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THE STRATIFICATION PRODUCED BY A DISTRIBUTED HEAT FLUX AND A PLUME IN A BOX.

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ABSTRACT

We investigate the convection and density stratification that form when buoyancy fluxes are simultaneously applied to a finite volume as both a turbulent buoyant plume from a small source and as a uniform heat flux from the same horizontal boundary. The turbulent plume tends to produce a stable density stratification, whereas the distributed flux from a boundary tends to force vigorous overturning and vertical mixing. Experiments show that steady, partially mixed and partially stratified states can exist when the plume buoyancy flux is greater than the distributed flux. The steady state involves a balance between the rate at which the mixed layer deepens due to encroachment and vertical advection of the stratified water far from the plume due to the plume volume flux acquired by entrainment. This may be relevant to the semi-enclosed seas of high latitude where there is commonly a de-stabilising heat flux from the sea surface as well as more localised and intense deep convection from the surface.

INTRODUCTION

A turbulent plume falling into a fluid volume of finite vertical and horizontal extent leads to the development of a stable density stratification, for which Baines & Turner (1969) obtained analytic solutions for the long time limit. On the other hand, when there is a pre-existing gravitationally stable density gradient in the water and a destabilising heat flux is imposed at a horizontal boundary, a mixed layer forms and increases in depth with time (Manins & Turner 1977). Hence if a layer is subjected to both a uniformly distributed boundary flux and an intense, localised flux, there is a competition between the tendency for the uniform flux to overturn the layer and the tendency for the turbulent plume to stratify the system.

The interaction of flows produced by two buoyancy sources at the bottom is shown diagrammatically in figure 1 (a). The plume, upon reaching the top of the layer, generates an outflow that forces a general downwelling of water. The downwelling and entrainment of this water back into the plume produces a stable stratification. However, the density gradient produced by the plume, if it were not continuously replenished would be overturned by the basal heating at a rate that can be calculated by applying the theory developed in Manins & Turner (1977). In the present situation we hypothesize that a steady state having a convecting layer depth less than the total depth of the water exists when the rate at which the top of the convecting layer is advected downwards by the plume-induced circulation is equal to the rate of deepening of the convecting layer by bottom heating.

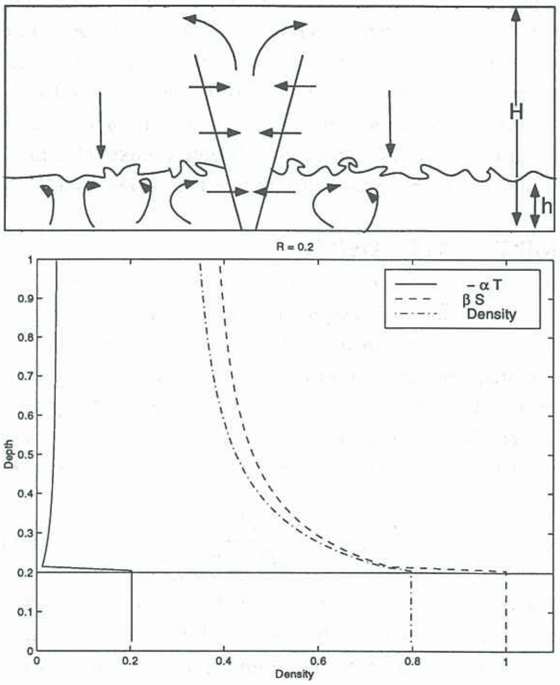

Figure 1. A diagrammatic illustration of a) the expected flow pattern and b) density, salinity and temperature profiles, for heating from the bottom boundary and a relatively fresh-water plume source at the base. The shape of these profiles remains constant while the density continues to decrease.

If there exists such a steady state, the system will consist of a well-mixed layer of depth h and an overlying density profile similar to that sketched in figure 1 (b). Since the two buoyancy sources provide a nonzero net flux into the chamber the density everywhere must decrease linearly with time.

When both heat and salinity are used to provide buoyancy fluxes for the distributed and localised sources, there will be concentration gradients of each component. However, the individual profiles will give density contributions of similar forms, both being stable above the mixed layer, and near uniform in the mixed layer. In the absence of diffusion the temperature will not be continuous across the top of the convecting layer as only a small fraction of the heat flux into the mixed layer is entrained into the plume to produce the overlying temperature profile. The plume transport necessarily produces a thermal contribution to the stable density gradient of the same form as the salinity gradient. Hence we expect a minimum in temperature just above the mixed layer.

