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EXCHANGE FLOWS THROUGH CONTRACTING CHANNELS: ENTRAINMENT AND MIXING

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ABSTRACT

Numerical techniques are used to investigate the role of entrainment and mixing in exchange flows through contracting channels. The use of numerical methods allows several of the simplifying assumptions inherent in two-layer hydraulic theory to be relaxed, at least for idealized situations. The results indicate the importance of mixing in the creation of an interfacial layer of intermediate density that carries a significant fraction of the horizontal transport and alters the scalar transport relative to two-layer predictions.

INTRODUCTION

Flow between two basins of different fluid densities is a long-standing problem of fundamental interest in geophysical fluid dynamics. Such flows occur frequently in nature, for example in straits, in channels between deep ocean basins, or between lagoons and coastal seas. Here, we investigate bi-directional exchange flows through a simple contracting channel via numerical simulation.

Theoretical understanding of exchange flows is based largely on equations describing the dynamics of steady, inviscid two-layer flows through idealized channels with slowly varying cross-sectional area (see e.g. Armi and Farmer, 1986 or Lawrence, 1990). Though two-layer hydraulic theory has proven extremely useful, a serious limitation to its applicability is the neglect of entrainment and mixing between the oppositely flowing water masses. The simulations discussed here are intended to illustrate the influence of vertical entrainment and mixing on the dynamics and horizontal transport of simple exchange flows.

NUMERICAL SIMULATIONS

The equations of motion for a density-stratified fluid are solved numerically over an orthogonal-curvilinear grid conforming to the variable width channel sidewalls. The simulations discussed here were conducted for the 120m long by 10m deep channel shown in Figure 2. The resolution of the numerical mesh was 129 $\times$ 17 $\times$ 65 grid points in the stream-wise x, spanwise y and vertical z directions respectively. Though the horizontal spacings are variable in curvilinear coordinates, this corresponds to nominal grid spacings of 93, 60 and 15cm. The numerical algorithm incorporates a fourth-order compact scheme for spatial differentiation, third-order Adams-Bashforth time stepping and a multi-grid projection method for pressure. The simulations were run using a closure scheme based on the Smagorinsky model, modified for stably-stratified flow.

The simulations are conducted in a variable-width channel with vertical, free-slip sidewalls. The free surface is treated as a stress-free rigid lid. Simulations were run both for maximal and submaximal exchange flows with a net barotropic component. The magnitude and direction of the barotropic transport was controlled by specifying a barotropic pressure drop across the channel. The maximal exchange cases presented here were computed with a free-slip bottom boundary. A no-slip condition was used for the submaximal case. At the solid walls, adiabatic conditions are prescribed for density. At the up- and downstream computational boundaries, the density of the inflowing fluid is prescribed to match the assumed values in the exterior reservoirs. Viscous sponge layers are employed within $20m$ of these open boundaries to minimize spurious reflections from imperfect treatment of the open boundaries. The solutions are analyzed only within a "test" section excluding these sponge layers. Solution quality is routinely monitored by assessing the balance of the potential energy equation and comparing the magnitude of the residal to the dominant transfer rates as shown in Figure 1(b).

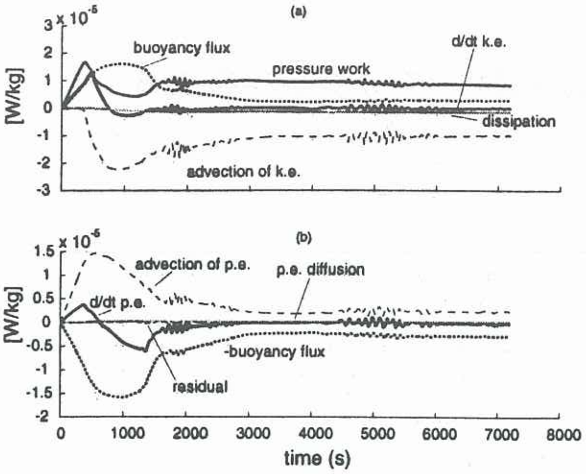

Figure 1: Components of the a) kinetic energy balance and b) potential energy balance for a simulation of maximal exchange with $q_r = 0.59$.

