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VECTOR ALIGNMENT PROPERTIES OF PARTICLE-PAIR DISPERSION IN ISOTROPIC TURBULENCE

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ABSTRACT

Vector alignment properties central to the process of relative dispersion of Lagrangian fluid particle pairs are studied in isotropic turbulence, through the use of both direct numerical simulations and stochastic modeling. We consider the statistics of particle-pair relative velocity projected along and perpendicular to the separation vector, where a non-uniqueness difficulty in modeling is briefly discussed. The directional dependence of dispersion characteristics is measured with respect to the initial separation vector.

INTRODUCTION

It is well known (e.g. Sawford 1985) that pollutant transport problems can be studied and modeled effectively in a Lagrangian framework following the trajectories of discrete fluid elements in turbulent flow. In particular, the relative motion between a pair of material fluid particles is the key in the formulation of stochastic models aimed at predictions of the variance of concentration fluctuations (Thomson 1990). The fundamental variables here are two vectors, namely the instantaneous separation $\underline{r}(t) = \underline{x}^{(1)} - \underline{x}^{(2)}$ between two particles (labeled one and two), and their relative velocity $\underline{w}(t) = \underline{u}^{(1)} - \underline{u}^{(2)}$.

In this paper we study the directional dependence and angular dynamics of $\underline{r}(t)$ and $\underline{w}(t)$. In general $\underline{r}(t)$ evolves by a change in length due to motion along $\underline{r}$, and by a change in direction due to motion in the plane orthogonal to $\underline{r}$. Consequently, we distinguish between projections of the relative velocity along and perpendicular to $\underline{r}$: i.e., respectively,

$$u_r \equiv (\underline{r} \cdot \underline{w})/r = |\underline{w}| \cos\theta_1 \qquad (1)$$

(where $\theta_1(t)$ is the angle between $\underline{r}(t)$ and $\underline{w}(t)$) and

$$u_\perp \equiv |\underline{w}||\sin\theta_1| \,. \qquad (2)$$

The "longitudinal" velocity u_r determines the rate of change of the separation distance, whereas the "transverse" component $u_\perp$ (which is by definition always positive) is equal in magnitude to the angular momentum vector defined as $\underline{r} \times \underline{w}/r$.

In isotropic turbulence the magnitude (r) of $\underline{r}$ is, of course, of primary interest, so that dispersion modeling in terms of u_r is a promising approach (Borgas & Yeung 1998, Kurbanmuradov 1997). However, the behavior of $u_\perp$ is important in anisotropic flows where the orientation of $\underline{r}(t)$ relative to specific directions in the flow is of interest. The change in orientation of $\underline{r}(t)$ over time is given by the angle $\theta_2(t)$ between the initial and instantaneous separation vectors $\underline{r}(0)$ and $\underline{r}(t)$, and the mean-squared separation in the direction of $\underline{r}(0)$ is

$$\langle s^2 \rangle \equiv \langle (\underline{r}(t).\underline{r}(0))^2/r_0^2 \rangle = \langle r^2 \cos^2\theta_2 \rangle \qquad (3)$$

Clearly, θ_2 is initially zero, but its subsequent distribution can give useful information on any anisotropy in dispersion characteristics.

In the following we consider the statistical properties of the variables u_r, $u_\perp$, θ_1 and θ_2 in isotropic turbulence. Results are obtained from direct numerical simulation (DNS) at Taylor-scale Reynolds number 230 for small and large initial separation and compared with stochastic modeling. Brief information on simulation and modeling strategies is first provided.

APPROACH

Lagrangian quantities are difficult to measure but can be obtained from DNS. In this work ensembles of particle pairs of prescribed initial separations ($r_0 \equiv |\underline{r}(0)|$) are tracked and analyzed in a manner similar to a previous study at lower Reynolds number (Yeung 1994). Simulations at 512^3 grid resolution are carried out on parallel computers. Our emphasis here is on the limits of small and large r_0, taken to be 1/4 and 256 times of the Kolmogorov scale (η) respectively, the latter being about 1.32 integral length scales at the prevailing Reynolds number. The ratio of the Lagrangian integral time scale to the Kolmogorov time scale is about 20. Care is taken to ensure an adequate ensemble size for the (non-dimensional) statistics presented.

