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TEMPERATURE DISSIPATION RATE STATISTICS IN DECAYING GRID TURBULENCE

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ABSTRACT

Measurements of all 3 components of the mean temperature dissipation rate $\langle\epsilon_\theta\rangle$ have been made in decaying grid turbulence with a 4 cold-wire probe in the region of $x_1/M = 20-80$. The resulting values of $\langle\epsilon_\theta\rangle$ agree to within $\pm10\%$ with the values inferred from the streamwise decay rate of the temperature variance for $x_1/M \leq 40$. The 3 components of $\langle\epsilon_\theta\rangle$ satisfy local isotropy also within $\pm10\%$ in this region. The assumption of statistical independence between θ and ϵ_θ (or either of the 3 components of ϵ_θ) is closely verified, reflecting the symmetry and local isotropy of the temperature field.

INTRODUCTION

The accurate determination of the temperature dissipation rate $\langle\epsilon_\theta\rangle$, defined by

$$\langle\epsilon_\theta\rangle_f = \alpha(\langle\theta_{,1}^2\rangle + \langle\theta_{,2}^2\rangle + \langle\theta_{,3}^2\rangle)\,, \qquad (1)$$

where α is the thermal diffusivity, angular brackets denote time averaging, $\theta_{,i}$ ($\equiv \partial\theta/\partial x_i$) is the spatial derivative of θ in the i^{th} ($i = 1, 2, 3$) direction and the subscript f has been used to distinguish estimates obtained via Eq. (1) from others (e.g. Eq. (2) given later), is important from both a theoretical viewpoint and in the context of turbulence modelling. By measuring all the 3 components in Eq. (1), the departure of the temperature field from local isotropy can be assessed. The modelling of $\langle\epsilon_\theta\rangle$ is important in a number of engineering situations involving heat and mass transfer. A knowledge of the joint probability density function (jpdf) between θ and ϵ_θ is important for predicting turbulent diffusion flames (e.g. Bilger, 1989).

There have been many published estimates of $\langle\epsilon_\theta\rangle$ in non-homogeneous turbulent flows (e.g. Freymuth and Uberoi, 1971; Sreenivasan et al., 1977; Anselmet and Antonia, 1985; Krishnamoorthy and Antonia, 1987; Anselmet et al., 1994; Mi et al., 1995, hereafter MAA). However, in only a few cases (Sreenivasan et al., 1977; Antonia et al., 1988; Anselmet et al., 1994) have all 3 components of $\langle\epsilon_\theta\rangle$ been estimated simultaneously. More commonly, a pair of parallel cold wires have been used in each of the 3 directions (e.g. Krishnamoorthy and Antonia, 1987; MAA) or in either the x_2 or x_3 direction with Taylor's hypothesis being used to estimate the x_1 derivative (e.g. Antonia and Browne, 1986). Results for the ratios $k_1 = \langle\theta_{,2}^2\rangle/\langle\theta_{,1}^2\rangle$ and $k_2 = \langle\theta_{,3}^2\rangle/\langle\theta_{,1}^2\rangle$ indicate varying degrees of anisotropy depending on the flow. For example, on the wake centreline, $k_1 = 1.9$ and $k_2 = 1.6$ (Antonia and Browne, 1986) whereas on the axis of a round jet (e.g. MAA), $k_1 = 0.98$ and $k_2 = 1.03$. MAA attributed the improved isotropy of the round jet, relative to either the plane jet or plane wake, to the more complex 3-D organization of this flow. Somewhat surprisingly, no measurements have been reported for the 3 components of $\langle\epsilon_\theta\rangle$ in grid turbulence although Mydlarski and Warhaft (1998) found that, for a grid flow with a constant transverse mean temperature gradient, $k_1 = k_2 = 1.4$. The inference is that the anisotropy of the small-scale temperature field must be caused by the mean temperature gradient since the small-scale velocity field is isotropic or very nearly so. One aim of the present study was to provide reliable "benchmark" data for both ϵ_θ and its mean value $\langle\epsilon_\theta\rangle$ in decaying grid turbulence without a mean temperature gradient. In this case, the transport equation for $\langle\theta^2\rangle$ simplifies to

