$, B^*, $\triangle$, A^*+B^*, $\square$, S^*, $A^*+B^*+S^*$ ($\bullet$) is to be compared with C^* (solid line).

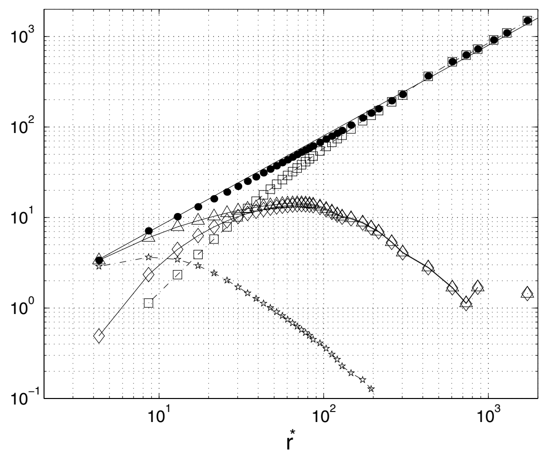

Figure 3: Terms in the generalized Kolmogorov equation. Same symbols as Fig. 2.

Figure 3 shows that, for longitudinal velocity structure functions, analyzed in terms of Kolmogorov's equation, the new term has the same kind of influence as that obtained for Yaglom's equation. However, Mydlarski and Warhaft's data, available for a large range of R_λ between 99 and 448, allow an assessment of the influence of the Reynolds number on the present analysis. Figure 4 shows that for all Reynolds numbers, the new term is about one half of $(4/5)r^*$ for $r = L$, where L is the integral length scale (estimated here as $L = 0.9u'^3/\langle\varepsilon\rangle$), even though larger contributions are obviously obtained for smaller Reynolds numbers since the maximum level attained by $-\langle\Delta u^3\rangle$ is then significantly smaller than $(4/5)r^*$.

It is also worth examining the limit trends, for very large and very small scales, of our new relations. We first consider the generalized Yaglom equation. For very large scales, typically for $r > L$, $-\langle\Delta u(\Delta\theta^2)\rangle$ tends to zero and $\langle\Delta\theta^2\rangle$ tends to the asymptotic value of $2\theta'^2$, so that one simply obtains the well-known decay law (e.g. [4]) for the temperature variance.

On the other hand, for very small scales, a Taylor series expansion for $\langle\Delta\theta^2\rangle$ must first be written,

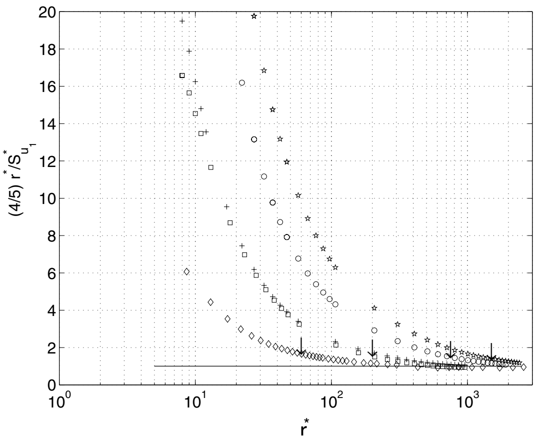

Figure 4: Ratio of the linear term $4/5r^*$ and the source term S_u^* for the present velocity data and those of Mydlarski and Warhaft, for various Reynolds numbers. Present data, $R_\lambda = 70$, $\diamond$. Mydlarski and Warhaft's data : $R_\lambda = 99$, $\square$, $R_\lambda = 134$, $+$, $R_\lambda = 319$, $\circ$, $R_\lambda = 448$, $\star$. Arrows indicate the value of L^* for each R_λ (except for $R_\lambda = 134$, for which L^* has almost the same value as for $R_\lambda = 99$).

$$\langle(\Delta\theta)^2\rangle \approx \langle\left(\frac{\partial\theta}{\partial x}\right)^2\rangle r^2 - \frac{1}{12}\langle\left(\frac{\partial^2\theta}{\partial x^2}\right)^2\rangle r^4, \qquad (5)$$

while the approximation

$$\frac{\partial}{\partial x}\langle(\Delta\theta)^2\rangle \approx \frac{r^2}{3}\frac{\partial}{\partial x}\langle\varepsilon_\theta\rangle \qquad (6)$$

is valid for locally isotropic turbulence. After substituting the above expressions into Eq. (4), using the approximation

$$-\langle\Delta u(\Delta\theta)^2\rangle \approx -\langle\frac{\partial u}{\partial x}\left(\frac{\partial\theta}{\partial x}\right)^2\rangle r^3 \qquad (7)$$

and equating terms in r^3, we obtain

$$-\frac{U}{15k_0}\frac{d\langle\varepsilon_\theta\rangle}{dx} = \langle\frac{\partial u}{\partial x}\left(\frac{\partial\theta}{\partial x}\right)^2\rangle + 2\frac{k_0}{3}\langle\left(\frac{\partial^2\theta}{\partial x^2}\right)^2\rangle, \qquad (8)$$

which characterizes the decay of $\langle\varepsilon_\theta\rangle$ behind the grid.