For Lagrangian modeling the time-dependent probability density function (PDF) for the separation vector $\underline{r}$, $P(\underline{r}(t), t; \underline{r}(0))$ for given initial separation is the fundamental quantity. The PDF of the magnitude of $\underline{r}$, i.e., $P(r, t; r_0)$ is a very important part, containing all the scale dependence of the process, and has been the focus of recent modelling. However for an anisotropic source of contaminant material, the vector orientation of relative dispersion will influence the evolution of the resulting concentration statistics, although ultimately an isotropic state develops. The mixing process is thus two-fold: the growth of separation distance in time describes how blobs of fluid from increasingly far apart mix together, while angular variables (like θ_2) describe how blobs from different parts of the sphere for fixed r mix to homogenize the concentration distribution on the sphere. Models of relative dispersion for the vector $\underline{r}$ (Thomson 1990; Borgas & Sawford 1994) have a key weakness, which is the arbitrary prescription of the 'spin' of the separation vector. This non-uniqueness difficulty is inherent for stochastic modeling in general (Borgas, Flesch & Sawford 1997) and is closely related to the angular momentum of fluid particle pairs.

Although models for Lagrangian statistics of θ_1 and $u_\perp$ are not unambiguously available, in recent work (Yeung, Borgas & Franzese 1998) we have devised an unique model for u_r and r. Here we attempt to model the PDF of $\underline{r}$ using a simple approximation:

$$P(\underline{r}, t; \underline{r}(0)) = P(r, t; r_0) p(\underline{r}/r; \underline{r}(0)) \ . \qquad (4)$$

We obtain the PDF for the direction cosines r_i/r by assuming a Gaussian distribution but remove the dependence on r by averaging over separations. We thus assume weak coupling between the magnitude of the separation vector and its orientation. Furthermore because a second angular variable, measuring the azimuth about the axis $\underline{r}(0)$, is always purely random and may be ignored, the orientation statistics of $\underline{r}$ can be described completely by the angle θ_2 defined in Eq. 3. Consequently, the second term on the r.h.s of Eq. 4 is represented by a modeled PDF for θ_2, $P(\theta_2)$. The input parameters are the non-dimensional quantities

$$\Delta(t) = \frac{r_0^2}{\langle r^2(t)\rangle} \ , \quad \zeta(t) = \frac{\langle s^2\rangle}{\langle r^2(t)\rangle} \qquad (5)$$

which we take from DNS. The functional form of this PDF (whose details are omitted here because of space) satisfies both limiting cases for small and large times. In particular at $t = 0$ we have $\Delta = 1$, $\zeta = 1$ and $P(\theta_2)$ takes the form of a delta function at zero. At large times $\Delta \sim 0$ and $\zeta \sim 1/3$ (because of isotropic relative displacements when the memory is lost) the PDF takes the asymptotic form $P(\theta_2) = \frac{1}{2}\sin\theta_2$ (which corresponds to a uniform distribution for $\cos\theta_2$). Note that if r_0 is small enough to lie within an inertial-range of scales, we expect that the the initial-orientation dependence will be rapidly lost, so that 'large' time may in fact cover most of the dispersion. The transition between these extremes in both simulation and modeling is shown below.

For practical dispersion purposes, the simple model of angular spin given in Eq. 4 is all that is required. However, of the input parameters Δ and ζ, only Δ can be predicted unambiguously by (quasi-one dimensional) stochastic models. The joint stochastic model process (in time) for (r, u_r) can in principle be formulated rigorously using a unique one-to-one relationship between the model and the Eulerian PDF for longitudinal velocity u_r (Borgas & Yeung 1998), but we do not have sufficient knowledge of Lagrangian stochastic principles to model the full process for $(r, u_r, u_\perp)$. Thus in future developments it will be necessary to use the Lagrangian statistics for $u_\perp$ and ζ (say) to discriminate between possible models.