$$\langle\epsilon_\theta\rangle_d = -U_1\frac{d(\theta^2/2)}{dx_1} \qquad (2)$$

where the subscript d is a reminder that $\langle\epsilon_\theta\rangle$ is obtained from Eq. (2) and U_1 is the mean velocity in the x_1 direction. Eq. (2) provides an important means of checking the accuracy of $\langle\epsilon_\theta\rangle_f$ or equivalently the accuracy with which the 3 components of $\langle\epsilon_\theta\rangle$ can be estimated with the 4 cold-wire probe.

The validation of $\langle\epsilon_\theta\rangle$ is obviously important before examining jpdfs of θ and ϵ_θ or θ and $\theta_{,i}^2$. Anselmet

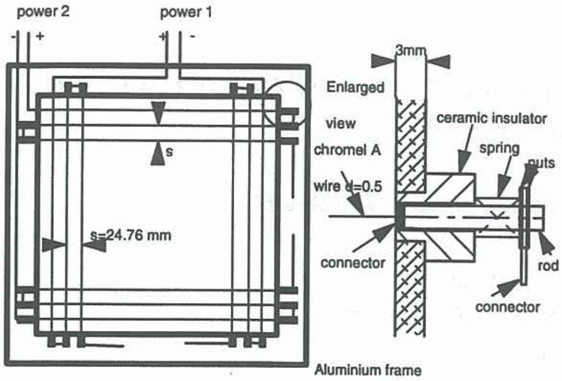

Figure 1: Sketches of the mandoline.

and Antonia (1985) noted that the main parameter influencing the joint statistics of θ and ϵ_θ is the asymmetry of θ. MAA suggested that both the asymmetry of $p(\theta)$, the probability density function (pdf) of θ, and the anisotropy of the small-scale turbulence influence the joint statistics of θ and ϵ_θ. The correlation between θ and ϵ_θ is important because the jpdf between these two quantities may be related to the average rate of creation and destruction of chemical species. Bilger (1976) suggested that θ and $\langle\epsilon_\theta\rangle$ may be statistically independent, viz. the jpdf $p_{\theta,\epsilon_\theta}$ can simply be replaced by

$$p_{\theta,\epsilon_\theta} = p(\theta)p(\epsilon_\theta)\ . \qquad (3)$$

Eq. (3) has been approximately verified in a plane jet (Anselmet and Antonia, 1985) and in a round jet (MAA).

EXPERIMENTAL DETAILS

Measurements were made on the centreline of the working section (350 mm × 350mm, 2.4 m long) of a non-return blower-type low turbulence wind tunnel downstream of a biplane grid in the range of $x_1/M = 20-80$ (where x_1 is measured from the grid plane and M is the mesh size of the grid). The mean velocity U_1 was 7 m/s. A square mesh ($M = 24.76$ mm with 4.76 mm × 4.76 mm square rods) grid, located at the entrance to the working section, was used with a solidity of 0.35.

