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MODELING TWO-PHASE FLOW IN A VOLCANO

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ABSTRACT

Volcanic eruptions present different regime, which can be understood and classified in the framework of a two-phase flow, either an annular flow such as in hawaiian fire fountains or a slug flow in strombolian explosions. The gas, exsolved at depth, carries physical information about the dynamics of strombolian activity, which in turn may lead to a better understanding of volcanic systems. Activity of Stromboli is characterised by a series of explosions, occurring with a regular intermittency, ≈ 1 h. Their origin lies in the breaking of a metric, overpressurised bubble $\Delta P \approx 0.24 \times 10^5$ Pa at the surface of the magma column (Vergniolle et al., 1996). Such gas pockets, almost as large as the volcanic conduit, are formed intermittently at depth by coalescence of a foam layer in a shallow magma chamber (Jaupart and Vergniolle, 1989). Here we study the motion of a slug initially overpressurised in a vertical tube of length ≈ 50 m and diameter ≈ 2 m. Because the rise time of a slug is small (40 s) compared to their formation time (1 h), there is only one large bubble at the time in the tube. Although Stromboli volcano can be viewed as a very large scale laboratory experiment remote measurements are needed in order to follow the bubble rise from shallow depth and their breaking at the top of a viscous liquid. Because bubbles oscillations have long been recognised as a source of sound either in underwater explosions or in the ocean (Leighton, 1994), acoustic measurements have been carried out at Stromboli volcano.

Because bubbles are formed with an initial overpressure and rise in a tube, they grow mainly by increasing their length (slug flow), overshoot their equilibrium position and the gas compressibility provides the restoring force for their oscillations. This mode of oscillation is a volume mode, analogous to the breathing mode which is excited when bubbles are formed at an orifice (Leighton, 1994). Meanwhile, ascending bubbles adjust their size to the decreasing pressure field. However in a magma, overpressure can be preserved inside the bubble because viscous forces significantly delay the growth to equilibrium. The change in the volume of the rising bubble pushes the liquid column up and down, vigourously enough to produce sound waves of low frequency ≈ 0.5 Hz (Vergniolle et al., 1996). Because the bubbles are relatively long (25 m) compared to their radius (0.8 m), their shape is assumed to the a cylinder whose length oscillates until it stops by viscous dissipation in the magma. By combining the bubble rise with acoustic measurements, the initial overpressure at the depth of the magma chamber is estimated $\approx 1.8 \times 10^7$ Pa for an initial length of 0.3 m. Acoustic pressure is radiated very strongly, 0.11 Pa at a distance of ≈ 250 m from the volcanic vent when the slug is formed at depth, 40 s before breaking at the surface.

INTRODUCTION

Visual observations at Stromboli volcano show that sound and fragments of molten magma, called ejecta, are produced simultaneously when bubbles break at the surface of the lava between explosions. Oscillations of bubbles have been proven to be a powerful source of sound (e.g. Leighton, 1994). Although the audible sound radiated by an explosion is very striking, most of the acoustic energy is infrasonic, below 10 Hz (Vergniolle et al., 1996). The strong acoustic pressure recorded during explosions has been associated with the breaking of a large bubble with a radius of a few meters and still overpressurised ($\approx 0.24 \times 10^5$ Pa) at the top of the liquid column.

Acoustic measurements were performed on Stromboli volcano in direct line with the vent. The set-up consists of a microphone, an amplifier and a DAT recorder (-3 dB at 1 Hz) and corrected numerically down to 0.4 Hz. The acoustic pressure (Fig 1), measured in air with perfect weather conditions (dry, sunny, and without wind), is only due to the volcanic source, which radiates like a monopole.

Here the rise of an overpressurised single gas pocket (a slug) in a vertical column of viscous liquid is modeled. Acoustic pressure, radiated by the up and down motion of the magma column which are induced by the oscillations of the rising bubble, is calculated and compared with the measurements carried out at

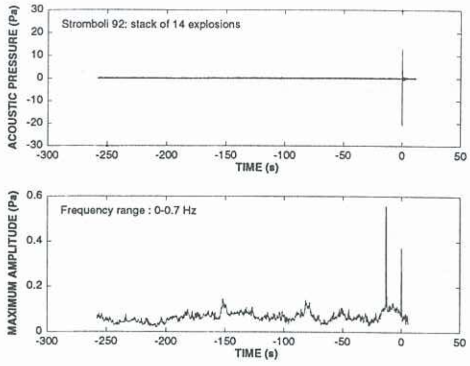

Figure 1: a) Acoustic pressure measured at 250 m from the source at Stromboli volcano (Stacking of 14 explosions. b) Maximum amplitude in the low frequency range.

