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A STOCHASTIC MODEL OF BURSTING PROCESS IN A ROTATING TURBULENT BOUNDARY LAYER

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ABSTRACT

Coherent structures or large scale motions in a rotating cylinder in a quiescent flow are considered by a conditional sampling procedure. As it is clear from the previous investigations that the coherent structure depends on the conditional technique of sampling, we discuss some dynamical aspect of them. Variable interval time average (VITA) technique is modified with a reasonable assumption, and the turbulent signal is conditioned. The random process generated by the Langevin equation is analyzed by the modified VITA technique. With comparing these results, we found that the coherent structures are well modeled by a Langevin equation as far as the conditional sampling is assigned.

INTRODUCTION

The turbulent shear flow around a rotating cylinder in a quiescent flow is a simple case of rotating turbulent flow field, where centrifugal force works (Bradshaw, 1969). The authors have investigated so far the mean and turbulent properties of this flow using hot-wires and flow visualization techniques (Nakamura et al., 1983, Ueki et al., 1992). In this paper we discuss the coherent structures or large scale motions in this complex flow field.

A great deal of information on coherent structures has been obtained from the visualization techniques, hot-wire measurements, and numerical simulations (Robinson 1991). While these studies have provided useful qualitative and quantitative information, most, if not all, results on coherent structures depend on the method adopted. It may be helpful to compare the different detection methods for establishing the accurate data (Yuan et al., 1994), however, a different approach is presented here. We modify the variable time average technique (VITA) (Blackwelder et al, 1976) with considering the physical meaning of the organized motion, and apply it to the turbulence signal and also to the random process generated by the Langevin equation. The conditioned coherent structures in these signals has a similar statistical features. So the key point is, the conditioned structures have the random property, or in other words their dynamics can be modeled by the Langevin equation.

EXPERIMENTAL FACILITY AND THE FLOW FIELD

The schematic flow field and symbols are indicated in Fig. 1. A cylinder of which diameter d equals $300mm$ was rotated around the y-axis. Reynolds number was set at $Re = U_w d/\nu = 6.2 \times 10^5$, where U_w is the velocity at the cylinder surface. Mean velocity and conventional turbulence quantities were measured using the usual I-type and X-type hot wire probes at a sampling frequency of 5 kHz.

In this flow field we have two different power law velocity distributions (Nakamura et al., 1983). Our experimental data was good agreement with the recent numerical result (Salhi et al., 1993). And also the data of Andersson et al. (1991) exhibits the same

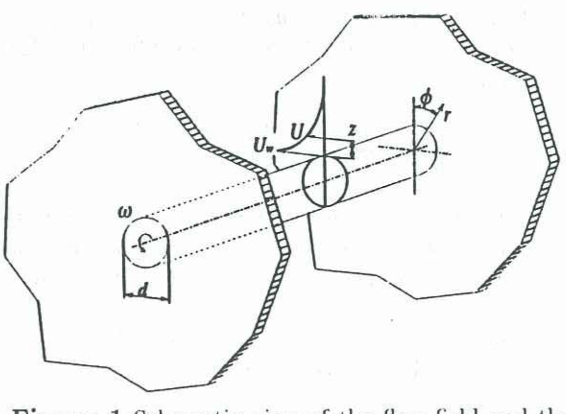

Figure 1 Schematic view of the flow field and the coordinate sysstem.

tendency with our obtained results. The authors have already calculated so far many statistical quantities, and revealed the features of this flow field. So the detailed explanations are given in the references (Nakamura et al., 1983, Ueki et al., 1992).

CONDITIONAL SAMPLING AND RANDOM PROCESS

The variable interval time average (VITA) technique is a kind of conditional sampling procedures (Blackwelder et al., 1983), which is well known to identify the organized structure in turbulence. We propose here the modified VITA technique. And the Langevin model is briefly explained for the analysis in the following section .