A mandoline similar in design to that of Warhaft and Lumley (1978) was used to heat the flow. For all measurements, it was fixed at $1.5M$ downstream of the grid. The mandoline was constructed fom fine Chromel-A wires of 0.5 mm diameter. It comprised two parts separated by 15 mm in the streamwise direction : the wires were horizontal in one and vertical in the other. Each part has a resistance of about 22Ω and was heated by a power supplier with a total power consumption of about 2 KW for the two. The mean temperature ΔT relative to ambient was about 3°C in the tunnel. The wire separation in each part was 24.76 mm, i.e. the same as M. To prevent sagging, small springs were used to keep each wire under tension. A sketch of the mandoline is shown in Figure 1. The probe comprised two pairs of parallel cold wires, one in the $x_1 - x_2$ plane and the other in the $x_1 - x_3$ plane. This allowed temperature derivatives to be determined in the x_3 and x_2 directions respectively. The wire separations Δx_2 and Δx_3 were about 1.15 mm. This corresponded to $\Delta x_2^* \equiv \Delta x_3^* = 5.3 - 2.3$ for $x_1/M = 20 - 80$ (the superscript * denotes normalization by the Kolmogorov length scale $\eta = \nu^{3/4}/\langle\epsilon\rangle^{1/4}$, ν is the kinematic viscosity of air, $\langle\epsilon\rangle$ is the mean turbulent energy dissipation rate and was determined by a separate experiment for the same experimental conditions using both the isotropic assumption and the decay rate of the mean turbulent energy). The spatial derivatives $\theta_{,2}$ and $\theta_{,3}$ were obtained from the finite difference approximations $\theta_{,2} = \Delta\theta/\Delta x_2$ and $\theta_{,3} = \Delta\theta/\Delta x_3$ respectively and $\Delta\theta$ is the difference between the temperature fluctuations from the parallel cold wires. $\theta_{,1}$ was obtained from $\partial\theta/\partial t$ using Taylor's hypothesis. This should be quite satisfactory since the turbulent intensity u_1'/U_1 is less than 2% in this experiment where u_1 is the velocity fluctuations in the longitudinal direction and the prime denotes root mean square (rms) value. The cold wires were etched from Wollaston Pt-10% Rh to an active length of about $800d_w$ ($d_w = 0.63\mu$m is the wire diameter). The output signals from the constant current anemometers were passed through buck and gain circuits and low-pass filtered at a cut-off frequency f_c close to f_K, the Kolmogorov frequency which can be estimated via $f_K = U_1/2\pi\eta$. The signals were then digitized into a personal computer using a 12 bit A/D converter at a sampling frequency of $2f_c$. The record duration was 52 s. The data were analysed on a VAX 780 computer.

EXPERIMENTAL RESULTS

The decays of the velocity and temperature variances downstream of a grid are given by

$$\left.\begin{array}{lcl}\left(\frac{U_1^2}{\langle u_1^2\rangle}\right) & \sim & \left(\frac{x_1}{M} - \frac{x_{0u}}{M}\right)^n \\ \left(\frac{\Delta T^2}{\langle\theta^2\rangle}\right) & \sim & \left(\frac{x_1}{M} - \frac{x_{0\theta}}{M}\right)^m\end{array}\right\} \qquad (4)$$

respectively, where $x_{0\theta}$ and x_{0u} are the effective origins for temperature and velocity. Sreenivasan et al. (1980) suggested that $x_{0\theta}$ and x_{0u} are equal. The velocity variance decay has been extensively studied (e.g. Comte-Bellot and Corrsin, 1966; Antonia et al., 1998), and there is reasonable concensus about the magnitude of the exponent n (about 1.25). The present velocity data (not shown here) without the mandoline indicated that $n \simeq 1.3$.

The decay of $\langle u_1^2\rangle$ was also checked after the mandoline was inserted. The value of n was unchanged.

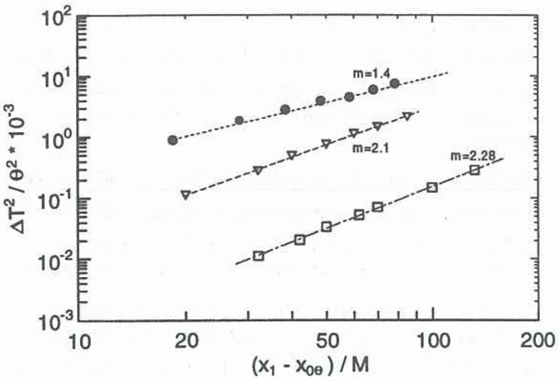

Figure 2: Streamwise decay of the temperature variance. ●, present; ▽, WL; □, Sreenivasan et al. (1980); straight lines are least-squares fits.