MIXED LAYER DEPTH

The steady state depth, h, of the convecting layer can be determined by requiring that the density change at the same rate in both the stratified and convecting regions, and by assuming that the densities in each region are separately controlled by the two buoyancy fluxes. The rate at which the density changes in the stratified region is determined by the plume flux, F. Thus

$$\frac{d\rho}{dt} = \frac{\rho F}{A(H-h)g}, \qquad (1)$$

where A is the base area, H is the water depth, g is the gravitational acceleration and F is the buoyancy flux from the plume. The density in the mixed layer changes due to the basal buoyancy flux B, and so

$$\frac{d\rho}{dt} = \frac{\rho B}{Ahg}. \qquad (2)$$

Assuming that in the steady state that $\frac{d\rho}{dt}$ is the same in both layers, the above two expressions can be equated to give

$$\zeta_{mixed} = \frac{R}{1+R}, \qquad (3)$$

where $\zeta_{mixed} = \frac{h}{H}$ is the normalised mixed depth and $R = B/F$ is the ratio of input buoyancy fluxes. This simple relation suggests that when there is no heating, the plume fills the whole box with a stable stratification ($h = 0$), and that as $R \to \infty$ the whole tank will overturn ($h = H$). If one includes the interfacial thickness due to the turbulent eddies (Deardorf *et al* 1980) then full tank mixing occurs for $R \approx 1$. For $0 < R < 1$, we predict a partially mixed and stratified system.

ENTRAINMENT - ADVECTION BALANCE

In a more accurate formulation which takes into account transport through the top of the convecting layer we assume that, if the convecting layer depth h is to be constant, then the rate of deepening, U, of the mixed layer must be equal to the downwelling velocity, V, of the stratified interior. We can determine U from Manins & Turner (1977), who derived a result for the time evolution of h for a convectively mixed layer beneath a constant density gradient as

$$h = \sqrt{6E^*}(\frac{B}{A})^{1/2}N^{-1}t^{1/2}, \qquad (4)$$

where $N = (\frac{g}{\rho}\frac{d\rho}{dz})^{1/2}$ is the buoyancy frequency. The value of the mixing efficiency constant, E^*, is related to the fraction of the kinetic energy of the convecting layer which is converted into potential energy by mixing less dense overlying water downward in to the mixed layer. If there is extensive penetrative convection and entrainment creating a sharp density step at the top of the convecting layer, then all the deepening is due to working of the convective motions against buoyancy forces and $E^* = 1$. On the other hand, if convection is less vigorous and the density profile remains continuous, mixed-layer deepening is by 'encroachment' only (heating of the mixed layer) and $E^* = 1/3$. We define $U = \frac{dh}{dt}$ and allow N to be a function of depth in (4), giving

$$U = 3E^*\frac{B}{A}N(z)^{-2}h^{-1}, \qquad (5)$$

where $N(z)$ is now the depth-dependent buoyancy frequency. Changes of the density in the filling-box stratification are due only to vertical advection (Baines & Turner 1969) so that

$$\frac{\partial \rho}{\partial t} = -V\frac{\partial \rho}{\partial z}. \qquad (6)$$

The density profile $\frac{\partial \rho}{\partial z}$ is constant at large times and $\frac{\partial \rho}{\partial t}$ is determined by the rate of addition of buoyancy to the chamber so we can determine the advection velocity as

$$V = \frac{\rho F(1+R)}{gAH}\left(\frac{\partial \rho}{\partial z}\right)^{-1}. \qquad (7)$$

Setting $U = V$ for a steady mixed layer depth gives

$$\zeta_{mixed} = \frac{R}{1+R}3E^*. \qquad (8)$$

If mixing into the convecting layer is due to encroachment alone we have $E^* = 1/3$ (Manins & Turner 1977) which is appropriate for small ζ. For larger ζ, the Richardson number of the convecting layer decreases and the results of Manins & Turner (1977) and Denton & Wood (1981), show that $E^* \to 0.46$ which would mean full tank mixing occurs at $R \approx 2.5$. Although our discussion concerns a plume from a point source, the result should apply equally well to a plume from a line source. If a periodically released plume from a point source was used, (6) still holds and the same density profiles evolve (Baines & Turner 1969; Killworth & Turner 1982) but now F will be time varying and (8) is expected to apply only to the mean position of ζ_{mixed}.

