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NUMERICAL MODELLING OF DOUBLE DIFFUSIVE CONVECTION FLOW PROBLEMS IN POROUS ROCK MASSES

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ABSTRACT

We use the finite element method to solve double diffusive convection flow problems of pore-fluid in porous rock masses when they are heated from below. In particular, we investigate how the horizontal gradient of species concentration affects the convective pore-fluid flow, which is driven by the vertical temperature gradient in a porous medium. The related numerical results have demonstrated that (1) the horizontal gradient of species concentration may strongly destabilize the vertical temperature-gradient driven convective pore-fluid flow; (2) the enhanced convective pore-fluid flow due to the double diffusion can further influence the distributions of both the temperature and species concentration in the porous medium.

INTRODUCTION

Numerical modelling of convective pore-fluid flow is very important in various scientific and engineering fields. With the exploration geoscience taken as an example, pore-fluid flow in porous rock masses is the sole agent to transport minerals from one place to another in the earth's crust, so that it usually plays a crucial role in the orebody formation and mineralization in the upper crust of the earth. Thus, to explore the economical minerals resources, one has to analyse and predict the pore-fluid flow patterns in the earth's crust consisting of porous rock masses. Since it usually takes millions or billions years for the formation of a giant orebody to complete, numerical modelling is often the natural choice to solve the pore-fluid flow problems in porous rock basins consisting of complicated geometry and material characteristics.

From the orebody formation point of view, the main driving force to result in pore-fluid flow in the earth's crust may be due to the pressure gradient in a mechanical process, the temperature gradient in a thermal process or the species concentration gradient in a chemical reaction process. Since the time scale involved in these three processes are usually quite different, it may be reasonable to consider these processes separately in a numerical computation. For this reason, we can treat the porous rock mass as a rigid medium by assuming that the consolidation of the rock mass caused by a mechanical process has been completed. This certainly allows us to focus this study on the pore-fluid flow generated by the temperature gradient and species concentration gradient in porous rock masses.

According to the classical theory (Nield and Bejan, 1992, Phillips, 1991), pore-fluid flow in the earth's crust is stable if the Rayleigh number of the system is subcritical. The convective pore-fluid flow occurs when the Rayleigh number of the system is either critical or supercritical. Since the convective pore-fluid flow can facilitate the transport and mixing of minerals, it may generate high grade, giant minerals deposits in the upper crust of the earth. For this reason, we have developed numerical methods/techniques to predict the temperature gradient driven convective pore-fluid flow, which is so called the single diffusion problem, in hydrothermal systems (Zhao, Mühlhaus and Hobbs, 1997, 1998, Zhao, Hobbs and Mühlhaus, 1998). However, if the thermal buoyancy effect due to the temperature gradient is comparable with the chemical buoyancy effect due to the species concentration gradient, the double diffusion caused by both the temperature gradient and the species concentration gradient must be considered in the numerical analysis. In terms of the effect of the temperature and species concentration gradients on the convective pore-fluid flow in a porous medium, the picture is clear when the vertical temperature gradient is parallel to the species concentration gradient. In this case, if the thermal buoyancy caused by the vertical temperature gradient is in the same direction as the chemical buoyancy caused by the species concentration gradient, consideration of the double diffusion results in an augmented convective pore-fluid flow, compared with consideration of the vertical temperature gradient alone. On the other hand, if the thermal buoyancy is opposite to the chemical buoyancy, consideration of the double diffusion results in a weakened convective pore-fluid flow. When the vertical temperature gradient is orthogonal to the species concentration gradient, the picture is not clear with regard to the effect of the temperature and species concentration gradients on the convective pore-fluid flow in a porous medium, because little research has been done on this particular problem.

In this paper, we use the finite element method to solve the above-mentioned double diffusive convection flow problem in a porous medium. The governing partial differentiation equations in the analysis are the continuity equation, Darcy's equation, the energy equation and mass transport equation. The Boussinesq assumption is employed to consider a change in pore-fluid density due to a change in pore-fluid temperature and a change in the species concentration within the pore-fluid. The related numerical solutions from the double diffusive convection flow problem will be compared with those from the single diffusive convection flow problem so as to gain a better understanding about whether consideration of the double diffusion affects the orebody formation in the earth's crust.