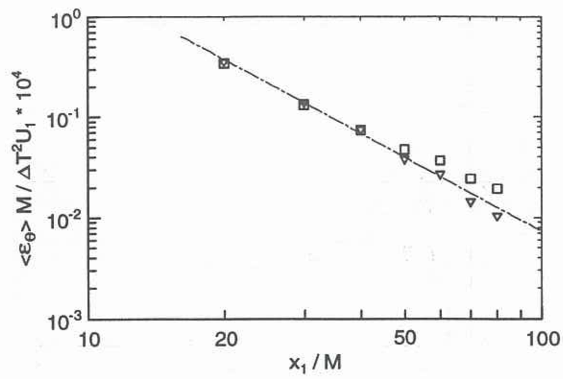

Figure 3: Comparison of $\langle\epsilon_\theta\rangle$ obtained from different methods, normalized by ΔT, M and U_1. □, $\langle\epsilon_\theta\rangle_f$; ▽, $\langle\epsilon_\theta\rangle_{iso}$; — - —, $\langle\epsilon_\theta\rangle_d$.

The same result was found when the mandoline was heated. In Figure 2, the decay of $\langle\theta^2\rangle$ is compared with the grid flow data of Sreenivasan et al. (1980) and Warhaft and Lumley (1978) [hereafter WL]. Using the trial and error method suggested by Comte-Bellot and Corrsin (1966), we estimated that $x_{0\theta}/M$ is 3 and m is 1.4. However, Sreenivasan et al. and WL reported $m = 2.2$ independently of ΔT. This difference probably reflects the different initial conditions in the different experiments.

Local isotropy of the small-scale velocity fluctuations has been verified in grid turbulence (Antonia et al., 1998). The temperature field satisfies homogeneity quite closely. ΔT is constant to within 1% in both the x_1 and x_2 directions. $\langle\theta^2\rangle$ is uniform to within 10% in the x_2 direction. Local isotropy of a scalar field requires k_1 and k_2 to be unity and the derivative skewness $S_{\theta_{,i}}$ $(= \langle\theta_{,i}^3\rangle/\langle\theta_{,i}^2\rangle^{3/2})$ to be zero. The present values of $S_{\theta_{,i}}$ (not shown here) are close to zero with $S_{\theta_{,1}}$ being the smallest. k_1 and k_2 are close to 1 ($\pm$10%) for $x_1/M \le 40$. For $x_1/M > 40$, the increase of k_1 and k_2 does not imply a departure from local isotropy. It is more likely due to an inappropriate spatial resolution in either Δx_2 or Δx_3. By varying the distance between the two cold wires (this was done in a separate experiment; results are not given here), it was found that $\langle\theta_{,2}^2\rangle$ and $\langle\theta_{,3}^2\rangle$ were overestimated for $x_1/M \ge 50$ where Δx_2^* or Δx_3^* was less than 2.8. The overestimation was about 10% at $x_1/M = 50$, resulting in $\langle\epsilon_\theta\rangle$ being overestimated by about 20% at this location (Figure 3). $\langle\epsilon_\theta\rangle_{iso}$ estimated from the isotropic assumption agrees well with $\langle\epsilon_\theta\rangle_d$, indicating the appropriate behaviour of the 4 cold-wire probe for $x_1/M \le 40$.

Figure 4 shows the jpdf between θ and ϵ_θ and between θ and $\theta_{,i}^2$. The jpdf $p_{\alpha,\beta}$ is defined such that $\iint_{-\infty}^{\infty} p_{\alpha,\beta}\, d\gamma\, d\delta = 1$, where γ and δ represent the centred and rms normalized values that α and β can take. Unlike the plane jet jpdf of Anselmet and Antonia (1985) [Figure 4 of their paper] which stretches out to negative values of θ, the jpds in Figure 4 shows good symmetry, indicating the independence between θ and ϵ_θ or $\theta_{,i}^2$.

