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TURBULENT ENERGY AND TEMPERATURE VARIANCE FLUCTUATIONS IN A CYLINDER WAKE

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ABSTRACT

Simultaneous measurements of all three components of the velocity vector and temperature fluctuations were made with a 5-wire probe (4 hot wires and a cold wire) in the self-preserving region of a slightly heated cylinder wake. The previously established similarity between ϕ_q and ϕ_θ, the spectra corresponding to the turbulent kinetic energy $\langle q^2 \rangle$ and temperature variance $\langle \theta^2 \rangle$ respectively, is better satisfied away from than on the centreline. In the central region of the wake, the spectral distribution $k_1\phi_\theta$ departs from $k_1\phi_q$ around the peak wavenumber location. This behaviour is attributed to a lack of mixing in this region. The joint probability density function (jpdf) between q^2 and θ^2 does not reflect the level of similarity exhibited by ϕ_q and ϕ_θ.

INTRODUCTION

In turbulent flows, the instantaneous temperature is convected by the instantaneous velocity vector with molecular diffusion smoothing out the small-scale temperature fluctuations (e.g. Corrsin, 1951; Lumley and Panofsky, 1964; Fulachier and Antonia, 1984). By analysing the transport equations for $\langle q^2 \rangle$ ($q^2 \equiv u_1^2 + u_2^2 + u_3^2$ is the turbulent kinetic energy, u_1, u_2 and u_3 are velocity fluctuations in the x_1, x_2 and x_3 directions; angular brackets denote time averaging) and $\langle \theta^2 \rangle$, Corrsin (1953) pointed to the analogous forms of the transport equations for $\langle q^2 \rangle$ and $\langle \theta^2 \rangle$, despite the absence of pressure in the $\langle \theta^2 \rangle$ equation. This analogy is more attractive than analogies between either $\langle u_1^2 \rangle$, $\langle u_2^2 \rangle$ or $\langle u_3^2 \rangle$ and $\langle \theta^2 \rangle$. Fulachier and Dumas (1976) measured the spectra of all the three velocity components and of the temperature fluctuations in a boundary layer. They proposed a similarity between the temperature spectrum ϕ_θ and the spectrum ϕ_q, which corresponds to the turbulent energy $\langle q^2 \rangle$

$$\phi_q = \frac{\langle u_1^2 \rangle}{\langle q^2 \rangle}\phi_{u_1} + \frac{\langle u_2^2 \rangle}{\langle q^2 \rangle}\phi_{u_2} + \frac{\langle u_3^2 \rangle}{\langle q^2 \rangle}\phi_{u_3} .$$

The spectral density ϕ_α is defined such that $\int_{-\infty}^{\infty} \phi_\alpha dk_1 = 1$. Fulachier and Antonia (1984)

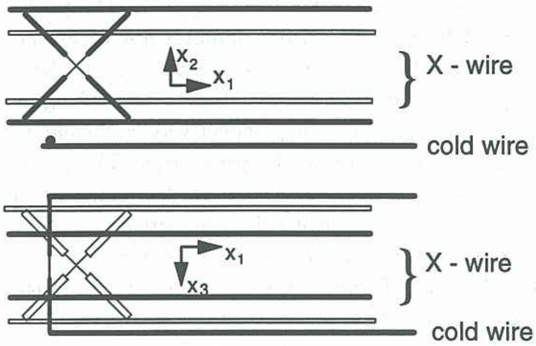

Figure 1: Sketch of the five wire probe and the coordinate system.

tested this spectral similarity over a wide range of turbulent shear flows. The similarity between q^2 and θ^2 in the physical domain was also considered by Antonia et al. (1996). These authors postulated an analogy between $\langle \delta q^2 \rangle$ [$\equiv \langle \delta u_1^2 \rangle + \langle \delta u_2^2 \rangle + \langle \delta u_3^2 \rangle$, where δu_i, i = 1, 2 or 3, is the velocity increment $u_i(x_1 + r) - u_i(x)$] and $\langle (\delta\theta)^2 \rangle$.

The main motivation for this work is the need to improve our understanding of the similarity between the invariants q^2 and θ^2. As well as comparing ϕ_q and ϕ_θ, we compare p_{q^2,θ^2}, the jpdf between q^2 and θ^2 with either $p_{u_1^2,\theta^2}$ or $p_{u_2^2,\theta^2}$. We also evaluate the expectation of $\alpha \equiv u_1^2$, u_2^2 or q^2 conditioned on θ^2.