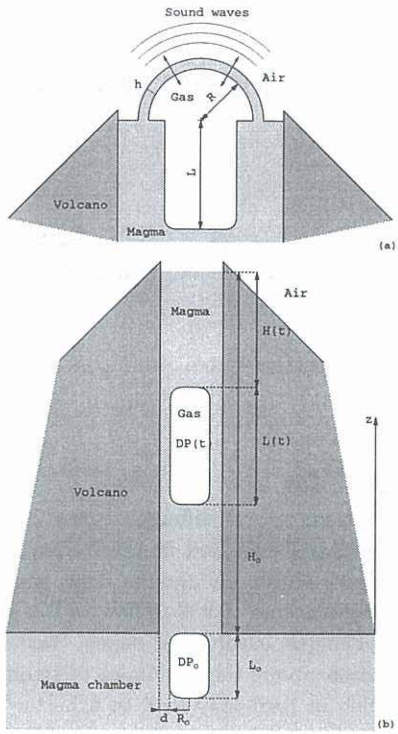

Figure 2: Sketch of a volcano.

Stromboli, in order to characterise the condition of slug formation, i.e. the bubble initial length and overpressure, at depth of $\approx$ 50 m.

THE MODEL

Assumptions

A typical strombolian bubble reaching the top of the magma column has a radius R_o of 0.8 m and a length L of 25 m (Vergniolle et al., 1996). When rising in a tube (Fig 2), the bubble shape consists in a cylindrical tail and a nose roughly hemispherical whose length depends on the viscosity of the surrounding liquid and on the radius of the tube (Batchelor, 1967). When the bubble approaches the top of the magma column, its nose may interact with the close by air-magma interface and may get distorted. However this effect will be neglected here and the calculation will be stopped at $\approx$ 1 m from the interface as shown later. When the thickness of the magma above the bubble is small, less than 1 m, the bubble expansion occurs also by increasing the radius of its hemispherical cap, which is the source of the main event in acoustic pressure (Vergniolle et al., 1996). In order to find the bubble length and overpressure at the depth of the magma reservoir, the solution for radius, length, overpressure at the air-magma interface (obtained from acoustic measurements by modeling the radial vibration of the bubble at the air-magma interface) is matched with the solution calculated for the bubble rise (Fig 2). The magma is assumed to be a Newtonian liquid of viscosity μ = 300 Pa.s and density $\rho_{\rm liq}$ = 2700 kg m^{-3}. Because the volcano has been active for more than 2000 years and releases a large gas pocket every hour, the wall of the volcanic conduit have probably been smoothen out over the years. Due to the "stirring-up" induced by bubble rise, the temperature is probably fairly homogeneous in the tube and the viscosity is assumed constant. Because the rise time is short (40 s) compared to the diffusion time (10^9 s), the number of gas moles inside the slug stays constant. The acoustic pressure recorded on 14 explosions at Stromboli gives the bubble radius for each explosion. The radius of the volcanic conduit R_c is estimated by assuming the thickness δ of the lateral film around the bubble. It is equal to its asymptotic value δ_∞ calculated when assuming potential flow (Batchelor, 1967):

$$\delta_\infty = 0.9 R_c \left(\frac{\mu^2}{\rho_{\rm liq}^2 R_c^3 g} \right)^{1/6} . \qquad (1)$$

At Stromboli δ_∞ = 0.26 m.

The radiation pattern of a rising bubble

The oscillations in the bubble length, while rising from depth, corresponds to oscillatory changes in the bubble volume. They induce changes in the magma level in the volcanic conduit. Because the magma above the bubble is mainly incompressible, contraction of the bubble reduces the level of the air-magma interface of an equivalent volume, while bubble expansion pushes it upwards. These changes in the level of the magma column in the volcanic tube follow the bubble oscillations and have the same frequency. Therefore the air-magma interface behaves as a vibrating body under the up and down pushes of the rising and vibrating bubble and radiates sound waves into air. These waves can be analysed to monitor the bubble rise.
Acoustic pressure $p_{\rm ac}$ emitted at the source at time t

will reach the microphones, at a time $t+r/c$, where r is the distance from the vent and c the sound speed in air. For such a monopole source, the excess pressure $p_{ac} - p_{air}$ at time t is (Lighthill, 1978)

$$p_{ac} - p_{air} = \frac{\dot{q}(t - r/c)}{4\pi r} = \frac{\rho_{air}}{2\pi r}\frac{d^2}{dt^2}[V_l(t - \frac{r}{c})] \quad (2)$$

