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COMPUTATIONAL MODELLING OF PARTICLE-LIQUID FLOW

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ABSTRACT

This paper presents Lagrangian and Eulerian models for particle-liquid flow. Model verification is made with well defined, published experimental data. Good agreement with the experimental data is found for mean velocity and turbulence behaviour at different concentrations. It is concluded that velocities profile for both phases become flatter with increasing particle concentrations, and the turbulent intensity of the fluid phase decreases with particle concentration.

INTRODUCTION

Two-phase flow exists in many areas of engineering & science, and some intriguing phenomena are often observed in such flow. Extensive studies have been carried out since 1950's, but a general analytical solution to the two-phase flow is still out of the question. To investigate two-phase flow, two numerical methods, the Lagrangian and Eulerian approaches, may have to be employed to reveal its fluid dynamics.

In the Lagrangian approach particle trajectories are obtained by solving a particle motion equation which can be derived from Newton's second law. Particle-particle collisions can also be simulated by introducing an artificial particle. In the Eulerian approach, the kinetic theory is used to model the characteristics of the high concentration particulate phase. The suppression of the turbulent energy of carrier phase caused by the presence of particulate phase is considered by modifying the standard k-ϵ turbulence model.

Verifications for both approaches have been made with two simple geometry piping particle-liquid flows. One is horizontal pipe flow given by Zisselmar & Molerus (1979), and another is downward pipe flow carried out by Nouri et al. (1987). All the experimental data were obtained at relatively high particle concentrations so that the effect of particle collisions can be assessed quantitatively.

THEORETICAL CONSIDERATIONS

The Eulerian Approach

After Reynold's averaging, the governing equations for both phases are given as follows:

fluid phase continuity:

$$\frac{\partial}{\partial x_i}(\alpha u_i) + \frac{\partial}{\partial x_i}\left(\overline{\alpha' u_i'}\right) = 0 \qquad (1)$$

particulate phase continuity:

$$\frac{\partial}{\partial x_i}(\alpha_p v_i) + \frac{\partial}{\partial x_i}\left(\overline{\alpha_p' v_i'}\right) = 0 \qquad (2)$$

fluid phase momentum:

$$\begin{aligned}
\frac{\partial}{\partial x_j}(\alpha \rho u_i u_j) &= -\frac{\partial}{\partial x_i}(\alpha p) \\
&+ \frac{\partial}{\partial x_j}\left[\mu\alpha\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \\
&- \frac{2}{3}\frac{\partial}{\partial x_i}\left(\alpha\mu\frac{\partial u_l}{\partial x_l}\right) \\
&+ \frac{\partial}{\partial x_j}\left[\mu\overline{\alpha'}\left(\frac{\partial \overline{u_i'}}{\partial x_j} + \frac{\partial \overline{u_j'}}{\partial x_i}\right)\right] \\
&- \frac{2}{3}\frac{\partial}{\partial x_i}\left(\overline{\alpha'}\mu\frac{\partial \overline{u_l'}}{\partial x_l}\right) \\
&- \rho\frac{\partial}{\partial x_j}\left(\alpha\overline{u_i' u_j'} + u_j\overline{\alpha' u_j'} + u_i\overline{\alpha' u_j'}\right) \\
&+ \rho\frac{\partial}{\partial x_j}\left(\overline{\alpha' u_i' u_j'}\right) + \alpha\rho g_i + F_i
\end{aligned} \qquad (3)$$

particulate phase momentum:

$$\begin{aligned}
\frac{\partial}{\partial x_j}(\alpha_p \rho_p v_i v_j) &= -\frac{\partial}{\partial x_i}(\alpha_p p) \\
&- \frac{\partial p_p}{\partial x_i} - \frac{\partial \overline{p_p'}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_p\alpha_p\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] \\
&+ \frac{\partial}{\partial x_j}\left[\mu_p\overline{\alpha_p'}\left(\frac{\partial \overline{v_i'}}{\partial x_j} + \frac{\partial \overline{v_j'}}{\partial x_i}\right)\right] \\
&- \frac{2}{3}\frac{\partial}{\partial x_i}\left[\left(\alpha_p\mu_p\frac{\partial v_l}{\partial x_l}\right) - \left(\overline{\alpha_p'}\mu_p\frac{\partial \overline{v_l'}}{\partial x_l}\right)\right] \\
&- \rho_p\frac{\partial}{\partial x_j}\left(\alpha_p\overline{v_i' v_j'} + v_j\overline{\alpha_p' v_j'} + v_i\overline{\alpha_p' v_j'}\right) \\
&- \rho_p\frac{\partial}{\partial x_j}\left(\overline{\alpha_p' v_i' v_j'}\right) + \alpha_p\rho_p g_i - F_i
\end{aligned} \qquad (4)$$