GOVERNING EQUATIONS OF THE PROBLEM

Basically, the problem considered in this study is a fully coupled problem between the pore-fluid flow, heat transfer and mass transport in a pore-fluid saturated porous medium. If the porous medium is isotropic and homogeneous, the governing equations of the problem in a steady state can be written as follows:

$$u_{i,i} = 0 \quad (1)$$

$$u_i = \frac{1}{\mu} K_{ii}(-p_{,i} + \rho_f g_i) \quad (2)$$

$$(\rho_0 c_p) u_j T_{,j} = \lambda^e_{jj} T_{,jj} \quad (3)$$

$$u_j C_{,j} = D^e_{jj} C_{,jj} \quad (4)$$

$$\rho_f = \rho_0[1 - \beta_T(T - T_0) - \beta_C(C - C_0)] \quad (5)$$

$$\lambda^e_{ii} = \phi\lambda_{ii} + (1-\phi)\lambda^s_{ii} \quad (6)$$

$$D^e_{ii} = \phi D_{ii} \quad (7)$$

where u_i is the pore-fluid velocity component in the x_i direction; p and T are pore-fluid pressure and temperature; C is the species concentration; ρ_0, T_0 and C_0 are the reference density of pore-fluid, the reference temperature and the reference species concentration used in the analysis; μ and c_p are the dynamic viscosity and specific heat of pore-fluid; λ_{ii} and λ^s_{ii} are the thermal conductivity for the pore-fluid and solid matrix in the x_i direction; ϕ and β are the porosity of the medium and the thermal volume expansion coefficient of the pore-fluid; K_{ii} is the permeability of the medium in the x_i direction; D_{ii} is the diffusivity of the species and g_i is the gravity acceleration component in the x_i direction.

Note that Equations (1) and (2) are the continuity equation and Darcy's equation, while Equations (3) and (4) are the energy equation and mass transport equation. Clearly, the source/sink terms are deliberately neglected in Equations (3) and (4).

Without loss of generality, the above governing differential equations can be written in the dimensionless form (Zhao, Mühlhaus and Hobbs, 1997, 1998). For the completeness of this paper, the resulting dimensionlized governing equations are given below.

$$u^*_{i,i} = 0 \quad (8)$$

$$u^*_i = -p^*_{,i} + Ra^T(T^* + NC^*)e_i \quad (9)$$

$$u^*_j T^*_{,j} = T^*_{,jj} \quad (10)$$

$$u^*_j C^*_{,j} = \frac{1}{Le} C^*_{,jj} \quad (11)$$

where the superscript * stands for the corresponding dimensionless variables; e_i is the gravity acceleration component in the x_i direction; Ra^T, Le and N are the following three dimensionless numbers of a system.

$$Ra^T = \frac{(\rho_0 c_p)\rho_0 g\beta\Delta T K_h H}{\mu\lambda_{e0}} \quad (12)$$

$$Le = \frac{\lambda_{e0}}{D_{e0}\rho_0 c_p} \quad (13)$$

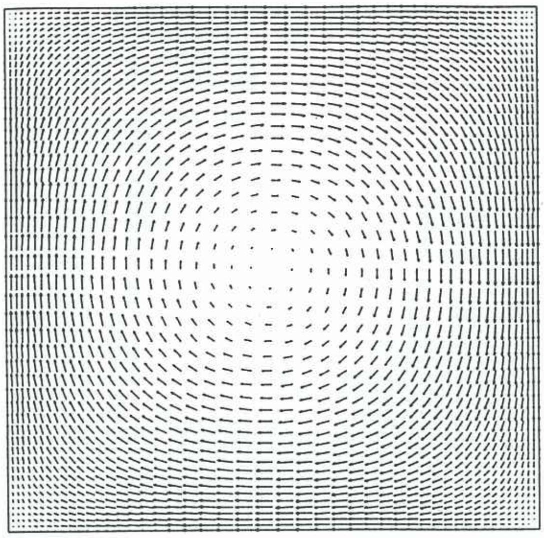
$$N = \frac{\beta_C \Delta C}{\beta_T \Delta T} \quad (14)$$

where λ_{eo} and D_{eo} are the reference thermal conductivity and the reference diffusivity of the species; ΔT and ΔC are the reference temperature difference and the reference species concentration difference; K_h and H are the reference permeability and length in the analysis.