where p_{air} and ρ_{air} are respectively atmospheric pressure and air density. The quantity $\frac{d}{dt}[V_l(t - r/c)]$ represents changes in the level of the air-magma interface and is equal to changes in the gas volume $\frac{d}{dt}[V_g(t - r/c)]$ existing in the vibrating bubble below. This gives

$$\frac{d^2}{dt^2}[V_l(t - r/c)] = \pi R_o^2 \ddot{L}(t - \frac{r}{c}) \quad (3)$$

for a cylindrical bubble of radius R_o and length L. Combining (2) and (3) gives

$$p_{ac} - p_{air} = \frac{\rho_{air} R_o^2}{2r}\ddot{L}(t - \frac{r}{c}) \quad (4)$$

Equations for the rising bubble

The upwards velocity U_b of a large bubble rising in a tube full of stagnant liquid depends mainly on the radius of the tube R_c but not on its length. When the viscous effects in the liquid are not negligible compared to the inertia effects, it becomes (Wallis, 1969)

$$U_b = 0.345\sqrt{2gR_c}(1 - e^{-(N_f/34.5)}) \quad (5)$$

$$N_f = \rho_{liq}\sqrt{8gR_c^3}/\mu. \quad (6)$$

For the large bubble at Stromboli,

$$U_b = 0.26\sqrt{2gR_c}. \quad (7)$$

Assuming that the changes in the level of the magma column are only due to contraction and expansion of the rising bubble, the volume of gas due to the bubble rise is exactly balanced by the loss of liquid along the bubble. Therefore the height of liquid H_l above the bubble decreases linearly with time t

$$H_l = H_o - U_b t \quad (8)$$

The motion of the rising bubble is deduced from the momentum conservation in the column of magma above the bubble. The forces exerted on the liquid column are the pressure applied by the oscillating bubble and the resisting motion of the viscous liquid against the solid walls. Its mass m_l decreases due to drainage along the bubble

$$m_l = \pi(R_o + \delta)^2 H_l \rho_l. \quad (9)$$

Because the magma is incompressible the velocity of the magma column U_l is equal to the expansion velocity of the bubble $\dot{L}$. Because the mass of magma column changes in time, the inertia force is

$$\frac{d}{dt}[m_l\dot{L}] = \pi(R_o + \delta)^2 H_l \rho_l \ddot{L} + \dot{L}\pi(R_o + \delta)^2 \rho_l \dot{H}_l. \quad (10)$$

The pressure force F_p applied on the magma column above a cylindrical bubble is

$$F_p = \pi R_o^2 (P_g - \rho_l g H_l - p_{air}), \quad (11)$$

where P_g is the internal pressure of the bubble. Because the heat transfer is adiabatic for large bubbles the pressure force becomes

$$F_p = \pi R_o^2 [P_o L_o^\gamma L^{-\gamma} - \rho_l g H_l - p_{air}], \quad (12)$$

where index o denotes the position of the bubble when leaving the magma chamber, and γ the ratio of specific heats, equal to 1.1 for hot gases (Lighthill, 1978). The last term to be calculated in the momentum equation is the friction exerted on the walls of the conduit due to the magma viscosity. The accurate calculation of this term is complex because it involves more than one coordinate. For laminar flow in a pipe, the pressure gradient $-[\frac{dP}{dz}]_f$ is:

$$-[\frac{dP}{dz}]_f = \frac{8\,\mu\, U_l}{R_c^2}. \quad (13)$$

with $U_l = \dot{L}$ and the friction force F_f becomes

$$F_f = -\pi 8\mu H_l \dot{L}. \quad (14)$$

The momentum equation is

$$\frac{d}{dt}[m_l\dot{L}] = F_p + F_f \quad (15)$$

and gives the evolution of the bubble length L in time

$$R_c^2\rho_l H_l \ddot{L} = -R_c^2 \rho_l \dot{H}_l \dot{L} - 8\mu H_l \dot{L} + R_o^2[P_o L_o^\gamma L^{-\gamma} - \rho_l g H_l - p_{air}]. \quad (16)$$

This has two initial conditions. The first one relates the initial overpressure ΔP_o at depth of the magma chamber, H_o, to the initial bubble acceleration $\ddot{L}_o$ giving

$$\ddot{L}_o = \frac{\Delta P_o R_o^2}{\rho_l H_o (R_o + \delta)^2}. \quad (17)$$