EXPERIMENTAL METHOD

Experiments were carried out in a a square perspex box with sides 50 cm, filled with water to a depth of between 15 and 35 cm to give aspect ratios of approximately 0.5 and 1. Base heating was provided by an electrical heating mat capable of running at powers of up to 2kW. Plumes were produced by using a strong salt solution in the tank and releasing fresh water from a source through the centre of the base with buoyancy fluxes of $F = 20$ to $40\ cm^4 s^{-3}$. The volume fluxes were sufficiently small that both the mass and momentum fluxes from the sources could be neglected. The temperature usually changed by 10^oC in the course of an experiment, hence the average value of the co-efficient of expansion could change by a factor of 2 and the buoyancy flux was no longer constant for a fixed heat flux. Thus no truly steady state was achieved in experiments with fixed heat flux. Instead, the system reached a quasi-steady state in which the depth of the mixed layer increased as B (and R) slowly increased with time.

Shadowgraph techniques were used since the sharp changes in refractive index gradient at the boundary of the convecting region offered an easy means to measure ζ_{mixed}. The conductivity and temperature were measured as functions of depth and the salinity and density profiles calculated from these.

EXPERIMENTAL RESULTS

The quasi-steady mixed layer depth measured from shadowgraphs is shown in figure 2 along with the theoretical prediction (8). Agreement with the theory is good for small R and the whole tank overturns for $R \approx 1$.The apparent difference at high ζ is due to the interface having a larger thickness due to the large convective eddies. The experimental uncertainties are of order of the thickness of the mixing region due to penetrative convection, as discussed by Deardorf *et al* (1980).

Measurements of the density profile in the steady state were made for the case of no base heating $R = 0$ and for $R = 0.4$. When there was no heating (figure 3 a), the predicted 'filling box' density profile of Baines & Turner (1969) developed quickly, approaching the constant asymptotic shape. For $R = 0.4$ (figure 3 b), we plot profiles taken after the mixed depth had reached a steady depth. The density profile retains the same constant shape with time and the transition from the stratified region to the convecting region occurs at a position that agrees well with the measurements from the shadowgraph. However, the entrainment of heat from the convecting layer gives rise to a characteristic stable temperature gradient in the stratified region, which gives a minimum in temperature just above the mixed layer. We note that these are instantaneous profiles (rather than time-averaged) and hence they ignore significant variability in the top of the mixed layer.

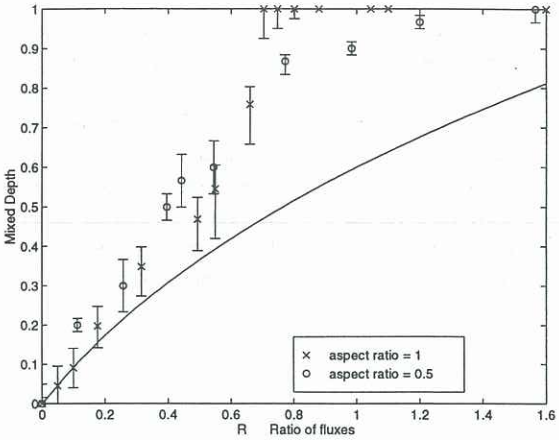

Figure 2. A comparison of experimental results and theory, (8), for the steady state mixed depth when the sources are on the same boundary, for aspect ratios of 0.5 and 1

APPLICATIONS

The surfaces of enclosed seas are commonly subjected to a destabilising buoyancy flux due to cooling, freezing or an excess of evaporation over precipitation. This flux maintains a convectively mixed surface layer. While the surface flux is generally wide-spread there are often also localised regions where the surface buoyancy flux is much larger, forcing more convection as a result of local winds. In this case the depth of the mixed layer may be given by (8). The density, temperature and salinity in the stratified region could be found using the equations of Baines & Turner (1969), once the rate at which the buoyancy transported from the convecting layer by entrainment of the plume is determined. Addition of rotation to the problem is expected to make only small quantitative differences and Pierce & Rhines (1996) have shown that for low rotation a turbulent plume generates the same density gradient as in the non-rotating filling box. Profiles from the Bering Sea, shown in figure 4, reveal a characteristic temperature profile with a maximum beneath the mixed layer, in the same way that our 'upside down' experiments show a minimum temperature above the mixed layer. The concentration profile of dissolved oxygen (an indication of how long the water body has been away from the surface) shows a minimum below the 'mixed' surface layer and a maximum at greater depths. Both of these effects are consistent with the circulation and stratification produced by the combined effects of deep plume-like convection and a widespread surface buoyancy flux. The weak stratification apparent within the surface layer may be a result of other factors, such an an input of fresh water at the surface from melting ice and precipitation or lateral advection of water masses.