Modified VITA Technique VITA finds peaks in the localized variance with short time averages based on the time for a structure to pass a fixed point. This time seems to be an interval in which a structure or an eddy passes the point. For a velocity field, $u(z,t)$, a localized variance is computed as follows,

$$\widehat{var}(z,t;T^+) = u^2(z,t;T^+) - \left[\hat{u}(z,t;T^+)\right]^2 , \quad (1)$$

$$\hat{u}(z,t;T^+) = \frac{1}{T^+}\int_{t-T^+/2}^{t+T^+/2} u(z,t)dt , \quad (2)$$

where T^+ is the time scale for structure passage, which is normalized by the inner variables. A detection occurs when the localized variance exceeds some predefined threshold level multiplied by the square of the rms velocity,

$$D(t) = \begin{cases} 1 & :\widehat{var} > k \cdot u_{rms}^2 , du/dt < 0 \\ 0 & : \text{otherwise} \end{cases} , \quad (3)$$

which is called a definition function hereafter.

In the Eq. (3) the sign of velocity fluctuation is judged for constructing the defining function. The derivative $du(z,t)/dt$ is a value depending on the one-point fixed time. On the other hand, however, $\widehat{var}$ is a representative of the interval of T^+. This is somewhat curious for physical interpretation. Our simple presentation is that the derivative du/dt should be changed to $d\hat{u}/dt$, which indicate the feature of the interval T^+ and obtained from Eq. (2) as follows,

$$\frac{d\hat{u}}{dt} = \frac{u(t+T^+/2) - u(t-T^+/2)}{T^+} . \quad (4)$$

If the structures with time scale T^+ passe the fixed point, both $\widehat{var}$ and $d\hat{u}/dt$ are essential criterion to identify them. Therefore the definition function is,

$$D(t) = \begin{cases} 1 & :\widehat{var} > k \cdot u_{rms}^2 , d\hat{u}/dt < 0 \\ 0 & : \text{otherwise} \end{cases} . \quad (5)$$

We call the above criterion a modified VITA (MVITA) technique in the following.

Langevin Equation and Random Process One of the simple models of Langevin equation is called Ornstein-Uhlenbeck process;

$$du = -\alpha u dt + \beta dW_t , \quad (6)$$

where the random process is a function of time, and W_t is the Winer process. Both α and β are constant. Equation (6) is solved as

$$u(t) = u_0 e^{-\alpha t} + \beta \int_0^t e^{-\alpha(t-s)} dW_s \quad , \quad u_0 = u(0) . \quad (7)$$

Here, Eq. (6) is simplified as follows,

$$u(t+\Delta t) = \rho u(t) + n(t) , \quad (8)$$

where ρ is constant, and $n(t)$ is the white noise. Correlation function is obtained as

$$C(\tau) \equiv \langle u(t+\tau)u(t)\rangle = C(0)e^{-\gamma\tau} , \quad (9)$$

$$\tau = m\Delta t \quad , \quad \gamma = (1-\rho)/\Delta t . \quad (10)$$

The turbulence signal looks very different from the random process generated by Eq. (8) at a glance. But what happens if the MVITA is conditioned to these signals. This is a key point of the following analysis.

RESULTS AND DISCUSSIONS

The difference between VITA and MVITA method is considered first, and then the statistical feature of the definition function $D(t)$ is investigated. We apply MVITA to the random process generated by the Langevin equation, and the obtained results are compared with experiments.

VITA and MVITA method The definition of MVITA has been mentioned by Eq. (5), in which the criterion of the local slope is different from that of VITA method. We will think here what is the feature of structures conditioned by these two methods.

The intermittency factor and frequency are defined as,

$$\Gamma = \frac{1}{n}\sum_{i=1}^{n} D(t_i) , \quad (11)$$

$$f_R = \frac{1}{n\Delta t}\sum_{i=1}^{n} (1 - D(t_i)) \cdot D(t_i) , \quad (12)$$

where Γ is the temporal average of the definition function and f_R expresses the frequency of turbulent lumps passing a probe over a sufficiently large number of data point. Continuous time is sampled with a sampling interval Δt, and it is expressed as the discrete time t_i. Figure 2 shows these two values for the threshold k against the distance from the wall. There is no significant difference between VITA and MVITA even at close to the wall.