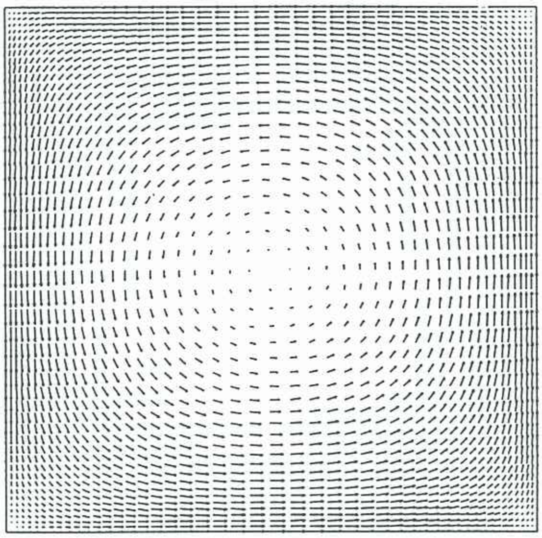
We have turned the above dimensionlized equations (i. e. Equations (8) to (11)) into the finite element formulations in the previous studies (Zhao, Mühlhaus and Hobbs, 1997, 1998). Here we will use the finite element method to investigate the effect of the double diffusion on the convective pore-fluid flow in a porous medium. The related numerical results from this investigation are reported in the next section.

EFFECT OF DOUBLE DIFFUSION ON THE CONVECTIVE PORE-FLUID FLOW IN A POROUS MEDIUM

As mentioned in the introduction, the main purpose of this study is to investigate how the horizontal species concentration gradient affects the convective pore-fluid flow resulted from the vertical temperature gradient alone in a porous medium. For this reason, we consider a square of one by one in size as the computational domain. This square computational domain is discretized into 2304 four-node quadrilateral finite elements with 2401 nodes in total. The dimensionless temperature at the bottom ($T^* = 1$) is hotter than that at the top ($T^* = 0$) of the computational domain. To simulate the horizontal species concentration gradient, the dimensionless species concentration at the left boundary ($C^* = 1$) is higher than that at the right boundary ($C^* = 0$). Both the top and the bottom boundaries are assumed to be impermeable in the vertical direction, while both the vertical boundaries are isolating and impermeable in the horizontal direction. The following dimensionless numbers are used in the computation. The Rayleigh number due to temperature only (Ra^T) is 40, which is just above the corresponding critical value for the convective pore-fluid flow driven by the temperature gradient alone. The Lewis number (Le) is 4 and the ratio of the chemical buoyancy to the thermal buoyancy (N) is -0.05.



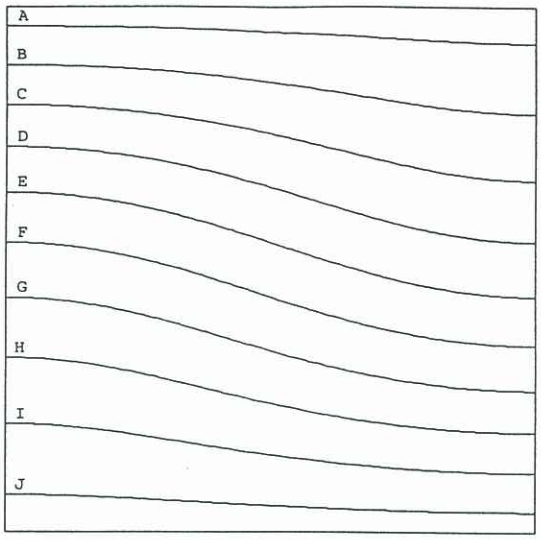
(Single diffusion)



(Double diffusion)

Figure 1 : The dimensionless pore-fluid velocity distributions due to the single and double diffusion

Figure 1 shows the dimensionless pore-fluid velocity distributions due to the single diffusion and double diffusion respectively. In this figure, the single diffusion stands for consideration of the vertical temperature gradient only in the computation, whereas the double diffusion stands for consideration of both the vertical temperature gradient and the horizontal species concentration gradient in the computation. It is observed that consideration of the vertical temperature gradient alone (i. e., the single diffusion) results in a clockwise convective pore-fluid flow, but consideration of both the vertical temperature gradient and the horizontal species concentration gradient (i. e., the double diffusion) results in an anti-clockwise convective pore-fluid flow. This indicates that consideration of the horizontal species concentration gradient has a profound effect on the convective pore-fluid flow, which is driven by the vertical temperature gradient alone. In addition, the maximum values of the dimensionless pore-fluid velocity are 2.04 and 4.61 for the single diffusion and double diffusion respectively. This means that consideration of the horizontal species concentration affects not only the pattern of the convective pore-fluid flow, but also the strength of the convective pore-fluid flow in a porous medium. Since both the pattern and strength of the convective pore-fluid flow may influence the formation of an ore deposit in the earth's crust, it is suggested that the horizontal species concentration gradient, if any, be considered in the analysis of orebody formation problems.