EXPERIMENTAL DETAILS

The experiments were conducted in a non-return blower-type wind tunnel with a working section of 350 mm × 350 mm × 2.4 m long. The floor of the tunnel was slightly tilted to get a zero pressure gradient in the streamwise direction. The free stream turbulence intensity was about 0.1%. The wake was generated by a stainless steel tube of outer diameter $d = 2.67$ mm mounted horizontally in the midplane, 20 cm downstream of the working section entrance. The tube was electrically heated so that temperature was a passive marker of the flow at sufficiently large distance from the cylinder (at $x_1/d = 420$, $\Delta T_0 = 0.5°$C, ΔT_0 is the temperature at the centreline relative to ambient). The measurement locations were in the far-wake ($x_1/d = 250$ and 420) with a free stream velocity of $U_1 = 6.7$ m/s, corresponding to Reynolds number $R_\theta = 1190$ ($R_\theta = U_1 d/\nu$, where ν is the kinematic viscosity of air). A probe, consisting of 2 X-wires and a cold wire was used to measure all three velocity fluctuations and temperature fluctuation simultaneously. One X-wire was in the $x_1 - x_2$ plane and the other in the $x_1 - x_3$ plane. The cold wire was a distance of 1.0 mm below the centre of the 2 X-wires. A sketch of the probe is shown in Figure 1. The probe comprised four $d_w = 2.5\mu$m diameter hot wires and one $d_w = 0.63\mu$m cold wire (Wollaston Pt-10% Rh). The hot and cold wires were etched to active lengths of about $200 d_w$ and $800 d_w$respectively, sufficient to minimise the heat loss to the prong tips. The probe was calibrated in the free stream at the centreline of the tunnel against a Pitot tube connected to a MKS Baratron pressure transducer (least count = 0.01 mm H_2O); the yaw calibration was performed over a range of $\pm 20°$ (using 4° steps) in the $x_1 - x_3$ and $x_1 - x_2$ planes for the two X-wires respectively. The hot wires were operated with constant temperature anemometers at an overheat ratio of 0.5. The cold wire was operated with a constant current anemometer supplying 0.2 mA. The temperature coefficient of the Wollaston wire was estimated to be 1.69×10^{-3} ($°C^{-1}$). The output signals from the anemometers were passed through buck and gain circuits and low-pass filtered at a cut-off frequency $f_c = 1600 - 2500$ Hz. The signals were then digitized into a personal computer using a 12 bit A/D converter at a sampling frequency of 5000 Hz. The record duration was 52 s. The data were analysed on a VAX 780 computer.

EXPERIMENTAL RESULTS

The performance of the present probe was checked by comparing velocity and temperature spectra with those obtained from separate experiments with a single X-wire and a single cold wire respectively. The measurements of Browne and Antonia (1987) and Antonia et al. (1987) indicated that self-preservation was satisfied approximately for $x_1/d \geq 200$. So, the present measurement locations ($x_1/d = 250$ and 420) were in the self-preserving region and only data at $x_1/d = 420$ will be presented. In order to check for possible interference with the flow and cross-contamination of velocity and temperature signals, the specta from the present probe have been compared (Figure 2) with those from either a single X-wire or a single cold wire. For ϕ_{u_1}, the agreement between different probes is excellent. A similar result was obtained for ϕ_{u_2} and ϕ_{u_3}. For ϕ_θ, the agreement between the present probe and a separate single cold wire measurement is also good, indicating that there is no contamination of velocity by temperature and vice versa.

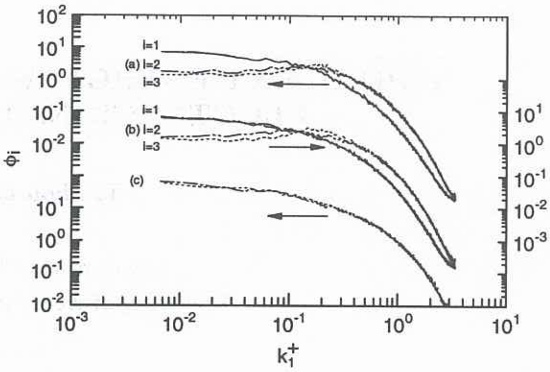

Figure 2: Comparison of velocity and temperature spectra obtained at the centreline with different probes. (a) present five-wire probe; (b) single X-wire; (c) ϕ_θ : — - —, present five-wire probe; - - -, single cold wire. The superscript + denotes normalization by the wake half-width L_0 and the free stream velocity U_∞.