A procedure called conditional averaging or phase averaging is introduced. The process is based on the idea that, what a large number of coherent events are aligned at a certain reference point. The contamination by background turbulence will be removed and the general feature of the detected events will be brought out. This is given by

$$\langle u \rangle = \frac{1}{N} \sum_{j=1}^{N} u(t_j + \tau) , \tag{13}$$

where t_j is the reference point obtained by the definition function. For each pulse where $D(t)$ is unit, the middle point of the pulse is defined as the reference point(see Fig. 3). The total number of the events is N and τ is the time lag relative to t_j.

Figure 4 shows the conditional averaging velocity based on VITA and MVITA. MVITA conditions the more larger structures than VITA.

Random Process and MVITA method The MVITA is applied to both the turbulent signal and the random process. Langevin model has control parameters, α and β, that is ρ in Eq. (8). When ρ is changed, the signal looks different, thus we call ρ the control parameter in the random process. What is the control parameter in experimental signal? We assume that the distance from the wall, z, is the significant parameter in this system. In order to obtain the relation between ρ and z, the integral time scale is computed for various ρ and z. And matching the integral time scale of the turbulence signal to that of random process, MVITA is applied to the random process.

Figure 5 shows the correlation function,

$$C(\tau) \equiv \langle D(t + \tau) D(t) \rangle . \tag{14}$$

While the threshold parameter k and averaging time T^+ are fixed, the distance from the wall z in the experiment and the ρ in the simulation are changed. For several values of z and ρ, they resemble each other very much.

The conditional average $\langle u \rangle$ is computed in Fig. 6 with several z and ρ value. Within the experimental accuracy they resemble each other. We also computed the probability density function of the gap size, ℓ_3, indicated in Fig. 3, which is a period of the structures passing the fixed point in space. The result (see Fig. 7) has a exponential distribution like

$$p(\ell) = \lambda_1 \exp(-\lambda_2 x) , \tag{15}$$

where λ_1 and λ_2 are constant. When $\lambda_1 = \lambda_2 = 1/\langle \ell_3 \rangle$, this is the Poisson process. But the results are slightly different from this.

Judging from these results, the simple langevin model shares the same statistical feature with the turbulence signal as far as the conditional sampling process is imposed. There are several inherent limitations in MVITA technique. First, the time scale to be used for averaging is not clear, and the arbitrariness in choosing the threshold level is the second one. However, Langevin model can predict the coherent structures independent of k and T^+.

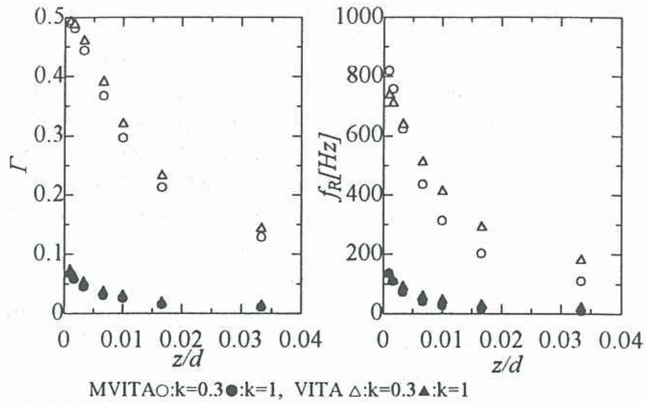

Figure 2 Frequncy of lump eddy and intermittency factor.