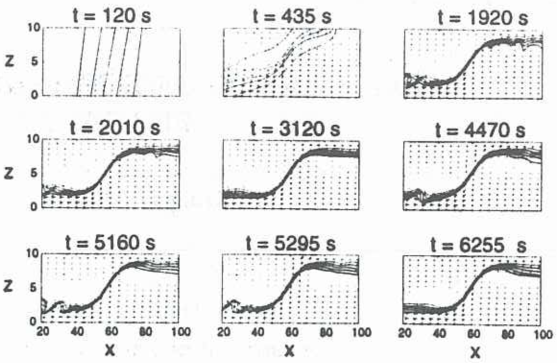

Figure 3: Snapshots of the velocity vectors, overlain with contours of the density field, for various times during the evolution of the initial value problem to steady state. For this case $q_r = 0.55$. Simulation data are taken from the mid-channel, vertical $x - z$ plane.

Figure 2 shows the channel geometry and representative velocity and density "interfaces" for simulated and predicted exchange flows. The simulations were run as lock-exchange initial value problems with a density difference of 0.5 kg/m^3.

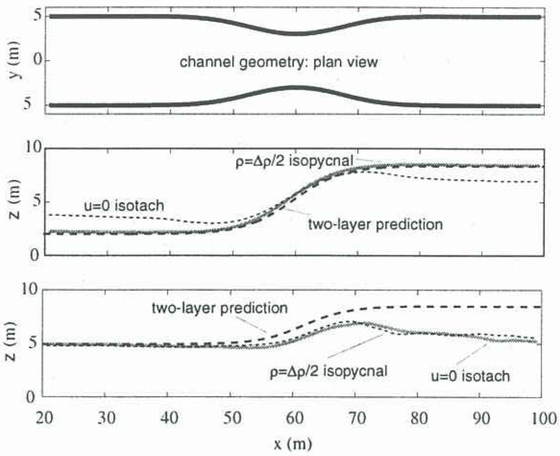

Figure 2: A comparison of the predicted and simulated interface heights as a function of streamwise position. The middle panel is typical of all the maximal exchange simulations. The lower panel shows the results for a submaximal exchange flow.

Figure 3 shows the time evolution of a maximal exchange simulation as it evolves toward a quasi-steady state. Here the ratio of right- to left-ward transport q_r is less than 1.

RESULTS

To permit comparison between the model results and inviscid theory we time- and cross-channel average the simulated fields once a quasi-steady state is achieved. For the purpose of forming layer Froude numbers, the zero isotach of the averaged flow is used to decompose the flow into two layers. Alternatively, the mid-isopycnal $\Delta\rho/2$ can be used to separate the flow. Note that these two surfaces are not co-located in the domain (Fig. 2). We use the zero isotach to define the layers because this definition seems more consistent with the spirit of the theory, i.e. two layers of fluid flowing in opposite directions; though the choice is somewhat arbitrary.

Comparison of the model results with the hydraulic predictions on a Froude number plane demonstrates that the addition of mixing and dissipation does not fundamentally change the maximal exchange solutions (Fig. 4a-c). Two supercritical regions bound the central subcritical region, satisfying the defining requirement of a maximal exchange flow. Quantitatively, however, much more of the flow is subcritical than predicted. For example, when $q_r \approx 1$ inviscid theory predicts that the two control points collapse to a single point and that the flow is essentially supercritical everywhere. The simulations show no such coalescence of the controls and, in general, simulated Froude numbers are systematically smaller than predicted.

A more careful examination of the continuous fields suggests a qualitative change in character of the flow as a result of entrainment and mixing between layers. The velocity and density profiles shown in the upper panels of Figure 5 suggest the flow might be more appropriately described by three-layers, with two bounding layers of approximately uniform properties being separated by a finite-thickness interface. Marked on the panels are the upper and lower limits of the interface layer, defined as a 0.05 kg/m^3 difference from the inflow densities. Overlaying these inter-