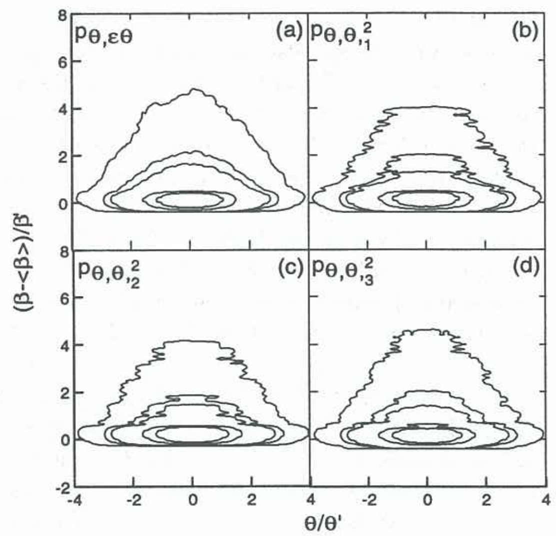

Figure 4: Jpdf between θ and β. (a) $\beta = \epsilon_\theta$; (b) $\theta_{,1}^2$; (c) $\theta_{,2}^2$; (d) $\theta_{,3}^2$. Outer to inner contours : 0.0004, 0.005, 0.05, 0.1, 0.4.

The correlation between θ and ϵ_θ or $\theta_{,i}^2$ can also be quantified by considering the expectations of $\langle\epsilon_\theta\rangle$ and $\langle\theta_{,i}^2\rangle$ conditioned on θ. These conditional expectations, normalized by the corresponding mean values, are shown in Figure 5 for $x_1/M = 40$. For $|\theta/\theta'| \le 2$, all the conditional expectations are equal to 1, indicating independence between either ϵ_θ or $\theta_{,i}^2$ and θ. Near $|\theta/\theta'| \simeq 4$ all the conditional expectations exhibit local maxima, indicating that large values of ϵ_θ or $\theta_{,i}^2$ may be associated with either small or large values of θ. The fluctuations of $\langle\epsilon_\theta|\theta\rangle$ or $\langle\theta_{,i}^2|\theta\rangle$ around this region almost certainy reflect the inadequate number of samples since there is only a 0.4%

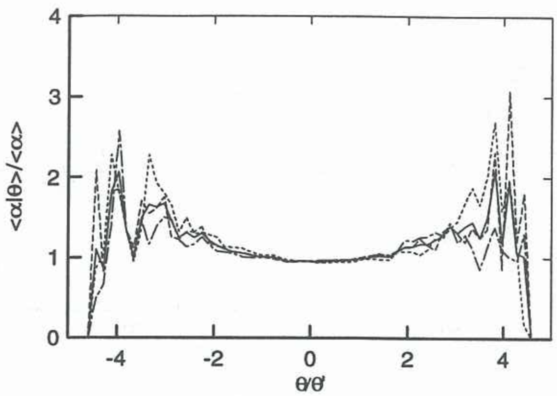

Figure 5: Expectation of α conditioned on θ. —, $\alpha = \epsilon_\theta$; - - -, $\theta^2_{,1}$; $\cdots$, $\theta^2_{,2}$; — - —, $\theta^2_{,3}$.

probability that the magnitude of θ exceeds $3\theta'$. Similar observations were also reported for a round jet by MAA at $x_2/R_u = 0$ and 0.53 (where R_u is the half-radius). An important difference between MAA and the present data is that the present values of $\langle\theta^2_{,1}|\theta\rangle$ peak at $|\theta/\theta'| = 4$ even though Taylor's hypothesis was used (the difference between $\langle\theta^2_{,1}|\theta\rangle$ obtained directly and by invoking Taylor's hypothesis was examined at by MAA). Jayesh and Warhaft (1992) calculated $\langle\epsilon_\theta|_\theta\rangle$, with ϵ_θ estimated using local isotropy and Taylor's hypothesis, at $x_1/M = 62.4$ and 82.4. At $x_1/M = 62.4$, $\langle\epsilon_\theta|\theta\rangle$ was strongly asymmetric about $\theta = 0$. At $x_1/M = 82.4$, the symmetry improves slightly, the distribution now suggesting a second local peak at $\theta = -3\theta'$. The distribution of $\langle\epsilon_\theta|\theta\rangle$ obtained by Jayesh and Warhaft (1992) in the same grid flow, but with a constant mean temperature gradient, indicated a strong dependence on θ, the largest dissipation rate being associated with the largest temperature fluctuation.