$$F_i = K_{ex}|v_i - u_i| \quad (5)$$

$$K_{ex} = \frac{3\alpha\alpha_p}{4v_t^2 D_p} C_D |v_i - u_i| \quad (6)$$

This set of equations (1) through (4) is suitable for all kinds of two-phase flows. In the case of dilute two-phase flow, the governing equations can be further simplified, i.e., all terms involving volume fraction fluctuations can be neglected. But this is not the case for high concentration flow in which the effects of volume fraction fluctuations should be accounted for (Elghobashi & Abou-Arab, 1983).

Turbulence Model

The standard k-ϵ turbulence model is employed in this work since it is widely used to simulate the two-phase flows. But some modifications are required to account for the effect of the particulate phase (Tu & Fletcher, 1994). The following k-ϵ turbulence model is used here:

$$\frac{\partial}{\partial t}(\alpha\rho k) + \frac{\partial}{\partial x_i}(\alpha\rho u_i k) = \frac{\partial}{\partial x_i}\left(\alpha\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \alpha G_k - \alpha\rho\epsilon + \Pi_e - k_w \quad (7)$$

$$\frac{\partial}{\partial t}(\alpha\rho\epsilon) + \frac{\partial}{\partial x_i}(\alpha\rho u_i \epsilon) = \frac{\partial}{\partial x_i}\left(\alpha\frac{\mu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_i}\right) + C_{1\epsilon}\alpha\frac{\epsilon}{k}G_k - C_{2\epsilon}\alpha\rho\frac{\epsilon^2}{k} + C_{3\epsilon}\frac{\epsilon}{k}\Pi_e - \epsilon_{\epsilon w} \quad (8)$$

$$G_k = \mu_t\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)\frac{\partial u_j}{\partial x_i} \quad (9)$$

$$\Pi_e = K_{ex}\left[\overline{u_i'(v_i' - u_i')} + (v_i - u_i)\overline{\alpha_p' u_i'}\right] \quad (10)$$

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \quad (11)$$

$$\sigma_k = 1.0, \quad \sigma_{1\epsilon} = 1.3, \quad C_{1\epsilon} = 1.44,$$
$$C_{2\epsilon} = 1.92, C_{3\epsilon} = 1.2, \quad C_\mu = 0.09 \quad (12)$$

where the additional source terms k_w, $\epsilon_{\epsilon w}$ added here are needed to balance the molecular diffusion of k which is finite at $y = 0$ and given by Chien (1980).

Following Simonin & Viollet's (1990) work, the correlations between the velocity fluctuations, $\overline{u_i'(v_i' - u_i')}$, are modeled as:

$$\overline{u_i'(v_i' - u_i')} = 2k\left[\frac{b + t_p/t_e}{1 + t_p/t_e}\right] - 2k \quad (13)$$

$$b = (1 + C_v)\left(\frac{\rho_p}{\rho} + C_v\right)^{-1} \quad (14)$$

in which $C_v = 0.5$ is the added-mass coefficient, t_p and t_e are the particle relaxation time and the turbulent Lagrangian integral time, respectively. Using gradient equations, the second fluctuation term on the left hand side of Eqs. (1) & (2) can be modeled as:

$$\overline{\alpha' u_i'} = -\overline{\alpha_p' u_i'} = \frac{\nu}{\sigma}\frac{\partial \alpha}{\partial x_i} \quad (15)$$

here σ is the turbulent Schmidt number and has a value of 0.67 in this work.

Kinetic Theory

By analogy with the molecular motion of gas, the kinetic theory is used to compute the viscosity and pressure of the high concentration particulate phase. Characteristics of the particulate phase are modelled as a function of granular temperature which accounts for the particles' turbulent motions, and is given as $\theta = v_i^2/3$. An additional equation for the granular temperature can be established based on the energy balance (Ding & Gidaspow, 1990).

$$\frac{3}{2}\left[\frac{\partial}{\partial t}(\alpha_p\rho_p\theta) + \frac{\partial}{\partial x_i}(\rho_p\alpha_p v_i\theta)\right] = \tau_s\frac{\partial v_i}{\partial x_i} - \frac{\partial}{\partial x_i}\left(k_\theta\frac{\partial\theta}{\partial x_i}\right) - \gamma + \phi_{fm} \quad (16)$$

$$\tau_s = \left[-p_p + \alpha_p\mu_p\frac{\partial v_i}{\partial x_i}\right]\delta_{ij} + 2\alpha_p\mu_{p,col}S_{ij} \quad (17)$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij} \quad (18)$$

in which $\tau_s\frac{\partial v_i}{\partial x_i}$ is the generation of energy by the solid stress tensor; $\frac{\partial}{\partial x_i}\left(k_\theta\frac{\partial\theta}{\partial x_i}\right)$ is the diffusion of energy; γ is the collisional dissipation of energy; θ_{fm} is the energy exchange between the fluid and the particulate phase. The detailed formulation is given by Ding & Gidaspow (1990).