A similar development for $\langle\Delta u^2\rangle$ inserted into our generalized Kolmogorov equation,

$$-\langle(\Delta u)^3\rangle + 6\nu\frac{d}{dr}\langle(\Delta u)^2\rangle + S_u = \frac{4}{5}\langle\epsilon\rangle r, \qquad (9)$$

gives

$$-\frac{U}{35\nu}\frac{d\langle\varepsilon\rangle}{dx} = \langle\left(\frac{\partial u}{\partial x}\right)^3\rangle + 2\nu\langle\left(\frac{\partial^2 u}{\partial x^2}\right)^2\rangle. \qquad (10)$$

This is the transport equation for $\langle\varepsilon\rangle$ or the enstrophy (for homogeneous turbulence), which was first derived by Batchelor and Townsend [1].

Therefore, our two generalized equations are compatible with the known relationships for decaying grid turbulence, while the "classical" Yaglom and Kolmogorov equations are not compatible, through their large scale limiting behaviour, with the decaying energy characteristic of grid turbulence. Further comments can be made with regard to our generalized equations. Figures (5) and (6) present, for the Yaglom and Kolmogorov equations respectively, the contributions associated with the terms of order r and order r^3 for the same data as for figures (2) and (3). The budget equations considered at the order r are obviously the same as those on figures (2) and (3), but, at the order r^3, the source terms in our generalized equations account exactly - at very small scales, $r* < 5$ - for the variation in x of the dissipation rates $\langle \varepsilon_\theta \rangle$ and $\langle \varepsilon \rangle$. This variation is the result of the "subtle" balance - at order r^3 - of the "classical" A and B terms, a feature which does not appear when only an order r balance is carried out.

It is interesting to note that results we have recently obtained in a channel flow [3] for Kolmogorov's equation show a physically similar result, namely that a large scale term must be added in order to account for the flow specific properties of this flow, turbulent diffusion, which balances at large scale the $\langle \varepsilon \rangle$ contribution. However, for this flow, it was found that the assumption of local homogeneity had also to be relaxed, so that, when deriving the generalized Kolmogorov equation from the Navier-Stokes equations, spatial "large-scale" derivatives needed to be incorporated in the expressions of the gradient and Laplacian operators.

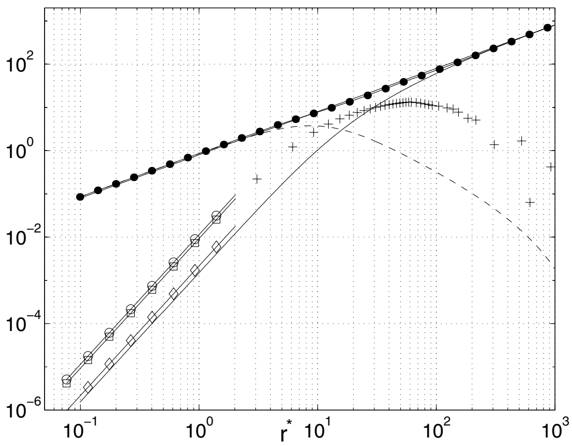

Figure 5: Contributions from the terms of order r and order r^3 in the generalized Yaglom equation. $+$, raw A^* data, - - -, raw B^* data, the line represents the source term S^*, $\bullet$, sum $A^* + B^* + S^*$, $\square$, normalized r^3 relation (7) approximating A, $\circ$, normalized r^3 relation derived from (5) approximating $-B$, $\diamond$, sum of the r^3 normalized terms in relations (7) and (5) (which appears in the right hand side of (8)).

Conclusions

We have demonstrated in this paper that Yaglom's and Kolmogorov's equations, in their "classical" form, cannot be verified over the intermediate to large-scale range, simply because they are not compatible with the kinetic energy budget (which sets the value of $\langle \varepsilon \rangle$ in each type of flow). This should not be a surprise since one should never forget that $\langle \varepsilon \rangle$ is also the rate at which energy is fed into the flow at large scale (at least for equilibrium flows). When the new terms (which only involve velocity second-order statistics) are considered, experimental data are in very good agreement with the generalized equations, so that these equations can be used to determine accurately the value of $\langle \varepsilon \rangle$, even at moderate Reynolds numbers.

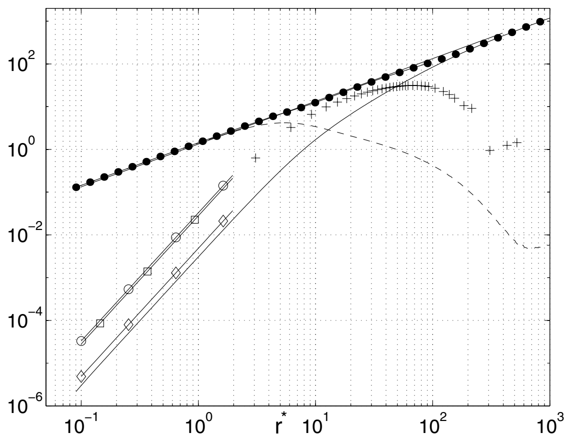

Figure 6: Contributions from the terms of order r and order r^3 in the generalized Kolmogorov equation. Symbols have the same meaning as in Fig 5.

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