In Figure 3 (a,b,c,d), $k_1^+\phi_{u_1}$ and $k_1^+\phi_q$ are compared with $k_1^+\phi_\theta$ at $x_2/L_0 = 0$, 1, 1.5 and 2. The ordinate was multiplied by the wavenumber k_1^+ to clearly identify the contributions from different wavenumbers to the spectrum. At all these locations, $k_1^+\phi_q$ is much closer to $k_1^+\phi_\theta$ than $k_1^+\phi_{u_1}$, supporting the spectral similarity between ϕ_θ and ϕ_q. At $x_2/L_0 = 0$ (Figure 3a), $k_1\phi_\theta$ is slightly lower than $k_1^+\phi_q$ around the peak location. This may be due to insufficient mixing in a region where the mean shear is small (it is zero at the centreline). As expected, the maxima of $k_1^+\phi_q$ and $k_1^+\phi_\theta$ at all four locations occur at k_0^+, the wavenumber where $k_1^+\phi_{u_2}$ has a peak value, underlining the importance of the organised motion in transporting $\langle q^2 \rangle$ and $\langle \theta^2 \rangle$. Figure 4 compares $k_1^+\phi_q$ with $k_1^+\phi_\theta$ at k_0^+ across the wake. While $k_1^+\phi_q$ is nearly constant, $k_1^+\phi_\theta$ approaches $k_1^+\phi_q$ as x_2 increases. At $x_2/L_0 = 1$, $k_1^+\phi_\theta \approx k_1^+\phi_q$. This behaviour can be related to the mean shear (also shown in Figure 4), which varies monotonically in the central part of the wake ($0 < x_2/L_0 < 1$). At $x_2/L_0 = 1$, it reaches a maximum and the difference between $k_1^+\phi_\theta$ and $k_1^+\phi_q$ has disappeared. Since the mean shear promotes mixing, the spectral analogy should work best when the mean shear is non-zero. It is expected that the similarity between ϕ_θ and ϕ_q will not hold as well in shearless flows as in sheared turbulence (unpublished results obtained in heated grid turbulence support this expectation).

Jpdf should provide a useful quantification of the correlation between two variables. The jpdf $p_{\alpha,\beta}$ is defined such that $\iint_{-\infty}^{\infty} p_{\alpha,\beta} d\gamma d\delta = 1$ where γ and δ represent centred variables normalized by the root-

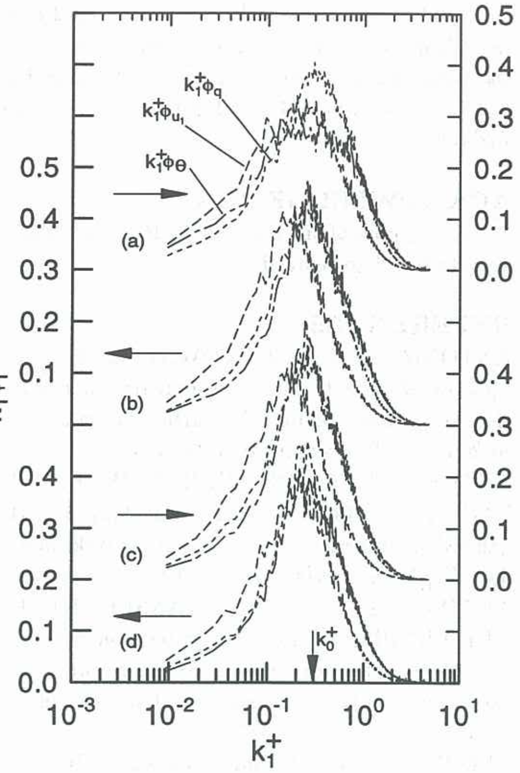

Figure 3: Comparison of $k_1\phi_{u_1}$ and $k_1^+\phi_q$ with $k_1^+\phi_\theta$ at four locations across the wake. (a) $x_2/L_0 = 0$; (b) 1; (c) 1.5; (d) 2. — - —, $k_1^+\phi_\theta$; ···, $k_1^+\phi_q$; - - -, $k_1^+\phi_{u_1}$. (k_0^+ is the wavenumber at which $k_1^+\phi_{u_2}$ has a peak value).

