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THE SWEEPING DECORRELATION HYPOTHESIS AND ANISOTROPY IN A TURBULENT JET AT MODERATE REYNOLDS NUMBERS

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ABSTRACT

High-order spectra and structure functions are used to investigate the random sweeping decorrelation hypothesis. Measurements in a turbulent round jet at moderate Reynolds numbers ($R_\lambda = 230-545$) indicate that higher-order (up to 4) velocity spectra and structure functions do not follow the Kolmogorov (1941a,b) scaling. The large scale governing parameter $\langle u_i^2 \rangle$ ($i = 1, 2$) appears to influence the dynamics of the inertial range. The assumption of statistical independence between large and small scales is examined through the correlation coefficient $\rho_{u_i^k \delta u_i^l}$.

INTRODUCTION

Higher-order spectra $\phi_{u_i}^{(m)}(k_1)$, or spectra of u_i^m, are defined such that

$$\langle (u_i^m - \langle u_i^m \rangle)^2 \rangle = \int_{-\infty}^{+\infty} \phi_{u_i}^{(m)}(k_1) dk_1 \quad i = 1, 2, 3 \tag{1}$$

where u_i denotes the velocity fluctuation. Following Kolmogorov (1941a,b), Dutton & Deaven (1972) obtained

$$\phi_{u_i}^{(m)}(k_1) = C_m \langle \epsilon \rangle^{2m/3} k_1^{-(2m+3)/3} , \tag{2}$$

where C_m are constants. Using a range of atmospheric turbulence measurements, Dutton & Deaven found that the higher-order ($2 \leq m \leq 4$) velocity spectra deviated significantly from (2), either retaining an approximate −5/3 power law or decreasing somewhat in slope. Van Atta & Wyngaard (1975) presented two theoretical attempts to determine the form of higher-order spectra $\phi_{u_i}^{(m)}(k_1)$. In the first, the appropriate dissipation rates in the transport equations for $\langle u^{2m} \rangle$ and $\langle u^{2m+1} \rangle$ were included in the dimensional analysis. In the second, the pdf and jpdf of velocity fluctuations were assumed to be Gaussian; the resulting higher-order spectra were

$$\begin{aligned} \phi_{u_i}^{(m)}(k_1) &= m^2 \times 1 \times 3 \times 5 \times \cdots \times (2m-3) \\ &\quad \langle u_i^2 \rangle^{m-1} \phi_{u_i}^{(1)}(k_1), \quad m \geq 2 . \end{aligned} \tag{3}$$

Eq. (3) shows that $\phi_{u_i}^{(m)}(k_1) \sim k_1^{-5/3}$ in the IR since $\phi_{u_i}^{(1)}(k_1) \sim k_1^{-5/3}$.

The measurements of Van Atta & Wyngaard (1975) in the atmospheric surface layer over land and ocean supported the above two attempts. These results confirmed (3) for $m \leq 9$ and refuted the extension of the original Kolmogorov (1941a,b) arguments to higher-order spectra.

Tennekes (1975) assumed that small scales are advected past a Eulerian observer by the energy-containing structures without dynamical distortion (referred to as random sweeping) and showed that if the large scale advection directly contributes to the kinetic energy in the IR, the Eulerian frequency spectra of u_i^2 may be described as

$$\phi_{u_i}^{(2)}(f) = \alpha_i \langle \epsilon \rangle^{2/3} \langle u_i^2 \rangle^{1/3} f^{-5/3} , \tag{4}$$

where f is the frequency, α_i are unknown constants. Eqs. (3) and (4) show that the quantity $\langle u_i^2 \rangle$, characteristic of the energy-containing eddies, can influence the dynamics of the IR. Comparing differences between Eulerian and Lagrangian correlation time microscales in approximately isotropic (grid) turbulence, Tennekes confirmed the sweeping hypothesis even at relatively low Reynolds numbers. Within the framework of the Renormalization Group Theory (RNG), Yakhot et al. (1989) showed that the wavenumber and frequency energy spectra differ substantially, and $\phi_{u_i}^{(2)}(k_1) \sim k_1^{-7/3}$. This seems to support (2) but is clearly inconsistent with the experimental measurements. Yakhot et al. (1989) explained that this discrepancy is due to the use of Taylor's hypothesis in the experiments. Chen & Kraichnan (1989) argued that the precise coherence between the energy and inertial range excitation is needed to inhibit sweeping effects; RNG discards sweeping effects from the outset.

Using measurements at relatively large values (2000 and 3000) of the Taylor microscale Reynolds number R_λ, Praskovsky et al. (1993) studied the higher-order velocity structure functions defined by

$$D_{u_i}^{(m)}(r) = \langle [u_i^m(x+r) - u_i^m(x)]^2 \rangle \quad i = 1,2,3, \quad m \geq 1 \qquad (5)$$

where r is in the x direction and lies within the IR. They found that $D_{u_i}^{(m)}(r) \sim r^{2/3}$ for $m \leq 4$. This scaling appears to support the random sweeping decorrelation hypothesis (RSDH). However, the strong correlation between the energy-containing scales and IR scales led them to conclude that RSDH cannot be exact.