RESULTS AND DISCUSSION

In general, continual increase of the mean separation distance with time implies that u_r has a positive mean value, which is associated with the probability of acute alignment between $\underline{r}(t)$ and $\underline{w}(t)$ being greater than 0.5 (Yeung 1994). Figure 1 shows the evolution of the mean values of $|\underline{w}|$, u_r and $u_\perp$ for both small and large initial separation. It is seen that $u_\perp$ has a larger mean value than u_r at all times. Initially $\langle u_r\rangle$ is small, but grows linearly according to the Eulerian mean acceleration $d\langle u_r\rangle/dt = \langle u_\perp^2\rangle/r$. This early-time behavior persists longer for large r_0 since it then takes longer for $\underline{r}$ to change significantly. For small r_0 we observe rapid growth in all three quantities shown, corresponding to larger relative velocities as r increases rapidly compared with its initial value. The simulation data extends to a time period of about 8 Lagrangian integral time scales. At later times the vectors $\underline{w}$ and $\underline{r}$ becomes less aligned, which causes $\langle u_r\rangle$ to decrease slowly (while remaining positive). In addition, as the particles become far apart the coordinate components of $\underline{w}$ behave increasingly as independent and Gaussian. Accordingly the statistics of $|\underline{w}|$ and $u_\perp$ become similar to those expected for $|\underline{w}|^2$ and $u_\perp^2$ to be chi-square distributed with three and two degrees of freedom respectively.

The distribution of the alignment angle θ_1 is a determining factor in the statistics of u_r versus $u_\perp$ as discussed above. Figure 2 shows the PDF of this angle at selected time instants for $r_0/\eta = 1/4$. Initially, because particle pairs begin their motion from randomly selected positions in a field of homogeneous turbulence, their statistics are essentially Eulerian. Under such circumstances isotropy suggests an approximately symmetric PDF for θ_1, although there is a small bias towards acute alignment in the $t = 0$

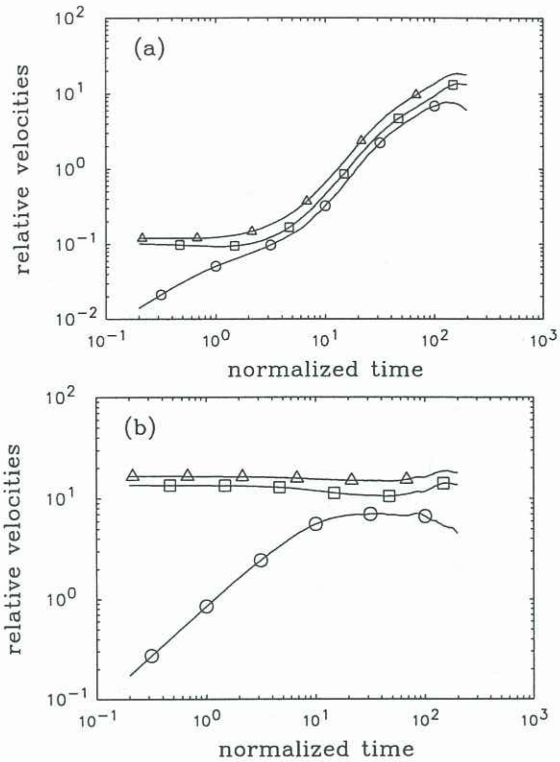

Figure 1(a,b): Evolution of $\langle|\underline{w}|\rangle$ (triangles), $\langle u_r\rangle$ (circles) and $\langle u_\perp\rangle$ (squares), normalized by Kolmogorov scales of time and velocity (τ_η, u_η). for (a) $r_0/\eta = 1/4$ and (b) $r_0/\eta = 256$.

data shown. The latter feature arises because (with $r_0/\eta \ll 1$) u_r is approximately proportional to a longitudinal velocity gradient (such as $\partial u/\partial x$) which is negatively skewed. However, it is clear that the PDF shifts noticeably towards increased acute alignment very quickly (in fact, in less than one Kolmogorov time scale). Furthermore, this trend is reversed at between 10 to 20 τ_η, and the PDF ultimately relaxes (line J) back towards a state of weaker alignment. The peak value of $\langle\cos\theta_1\rangle$ was found to be slightly greater than 0.8, which is close to the value observed at lower Reynolds number (Yeung 1994).