The Lagrangian Approach

The trajectories of particles in the ith direction of Cartesian coordinates can be computed via the following equations (Yang et al. 1996)

$$\frac{\partial x_i}{\partial t} = v_i \quad (19)$$

$$\frac{\partial v_i}{\partial t} = F_d(u_i - v_i) + g_i(\rho_p - \rho)/\rho_p + F_{xi} + F_l \quad (20)$$

$$F_d = \frac{18\mu C_d R_{ep}}{24\rho D_p^2} \quad (21)$$

$$C_d = a_1 + a_2/R_{ep} + a_3/R_{ep}^2 \quad (22)$$

$$F_l = \frac{1}{2}\rho V_r^2\frac{\pi D_p^2}{4}C_l\frac{\omega_r \times V_r}{|\omega_r||V_r|} \quad (23)$$

$$C_l = 0.35\frac{|\omega_r|D_p}{|V_r|}|\omega_r \times V_r| \quad (24)$$

$$I\frac{d\omega}{dt} = -C_t\frac{1}{2}\rho\frac{D_p^5}{2}|\omega_r|\omega_r \quad (25)$$

in which $F_d\,(u_i - v_i)$ is the drag force per unit particle mass; F_l is additional force due to the particle collisions; a is constants that apply over several ranges of R_{ep}, as given by Morsi & Alexander (1972); the additional forces, F_{xi}, denote any possible external forces, such as "virtual mass" etc.

Turbulent effects on the particulate phase can also be considered in the Lagrangian approach by using the Random Walk (Zhou & Leschziner, 1991) or Continuous Random Walk (Thompson, 1987) models. But before using these models, the Stokes number and ratio of the particle size to the characteristic length of eddy should be checked (Gore & Crowe, 1985). For large values of both numbers, particle will not follow every motion of the fluid phase and the turbulence will not have significant effect on the particle. Otherwise these effects may have to be taken into account.

Because of computer power limitations, it is impossible to simultaneously compute all particles trajectories for a dense particulate flow. A statistical method may have to be used to account for the particle-particle interaction effects. Here, the particle collision dynamics is modeled by introducing an artificial particle during each time integration of the particle motion equation. The artificial particle is introduced to hit (or be hit by) the traveling particle during the trajectory calculation; the probability of these collisions depends on local particle concentrations and velocity. The moment equations have to be solved to determine the traveling particle velocity and direction. A mean free path of the particle is introduced based on the kinetic theory. During the time integration of the particle motion equation, a comparison of the accumulated time with the mean particle free traveling time will be made. If a large value is found, then a binary collision of the particles is allowed. Otherwise, no collision is considered. The artificial particle velocity vector is controlled using a random function, which is a Gaussian (normal) random number with a mean 0.0 and a standard deviation of 1.0, and is generated during the particles collision. In the same direction as the traveling particle, it will have a positive value; otherwise negative.