The above considerations have motivated the present study. We examine the scaling behaviour of high-order spectra and structure functions (both u_1 and u_2) which is the main consequence of RSDH. Implicit in RSDH and local isotropy is the statistical independence between large scales and small scales. This assumption and its systematic dependence on R_λ is also examined.

EXPERIMENTAL DETAILS AND CONDITIONS

Measurements were made in a circular jet facility. It is an open circuit wind tunnel where air is supplied by a centrifugal fan through a diffuser, settling chamber and a 10:1 contraction. The exit diameter d of the nozzle is 55 mm. Measurements of u_1 and u_2 were made at $x/d = 40$ both on the centreline and at a half of jet local half-velocity width. The exit jet velocities U_j varied between 11.5 and 46.6 m/s produce the five R_λ between 235 and 545 on the centreline, and the five R_λ between 230 and 530 at a half of jet local half-velocity width. An X-wire was used to measure the longitudinal and lateral velocity fluctuations. The spanwise separation s_w between the two wires of the X-probe was 1 mm. The sensing elements were made of 2.54 μm Pt-10% Rh Wollaston wire approximately 0.51 mm in length. The hot wires were operated with an in-house constant temperature circuit at an overheat ratio of 1.5. The integral scale L, Taylor microscale λ and Kolmogorov length scale η vary between 11.5–12 cm, 4.81–8.64 mm, 0.105–0.290 mm respectively.

Output voltages from the anemometer were passed through buck and gain circuits and low-pass filtered at f_c which was determined by using a two-channel spectrum analyser (HP3582A). The signals were then digitised on a PC using a 12 bit A/D converter at a sampling frequency f_s $(= 2f_c)$. The signals were subsequently transferred to a VAX 780 computer for further analysis.

RESULTS

Higher-order (up to 4) velocity spectra (Figure 1) exhibit a well defined −5/3 power law while the higher-order (up to 4) velocity structure functions (Figure 2) scales as $r^{2/3}$ in the IR. Neither spectra nor structure functions follow the Kolmogorov (1941a,b)

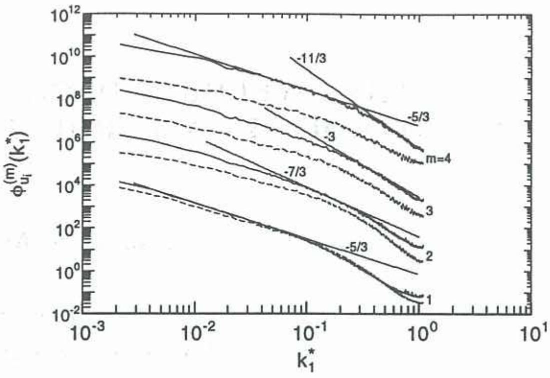

Figure 1: Higher-order spectra for $R_\lambda = 545$. solid curve, u_1 spectra; dashed curve, u_2 spectra.

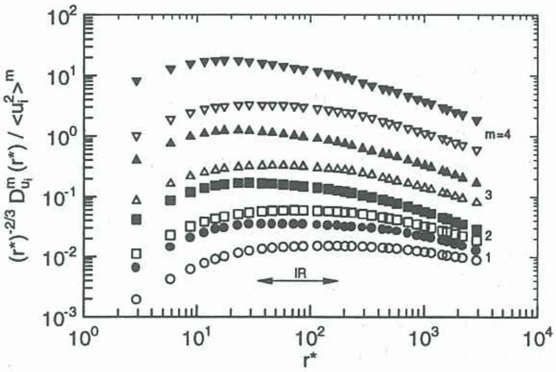

Figure 2: Non-dimensional higher-order structure functions for $R_\lambda = 545$. Open symbols, u_1; solid symbols, u_2.

scaling. These agree with previous results for the atmospheric boundary layer (Dutton & Deaven, 1972; Van Atta & Wyngaard, 1975) and high Reynolds number laboratory flows (Praskovsky et al., 1993). The −5/3 scaling region begins at a higher wavenumber, while the 2/3 scaling region shifts to a smaller r^* as the order m increases. The extent of these scaling regions decreases as m increases, and increases with increasing R_λ. The transverse fluctuation u_2 exhibits a different behaviour from that of the longitudinal fluctuation u_1. The extent of the −5/3 and 2/3 scaling ranges of u_1 is larger than that of u_2 for the same order m. The compensated second-order spectra of u_1 [$m = 2$ in Eq. (3)] collapse well for different R_λ at high wavenumbers ($k_1^+ > 0.1$). This indicates that $\langle u_i^2 \rangle$ in Eq. (3) influences the dynamics of the IR. As m increases, the collapse of the compensated higher-order spectra (3) impairs. For the same m, the higher-order spectra of u_1 collapse much better than those of u_2. This suggests that the formulation relation (3) may have to be improved by including other factors, e.g. R_λ, besides $\langle u_i^2 \rangle$; the departure of higher-order u_2 spectra from (3) also needs to be investigated.