The second one is to set the initial bubble length L_o at depth H_o to be at its maximum ($\dot{L}_o = 0$ at $t = 0$). Equations (16) shows that the viscosity of the liquid damps the bubble oscillations while the drainage of liquid above the bubble enhances oscillations. For most of the rise however (until 5 m below the surface), the term due to viscous damping is larger than the term due to liquid drainage above the bubble by a factor 4. In these conditions, a "viscous" bubble can be calculated by assuming no inertia effects and its viscous overpressure ΔP_v is

$$\Delta P_v = \frac{8\mu H_l \dot{L}}{R_o^2}. \quad (18)$$

Behaviour of a rising bubble overpressurised at depth

Equation (16) is solved numerically with a second and third order Runge and Kutta method. Here the bubble behaviour is described by considering a bubble similar to the one at Stromboli volcano, i.e. with a radius (0.8 m) and initial length (0.3 m) and overpressure (1.8×10^7 Pa) rising in a liquid column of length 50 m. A "lithostatic" bubble is defined as being a bubble which rises infinitively slowly and has enough time at each step to adapt to the weight of the liquid column above. Except at the early beginning when the bubble is strongly vibrating, the length of the "real" bubble is smaller than the "lithostatic" bubble which shows that overpressure can be stored into the bubble during its rise and also at the top of the liquid column (Fig 3). Oscillations stops after one third of the rise. The bubble expansion velocity is quite large just after the formation of the slug at depth and the space diagram shows that the bubble vibrates only for 2 cycles before being stopped by viscous damping (Fig 4). The acoustic pressure radiated by the oscillations of the rising bubble is maximum (≈ 0.11 Pa) when the bubble is formed at depth (Fig 4). The bubble overpressure is very close to the viscous overpressure almost all the way up, except very close to the bubble formation or to the air-magma interface, where inertia effects are important.

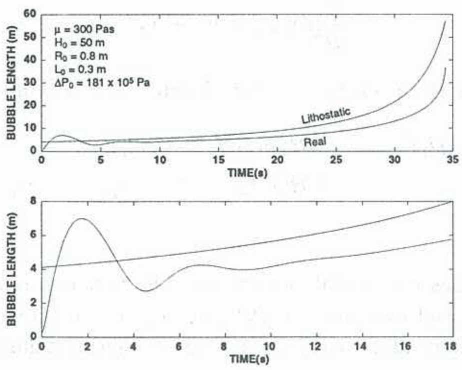

Figure 3: Bubble length versus time. a) from bottom to top of magma column b) beginning of rise at depth.

APPLICATIONS TO STROMBOLI VOLCANO

This model shows that in the conditions of Stromboli volcano the maximum in acoustic pressure should occur around 40 s before the onset of explosion. Because the frequency of the bubble length mode is low, ≈ 0.5 Hz, the maximum amplitude of the acoustic pressure is presented in this low frequency range in order to reduce the ambient noise produced by the volcano. Figure 1 shows that three peaks in acoustic pressure exist before explosion, the closest to the explosion being 40 s before its onset as expected. Furthermore, the very good agreement between its measured amplitude and the theoretical prediction (≈ 0.11 Pa; Fig 1 and 4) shows that a large bubble can sustain a very high pressure for a very short time. Although some amplification might occur in the volcano, results suggest that a strombolian bubble is formed with a large initial overpressure $\approx 1.8 \times 10^7$ Pa for an initial length around 0.3 m. The large bubble expansion velocity, occuring just after the bubble formation, might also produce waves at the top of the magma column with frequency above 1 Hz, which have be measured on acoustic records. The effects of the interface on the nearby bubble and of waves at the top of the liquid column are left for further studies. Although the model is very simple, it reproduces quite well the two key features which have been measured on a volcano (bubble rise time, amplitude of acoustic pressure). Therefore, measurements of acoustic pressure can provide valuable informations on the physics of volcanic activity as well as on the behaviour of bubbles.

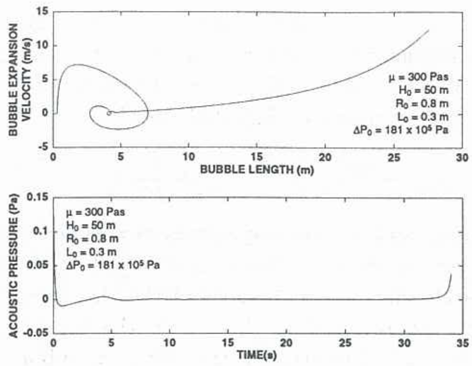

Figure 4: Space diagram (top). Theoretical acoustic pressure (bottom).

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