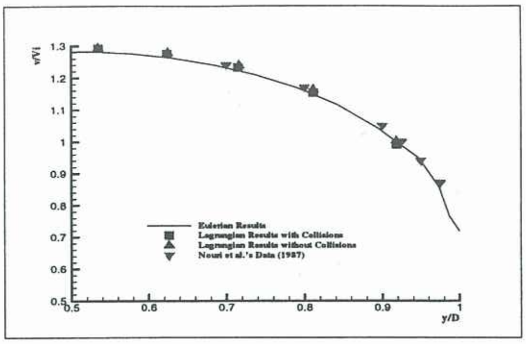

Figure 1: Comparison of Velocities at Volume Concentration of 6%

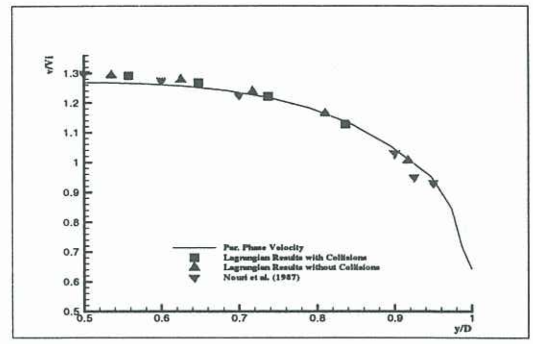

Figure 2: Comparison of Velocities at Volume Concentration of 14%

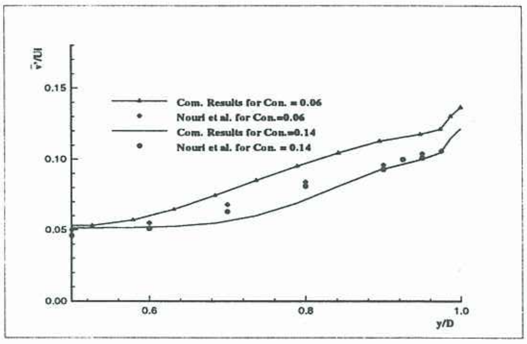

Figure 3: Particulate Phase Turbulent Intensity at Different Volume Concentrations

COMPUTATIONAL RESULTS & DISCUSSION

The two sets of experimental data were used to compare with the computational results, and some results are shown in Figs. 1-6. In case of vertical downward pipe flow (Nouri et al., 1987), the computational velocity results obtained by using both approaches agree well with the experimental data (Figs. 1 & 2). Fair agreement for the turbulent intensity of the particulate phase is found in Fig. 3, and it is observed that this turbulent intensity or granular temperature decreases with particle concentration.

Some comparisons (Figs. 4-6) are also made with the experimental data given by Zisselmar & Molerus (1979) for a horizontal pipe flow. Fig. 4 shows that a significant improvement is made if the particle collision dynamics is considered, and very good agreement is found for the Eulerian approach at a concentration of 3.9%. The velocity profile of the fluid phase is becoming flatter with increasing particle concentration (Fig. 5); while the turbulent intensity of the fluid phase decreases with particle concentration (Fig. 6).

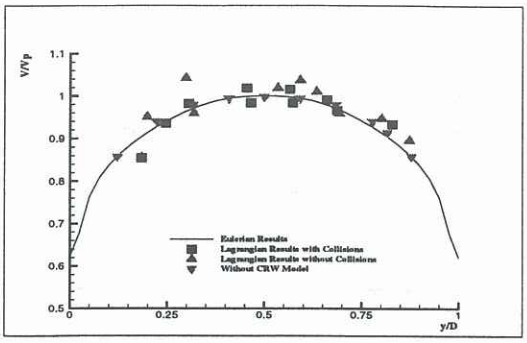

Figure 4: Velocity Profile by Lagrangian Approach at Volume Concentration of 3.9%

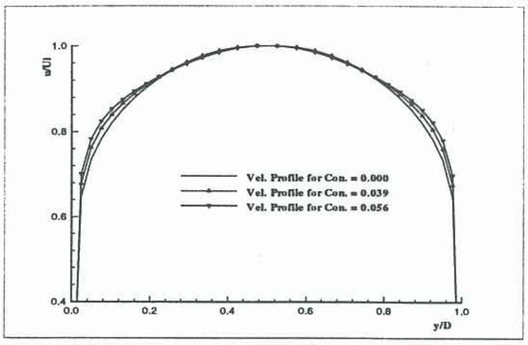

Figure 5: Velocity Profiles at Different Volume Concentrations

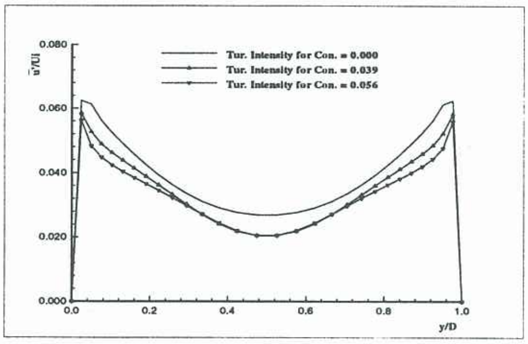

Figure 6: Turbulent Intensity Profiles at Different Volume Concentrations

CONCLUSIONS

Conclusions drawn from this work are: 1) velocities profile for both phases become flatter with increasing particle concentrations; 2) the granular temperature decreases with particle concentration; 3) the turbulent intensity of the fluid phase decreases with particle concentration; 4) the viscosity of the particulate phase increases with particle concentration; 5) particle collisions have significant effect on the horizontal pipe flow, while this effect on the vertical downward flow is insignificant due to a strong influence of the gravity; 6) the Lagrangian and Eulerian models described here provide accurate predictions of both mean flow and turbulence behaviour.

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