CONCLUSION

The performance of the 4 cold-wire probe was tested in decaying grid turbulence. The jpdfs between θ and ϵ_θ or $\theta^2_{,i}$ are symmetrical. These features are consistent with the independence between either ϵ_θ or $\theta^2_{,i}$ and θ. Expectations of ϵ_θ or $\theta^2_{,i}$ conditioned on θ support this independence, at least for $-2 < \theta/\theta' < 2$, reflecting the good symmetry and local isotropy of the temperature field.

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REFERENCES

ANSELMET, F. and ANTONIA, R. A. 1985. Joint statistics between temperature and its dissipation in a turbulent jet, *Phys. Fluids*, **28**, 1048-1054.

ANSELMET, F., DJERIDI, H. and FULACHIER, L. 1994. Joint statistics of a passive scalar and its dissipation in turbulent flows, *J. Fluid Mech.*, **280**, 173-197.

ANTONIA, R. A. and BROWNE, L. W. B. 1986. Anisotropy of the temperature dissipation in a turbulent wake, *J. Fluid Mech.*, **163**, 393-403.

ANTONIA, R. A., SHAH, D. A. and BROWNE, L. W. B. 1988. Dissipation and vorticity spectra in a turbulent wake, *Phys. Fluids*, **31**, 1805-1807.

ANTONIA, R. A., ZHOU, T. and ZHU, Y. 1998. Three-component vorticity measurements in a turbulent grid flow, *J. Fluid Mech.* (to appear)

BILGER, R. W. 1976 Turbulent jet diffusion in flames, *Prog. Energy Combust. Sci.*, **1**, 87-109.

BILGER, R. W. 1989. Turbulent diffusion in flames, *Ann. Rev. Fluid Mech.*, **21**, 101-135.

COMTE-BELLOT, G. and CORRSIN, S. 1966. The use of a contraction to improve the isotropy of grid-generated turbulence, *J. Fluid Mech.*, **25**, 657-682.

FREYMUTH, P. and UBEROI, M. S. 1971. Structure of temperature of fluctuations in a turbulent wake behind a heated cylinder, *Phys. Fluids*, **14**, 2574-2580.

JAYESH and WARHAFT, Z. 1992. Probability distribution, conditional dissipation, and transport of passive temperature fluctuations in grid-generated turbulence, *Phys. Fluids A*, **4**, 2292-2307.

KRISHNAMOORTHY, L. V. and ANTONIA, R. A. 1987. Temperature dissipation measurements in a turbulent boundary layer, *J. Fluid Mech.*, **176**, 265-281.

MI, J., ANTONIA, R. A. and ANSELMET, F. 1995. Joint statistics between temperature and its dissipation rate components in a round jet, *Phys. Fluids*, **7**, 1665-1673.

MYDLARSKI, L. and WARHAFT, Z. 1998. Passive scalar statistics in high-Péclet-number grid turbulence, *J. Fluid Mech.*, **358**, 135-175.

SREENIVASAN, K. R., ANTONIA, R. A. and DANH, H. Q. 1977. Temperature dissipation fluctuations in a turbulent boundary layer, *Phys. Fluids*, **20**, 12378-1249.

SREENIVASAN, K. R., TAVOULARIS, S., HENRY, R. and CORRSIN, S. 1980. Temperature fluctuations and scales in grid-generated turbulence, *J. Fluid Mech.*, **100**, 597-621.

WARHAFT, Z. and LUMLEY, J. L. 1978. An experimental study of the decay of temperature fluctuations in grid-generated turbulence, *J. Fluid Mech.*, **88**, 659-684.