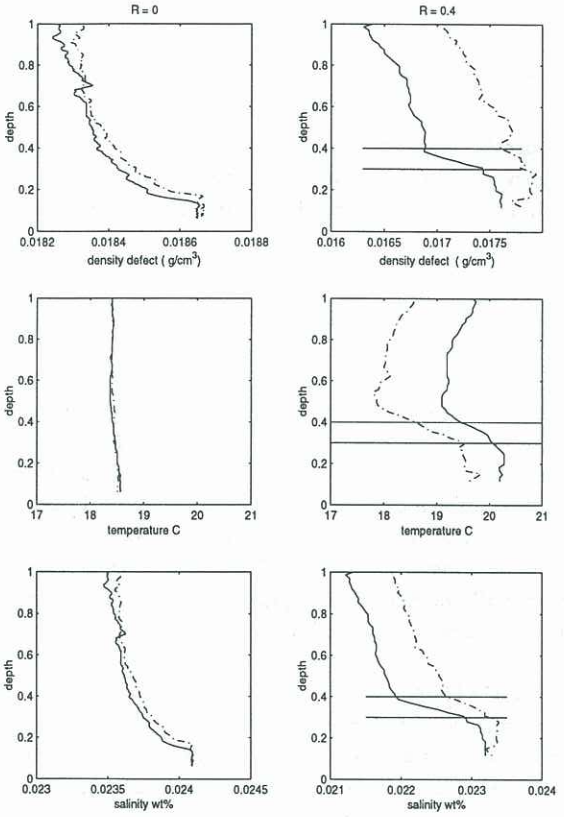

Figure 3. Density defect ($\rho - \rho_0$), temperature and salinity profiles profiles for a) $R = 0$, and b) for $R = 0.4$. The aspect ratio is 1/2 and the horizontal lines in the graph represent upper and lower limits to the depths of convection measured independently from the shadowgraph. The shape of the profiles remains constant with time while the density decreases.

CONCLUSION

When there is a uniform destabilising buoyancy flux through the horizontal boundary at which the plume source is located, we have shown that the long-time steady state of the finite chamber may be a partially mixed and partially stratified, depending on the ratio of the two buoyancy fluxes. The convecting layer depth adjusts until the rate of encroachment into the stratification is equal and opposite to the vertical advection driven by the plume 'filling box' process. For flux ratios $R > 0$ the normalised mixed layer depth, $\zeta_{mixed} = h/H$, is approximately $\zeta_{mixed} = R/(R+1)$ and for $R \geq 0.8$ the whole of the water column is convectively stirred. The stable density gradient was further strengthened by entrainment of buoyancy (heat) from the mixed layer. One result of this process is the production of a temperature minimum just above the convectively mixed layer.

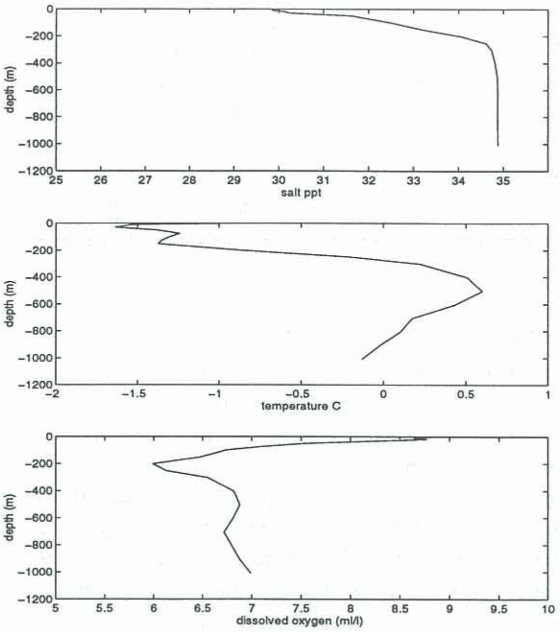

Figure 4. Measurements of density, temperature and oxygen profiles taken in the the Bering Sea from the Levitus oceanographic data set (1987).

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