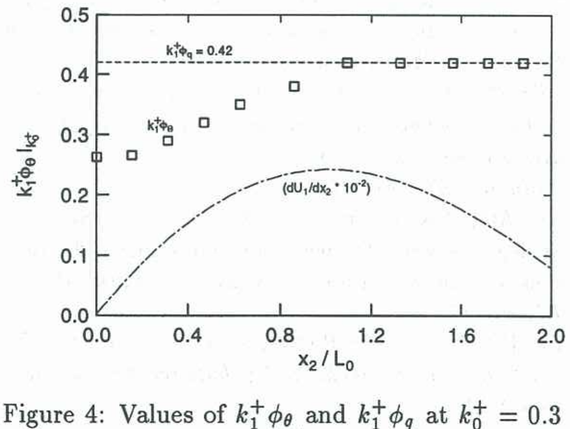

Figure 4: Values of $k_1^+\phi_\theta$ and $k_1^+\phi_q$ at $k_0^+ = 0.3$ across the wake. - - -, $k_1^+\phi_q$; □, $k_1^+\phi_\theta$; — - —, mean shear.

mean-square (rms) values of α and β. Figure 5(a,b) shows the jpdfs $p_{u_2q^2}$ and $p_{u_2\theta^2}$ at $x_2/L_0 = 0$ and 1.5, respectively. At $x_2/L_0 = 0$ (Figure 5a), the correlations between u_2 and either q^2 or θ^2 are quite small ($\rho_{u_2q^2} = 0.001$, $\rho_{u_2\theta^2} = 0.067$). The axes of the jpdfs are nearly parallel to the ordinate. There is practically no similarity between $p_{u_2q^2}$ and $p_{u_2\theta^2}$. At $x_2/L_0 = 1.5$ (Figure 5b), u_2 is positively correlated with either q^2 or θ^2; the jpdf major axes are in the

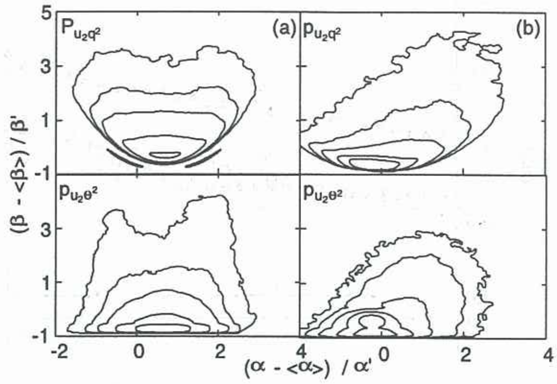

Figure 5: Jpdfs $p_{u_2q^2}$ and $p_{u_2\theta^2}$ at $x_2/L_0 = 0$ and 1.5. (a) $x_2/L_0 = 0$; (b) 1.5. Outer to inner contours : 0.0005, 0.005, 0.05, 0.1, 0.5.

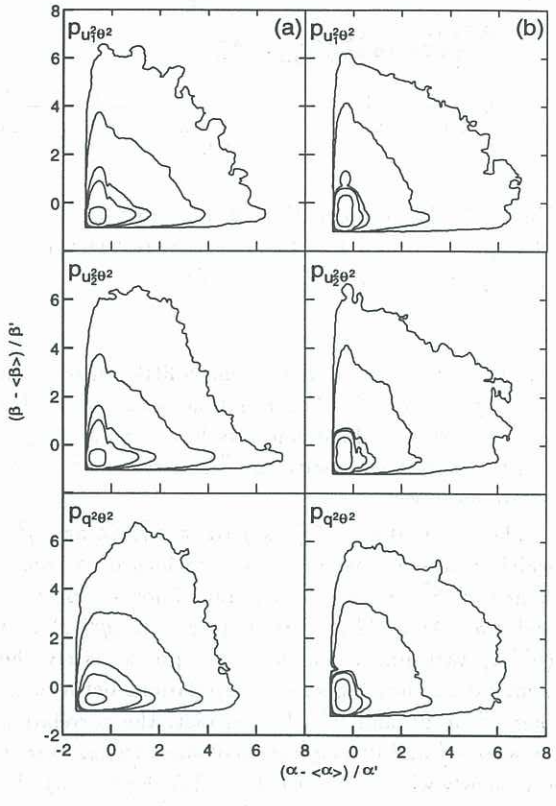

Figure 6: Jpdfs $p_{u_1^2\theta^2}$, $p_{u_2^2\theta^2}$ and $p_{q^2\theta^2}$ at $x_2/L_0 =$ 0 and 1.5. (a) $x_2/L_0 = 0$; (b) 1.5. Outer to inner contours : 0.0005, 0.005, 0.05, 0.1, 0.5.

second and fourth quadrants. The correlation coefficients are $\rho_{u_2q^2} = 0.5$, $\rho_{u_2\theta^2} = 0.42$ respectively. For smaller contour values, $p_{u_2q^2}$ and $p_{u_2\theta^2}$ reveal some similarity.