The fact that $\langle u_\perp\rangle$ is greater than $\langle u_r\rangle$ implies that there is considerable relative motion in directions perpendicular to $\underline{r}(t)$. In time, this effect tends to change the orientation of $\underline{r}(t)$ and hence would influence the directional dependence of dispersion. For isotropic turbulence ultimately the vector $\underline{r}(t)$ hould become randomly oriented in space. Here each coordinate axis has equal significance but it is useful to consider dispersion in the direction of $\underline{r}(0)$, via the quantities $\langle s^2\rangle$ and ζ (as defined in Eq. 5). Figure 3 shows the evolution of the ratio ζ for both small and large initial separations. Clearly, as expected this ratio decreases from unity at $t = 0$ towards the value 1/3 at large times. The time taken to reach this asymptote depends on r_0, since if $\underline{r}$ is initially a

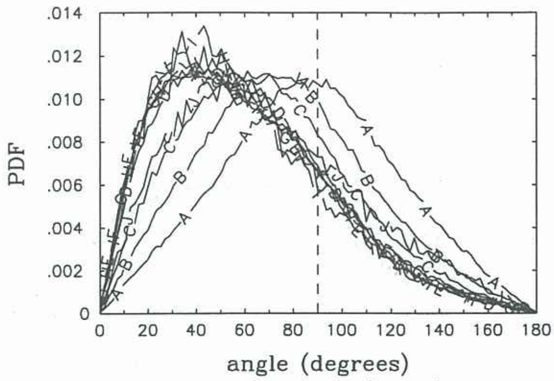

Figure 2: PDF of θ_1 (in degrees) at t/τ_η=0, 0.4, 1, 2, 4, 8, 16, 32, 64, 196 (lines A to J), for small initial separation ($r_0/\eta = 1/4$).

long vector then it takes much longer for the direction to change significantly. Still, there is remarkable similarity in shape between the curves for small and large r_0, with ζ showing an intermediate period of logarithmic decrease in both cases.

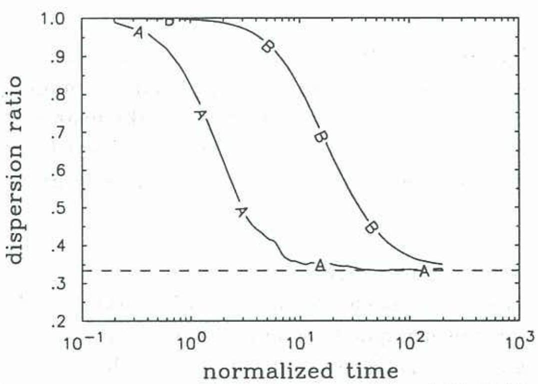

Figure 3: Evolution of the ratio $\zeta = \langle s^2\rangle/\langle r^2\rangle$, for $r_0/\eta = 1/4$ (A) and 256 (B).

The PDF of θ_2 giving the angular orientation of $\underline{r}$ is shown in Figure 4. The key issues are how quickly this PDF evolves towards the shape for neutral alignment, and whether its functional form at intermediate times can be modeled accurately. Comparison with Figure 3 shows that most of the transition occurs at times when ζ has decreased close to 1/3: for instance at $t/\tau_\eta = 2$ for $r_0/\eta = 1/4$ ζ has dropped to 0.6 but the probability of θ_2 being acute is still almost unity. For small r_0 the PDF at later times effectively coincide with the theoretical asymptote. By contrast, as stated earlier the alignment for large r_0 changes rather slowly, with the PDF being still asymmetric and evolving at the end of the simulations.