The basic assumption of statistical independence

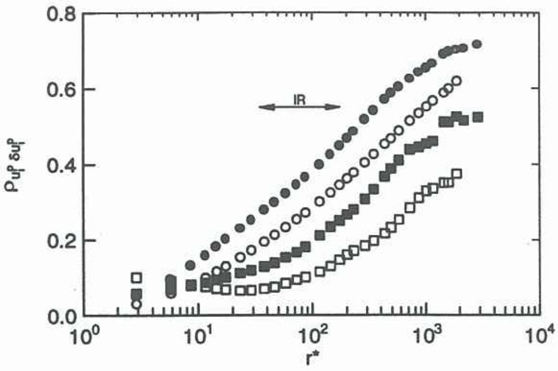

Figure 3: Correlation coefficient between u_i and δu_i for $R_\lambda = 545$. Circular symbols, $p = 1$; square symbols, $p = 2$. Open symbols, u_1; solid symbols, u_2.

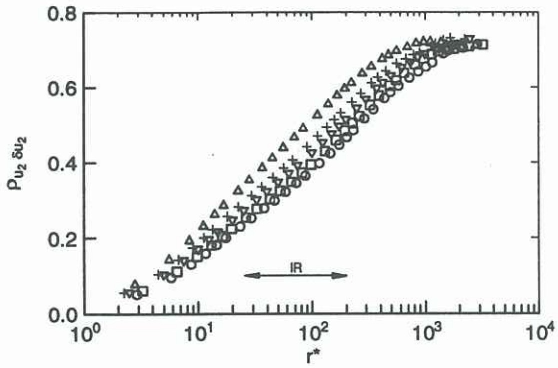

Figure 4: Dependence of correlation coefficient between u_2 and δu_2 on R_λ. $\triangle$, $R_\lambda = 235$; +, 305; $\triangledown$, 365; $\square$, 495; $\circ$, 545.

between large and small scales is examined through the correlation coefficient $\rho_{u_i^k \delta u_i^l}$ between the characteristic velocity u_i of large scales and the characteristic velocity difference δu_i (= $u_i(x+r) - u_i(x)$, r being small) of small scales. Measured values of $\rho_{u_i^k \delta u_i^l}$ show that this assumption is not valid for the present moderate values of R_λ. The magnitude of $\rho_{u_i^k \delta u_i^l}$ is significant throughout the whole IR (Figure 3). There are very strong interactions between large and small scales in the IR. RSDH is not verified in the present moderate R_λ flow. This is in agreement with the high R_λ data reported by Praskovsky et al. (1993). The magnitude of $\rho_{u_2^p \delta u_2^p}$ is larger than that of $\rho_{u_1^p \delta u_1^p}$ in the IR. Energy-inertial interactions are stronger for u_2 than u_1. The trend that $\rho_{u_2,\delta u_2}/\rho_{u_1,\delta u_1} > 1$ may continue to high R_λ. The magnitudes of both $\rho_{u_1,\delta u_1}$ and $\rho_{u_2,\delta u_2}$ decrease with increasing R_λ (Figure 4 only for u_2, u_1 not shown). Our measured values of $\rho_{u_i^p \delta u_i^p}$, together with those of Paskovsky et al. at higher R_λ, confirm that the energy-inertial scale interactions weaken as R_λ increases.

Several time scales were estimated : the sweeping time $\tau_s = (\langle u_1^2 \rangle^{1/2} k_1)^{-1}$, the viscous dissipation time $\tau_d = (\nu k_1^2)^{-1}$ and the local eddy-turnover time $\tau_e = [k_1\{k_1 E(k_1)\}^{1/2}]^{-1}$, where $E(k)$ is the three-dimensional energy spectrum. The sweeping time is the smallest of the three over the entire high wavenumber range and a large portion of the low wavenumber range. This agrees with that, obtained for DNS data, by Sanada & Shanmugasundaram (1992). The sweeping time decreases with increasing R_λ. The smaller the sweeping time, the smaller the probability of interaction between large and small scales. At very high R_λ, sweeping becomes more effective.

CONCLUSIONS

Higher-order ($2 \leq m \leq 4$) velocity spectra and structure functions do not follow the Kolmogorov (1941a,b) scaling. RSDH is not verified for the present moderate R_λ flow. The transverse fluctuation u_2 exhibits a different behaviour from that of the longitudinal fluctuation u_1. The large scale governing parameter $\langle u_i^2 \rangle$ influences the dynamics of the IR. The strong interaction between large and small scales in the IR weakens as R_λ increases. The sweeping time decreases with increasing R_λ, so that sweeping becomes more effective as R_λ increases.

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