The jpdfs $p_{u_1^2\theta^2}$, $p_{u_2^2\theta^2}$ and $p_{q^2\theta^2}$ are shown in Figure 6 at $x_2/L_0 = 0$ and 1.5, respectively. At $x_2/L_0 = 0$ (Figure 6a), $p_{u_1^2\theta^2}$ and $p_{u_2^2\theta^2}$ are similar, large values of θ^2 are mainly correlated with small values of either u_1^2 or u_2^2 and vice versa. For $p_{q^2\theta^2}$, small q^2 is mainly correlated with large θ^2. The cor-

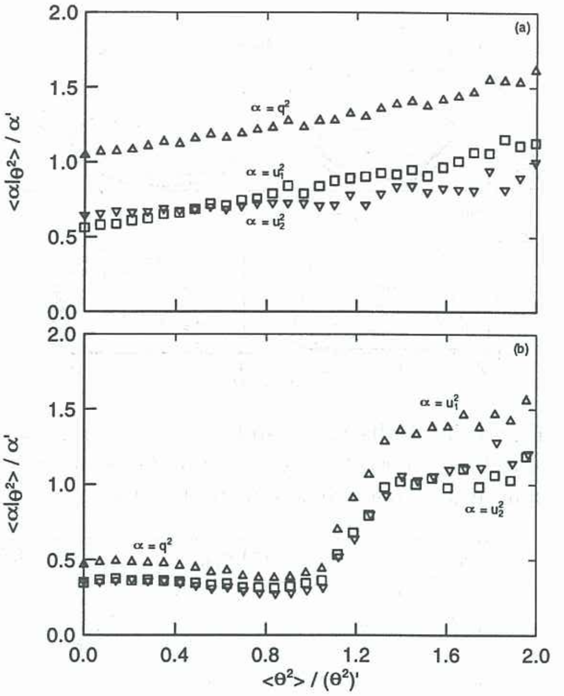

Figure 7: The expectation of α conditioned on θ^2 at $x_2/L_O = 0$ and 1.5. (a) $x_2/L_O = 0$; (b) 1.5. Δ, $\alpha = \langle q^2/\theta^2 \rangle$; $\square$, $\langle u_1^2/\theta^2 \rangle$; ∇, $\langle u_2^2/\theta^2 \rangle$.

relation between u_1^2 and θ^2 seems a little bigger than either $q^2 - \theta^2$ or $u_2^2 - \theta^2$ correlation. At $x_2/L_O = 1.5$ (Figure 6b), the three contours have similar shapes and the correlations between θ^2 and either u_1^2, u_2^2 or q^2 are improved.

The distributions of $\langle \alpha|_{\theta^2} \rangle$ ($\alpha = u_1^2$, u_2^2 and q^2), which represent the mean expectations of α conditioned on θ^2 are shown in Figure 7 for $x_2/L_O = 0$ and 1.5. At $x_2/L_O = 0$ (Figure 7a), $\langle q^2|_{\theta^2} \rangle$ and $\langle u_1^2|_{\theta^2} \rangle$ vary linearly with θ^2 at approximately the same rate, indicating similar correlations between either q^2 or u_1^2 and θ^2. In contrast, the correlation between u_2^2 and θ^2 is quite small since $\langle u_2^2|_{\theta^2} \rangle$ varies very slowly with θ^2. At $x_2/L_O = 1.5$ (Figure 7b), the conditional expectations $\langle u_1^2|_{\theta^2} \rangle$, $\langle u_2^2|_{\theta^2} \rangle$ or $\langle q^2|_{\theta^2} \rangle$ are nearly constant for $\theta^2/(\theta^2)' < 1.2$ (the superscript denotes root-mean-square value). There is negligible correlation between either u_1^2, u_2^2 or q^2 and θ^2. For $\theta^2/(\theta^2)' \geq 1.2$, these correlations increase, that between q^2 and θ^2 being largest.

CONCLUDING REMARKS

A five-wire probe was used to measure all three velocity fluctuations with the temperature fluctuation in the far-wake of a cylinder. The spectra ϕ_q and ϕ_θ are closer to each other away from than on the centreline. This behaviour is attributed to an insufficient mixing in the centreline region. The similarity between the invariant quantities q^2 and θ^2 as inferred from jpdfs (Figures 5 and 6) or conditional expectations (Figure 7) is not as good as that between ϕ_q and ϕ_θ. The similarity between q^2 and θ^2 may be considered reasonable only for large amplitudes of these variables.

ACKNOWLEDGEMENT

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