Figure 5(a) shows model results for θ_2 for small r_0. The best agreement with DNS (Figure 4a) is seen at larger times after the effects of highly non-Gaussian viscous straining on the particle pair have relaxed. Nevertheless, overall the results are quali-

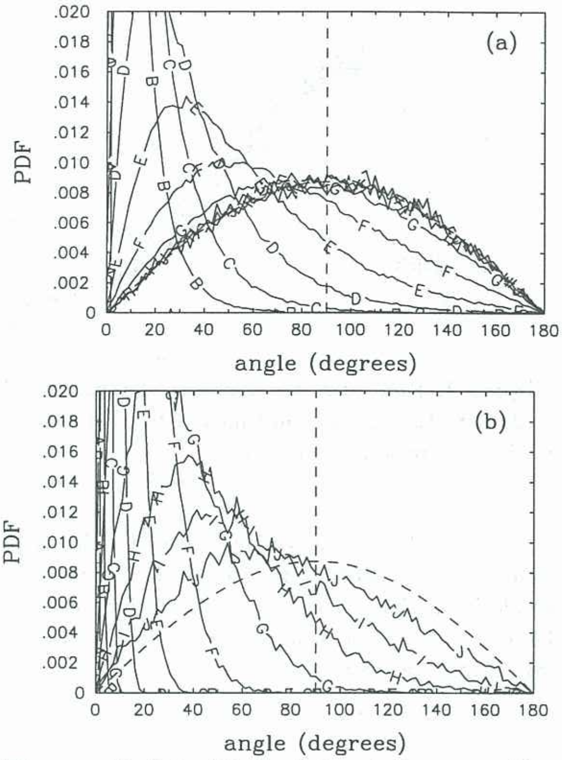

Figure 4(a,b): PDF of θ_2 at the same time instants as in Figure 2, for (a) $r_0/\eta = 1/4$ and (b) $r_0/\eta = 256$. Some of the high peaks near $\theta_2 = 0$ for early times are truncated. Dashed curve denotes scenario of purely random orientation.

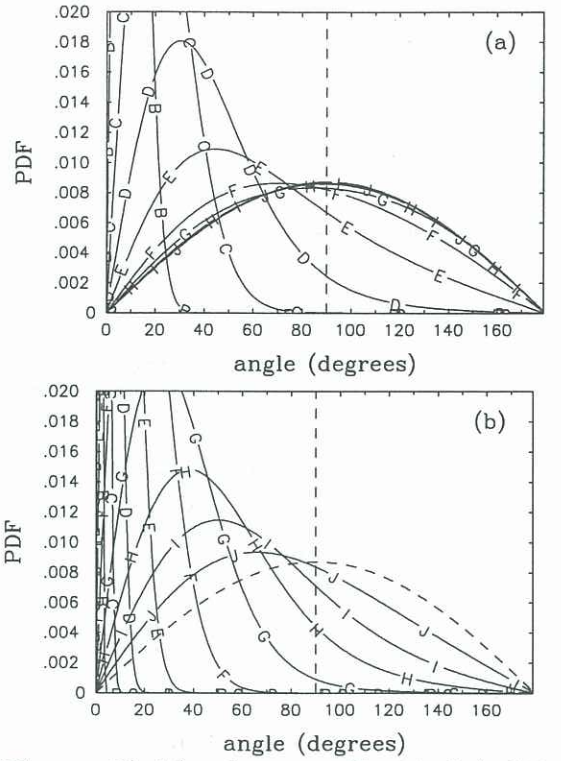

Figure 5(a,b): Same as Fig. 4, but from stochastic model.

tatively good. Figure 5(b) shows results for large r_0 where viscous effects are not as important, and the agreement is quantitatively good for all times and the slower relaxation to isotropy for $\underline{r}$ can be seen.

CONCLUSION

In summary we have studied important vector alignment properties in turbulent dispersion, including acute alignment between separation and relative velocity (which is favored at all times) and the tendency of the the separation vector to become randomly oriented relative to its initial direction. Results in isotropic turbulence from high-resolution numerical simulations and a simple new model for statistics of separation-vector rotation are in broad agreement. New understanding of processes in the plane perpendicular to the separation vector will be important for the development of future stochastic dispersion models in three dimensions and in anisotropic flows.

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