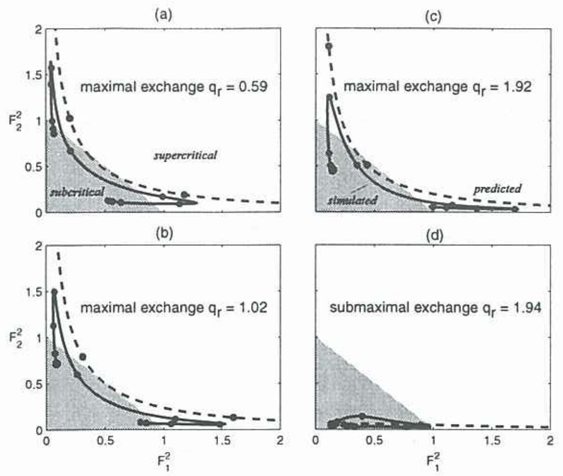

Figure 4: Simulated (solid) and predicted (dashed) solution curves plotted on a Froude number plane for a) $q_r = 0.59$ b) $q_r = 1.02$ c) $q_r = 1.92$ and d) $q_r = 1.73$. In the shaded region of the Froude number plane the flow is subcritical. Dots along the curves mark approximately 6 m intervals along the channel. Plots a-c are maximal exchange solutions and plot d is a submaximal exchange solution.

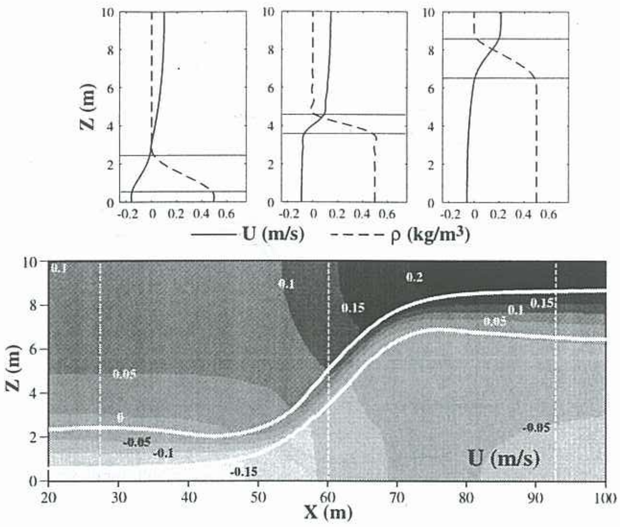

Figure 5: (above) Profiles of U and ρ from the three streamwise positions marked with dashed white lines on the contour plot. Horizontal lines mark the positions of the upper and lower bounds of the interfacial layer. (below) Contours of U overlain with the upper and lower boundaries of the interfacial layer for $q_r = 1.92$.

faces on contours of the along-channel velocity field (Fig 5b) shows that the interface layer is thinnest in the contraction and thicker to either side. It is also apparent that the zero isotach is not centered in the interfacial layer; rather, the interface is moving with the lower layer to the left of the contraction and with the upper layer to the right.

Although two-layer inviscid theory gives reasonable predictions of the layer transports, even when frictional effects are included , it provides no guidance on rates of entrainment and mixing between the layers. Bray et al (1995), in their analysis of observations from the Strait of Gibraltar, emphasize the importance of the interface in carrying horizontal transport, noting that its existence implies strong vertical exchange between layers. The computed solutions were time and cross-channel averaged after the flows reached quasi-steady state. The transports and the change in transport in the streamwise directions were used to quantify the exchange between layers. The results are represented schematically as transports in and between layers at the ends and middle of the channel (Fig. 6). In each plot the transport values have been normalized by the maximum layer transport. A number of features are common to all cases. Entrainment into the interfacial layer is preferentially from the faster of the bounding layers. In the vicinity of the hydraulic controls, this is the thinner of the two layers. The rates of entrainment are large, being as high as 30% of the transport of the faster layer, and as much as 20% of the slower moving layer. In the case of submaximal exchange the values are even more extreme, with as much as half of the transport carried by the upper layer being entrained into the interfacial layer.

We next consider the effect of bottom friction on the steady solution by changing the bottom boundary condition from free- to no-slip. Early in the temporal development there are obvious differences in the character of the flow. Instabilities associated with the propagating bores are more frequent and more intense than for the maximal exchange runs. In steady state the flow strongly resembles the submaximal exchange flows in Armi and Farmer (1986). For the case shown, with $q_r > 1$, the interface is nearly flat and at mid-depth to the left of the contraction, and displaced upward to the right (Fig. 7). Where the upper layer is thinnest, the interface is extremely active with near continual formation of billows that grow as they are advected downstream. Velocities in the lower layer are approximately uniform along the channel, whereas velocities in the upper layer peak downstream of the contraction. Shear stress, estimated as $K_m(\partial U/\partial z)$ (not shown), is enhanced in the bottom boundary layer, though the bed stress is less than the interfacial stress associated with the billows downstream of the contraction.