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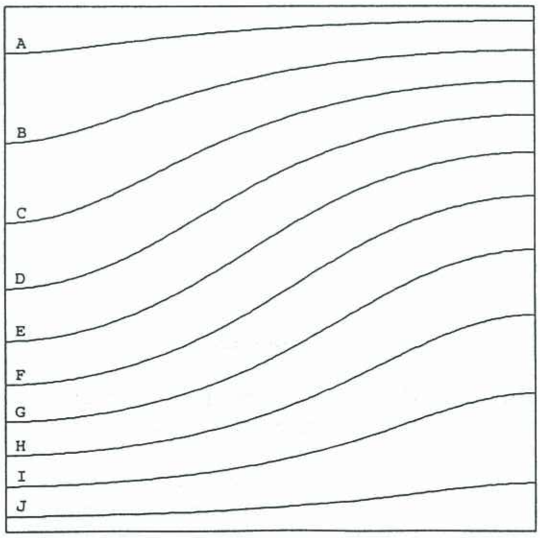
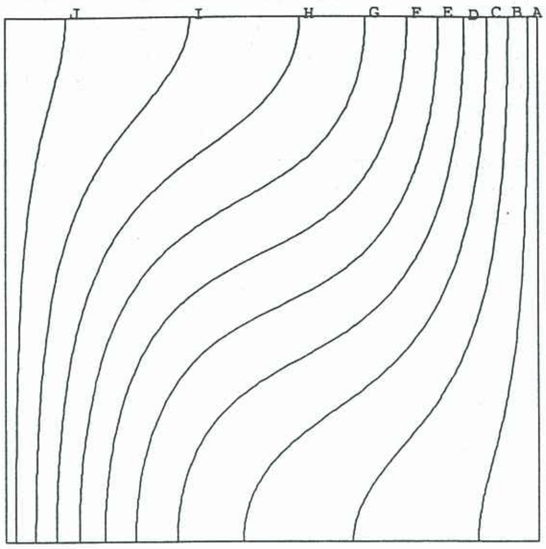


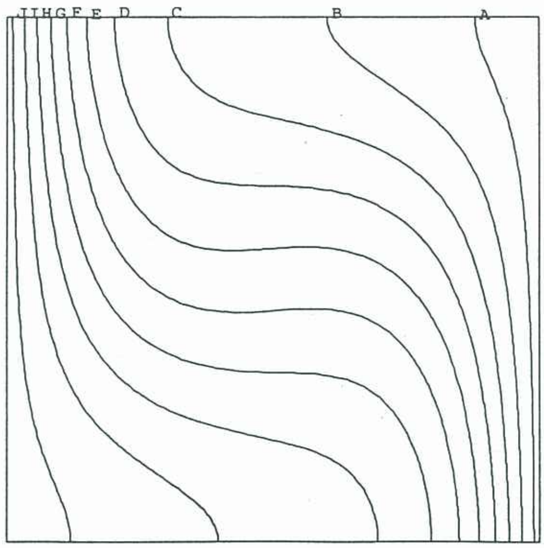
(Double diffusion)

Figure 2 : The dimensionless temperature distributions due to the single and double diffusion

Figure 2 shows the dimensionless temperature distributions for the single diffusion and double diffusion respectively. Clearly, the distribution pattern of the dimensionless temperature for the single diffusion is different from that for the double diffusion. The reason for this is that the pore-fluid patterns are totally different in the single and double diffusion analyses, as shown in Figure 1. This indicates that the enhanced convective pore-fluid flow due to the double diffusion can influence the temperature distribution in the porous medium.



(Single diffusion)



(Double diffusion)

Figure 3 : The dimensionless species concentration distributions due to the single and double diffusion

Figure 3 shows the dimensionless species concentration distributions for the single diffusion and double diffusion respectively. Not surprisedly, the distribution pattern of the dimensionless species concentration for the single diffusion is obviously different from that for the double diffusion because the pore-fluid pattern in the single diffusion analysis is totally different from that in the double diffusion analysis. This indicates that the enhanced convective pore-fluid flow due to the double diffusion can also strongly influence the species concentration distribution in the porous medium.

CONCLUSIONS

The numerical results from this study have demonstrated that (1) the horizontal species concentration gradient may strongly destabilize the vertical temperature-gradient driven convective pore-fluid flow; (2) the enhanced convective pore-fluid flow due to the double diffusion can further influence the distributions of both the temperature and species concentration in a porous medium.

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