Because of the obvious change in flow regime, we compare this simulation with hydraulic predictions for submaximal exchange flow. The predicted interface (see Fig. 2) is flat to the left of the contraction, rising through the throat. The upper layer thins and

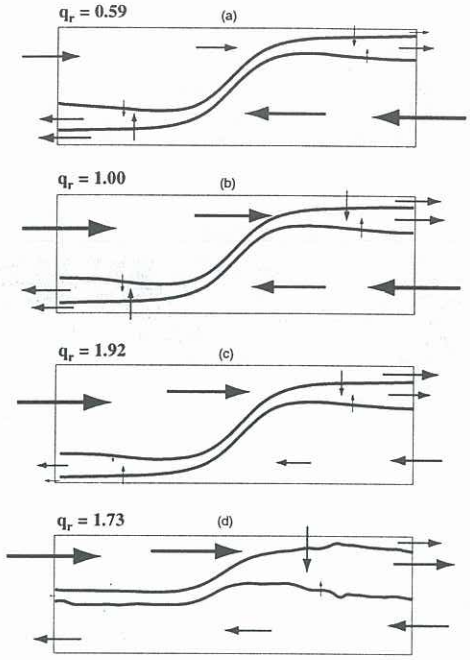

Figure 6: Schematic representation of the horizontal and vertical transports in a three-layer system for a) $q_r = 0.59$ b) $q_r = 1.02$ c) $q_r = 1.92$ and d) $q_r = 1.73$. Arrows show the magnitude and direction of the transports. Magnitudes have been normalized by the maximum transport in each panel.

supercritical flow is predicted to the right of the contraction. This is a reasonable qualitative representation of the simulated flow. The prediction corresponds well with the simulated interface height to the left of the contraction, but in contrast with the predictions, critical conditions occur only briefly and well downstream of the throat. (see Figure 4 (d))

CONCLUSIONS

The net transport in the interfacial layer is always away from the throat of the contraction. Although the flow within the layer may be bidirectional in places, the mean transport is in the same direction as the thinner, faster moving layer. The fraction of the transport carried by the interface is typically large, being equal to or greater than the transport carried by the bounding layer that is moving in the same direction. This indicates that more than half of the fluid moving away from the contraction has undergone mixing with the opposing stream or, to a lesser extent, is recirculating fluid from the slower moving layer. The one exception is the submaximal exchange case in which there is very little transport to the left of the contraction. The absence of a virtual control results in a stable flow upstream of the contraction with relatively little fluid entrained from the bounding layers.

The existence of a net (cross-channel averaged)

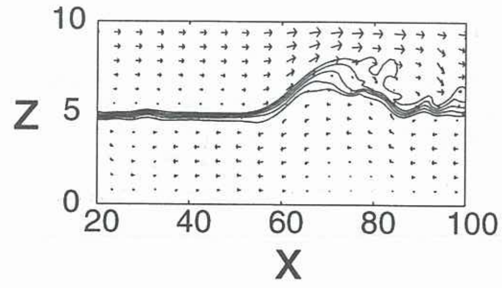

Figure 7: Isopycnals and subsampled velocity arrows in the center-channel plane for submaximal exchange in approximately steady state. Note the quiescent interface to the left of the contraction and the active billowing to the right.

flow, i.e. $q_r = 1$, leads to maximum entrainment on the downstream side of the contraction (to the left when $q_r < 1$ and to the right when $q_r > 1$). The interface carries less than half of the transport downstream of the contraction, but as much as 2/3 of the transport on the upstream side for the maximal exchange cases.

The interlayer transports can be interpreted in terms of mixing and recirculation. For example, consider a passive tracer C entering in the lower layer in Fig. 6a. Before passing through the contraction 15% of the flux of C would be lost to the interfacial layer and recirculate out the right end of the channel. Of the 85% passing through the contraction, about 30% would be diluted by mixing with overlying fluid, and the remaining 55% would leave the left end of the domain unaltered. In our simulations, these numbers are fairly typical, in that roughly half the fluid in a bounding layer that at some point is supercritical is lost to the interfacial layer. As such, this suggests mixing is of considerable importance in defining the circulation and streamwise tracer transport in hydraulically controlled flows.

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