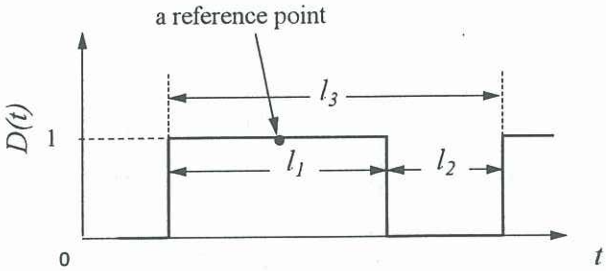

Figure 3 Definition function and the interval ℓ_1, ℓ_2, and ℓ_3. ℓ_3 is a period of the structures passing the fixed point in space.

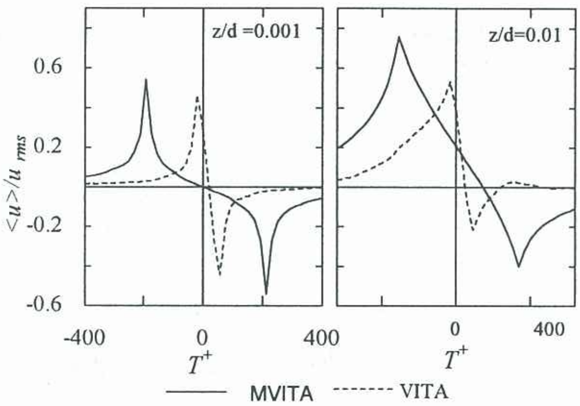

Figure 4 Conditional averaged streamwise velocities with T^+=404 and k=0.3.

CONCLUSION

Coherent structure is considered from a point of conditional sampling procedure. We modified the VITA technique (MVITA), and apply it to both the turbulence signal and the random process. These two signals look different each other, however, the statistical feature of the definition functions resemble each other significantly. Therefore, the turbulence signal has the same statistics with the random process as far as the MVITA technique is conditioned. Or in other words, the coherent structures defined by MVITA are well modeled by the Langevin equation.

We are still under investigating the detailed statistics of MVITA, and considering the physical grounds of it. MVITA will be applied to the zero-pressure-gradient boundary layers.

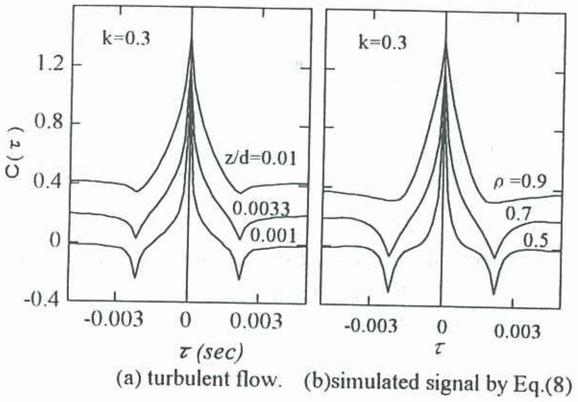

Figure 5 Correlation function of the definition function with several z and ρ values.

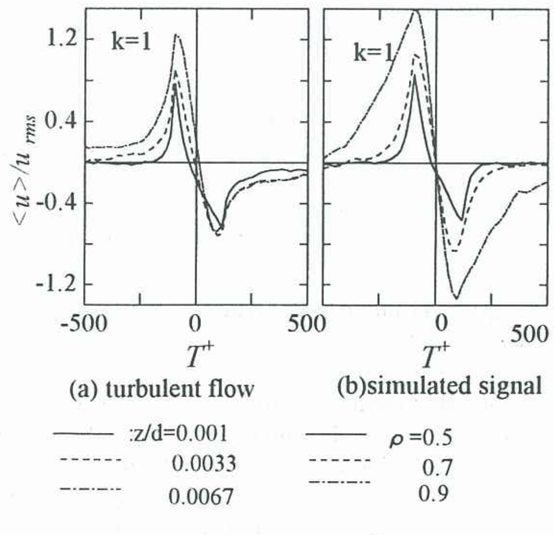

Figure 6 Conditional averaging profile by MVITA.

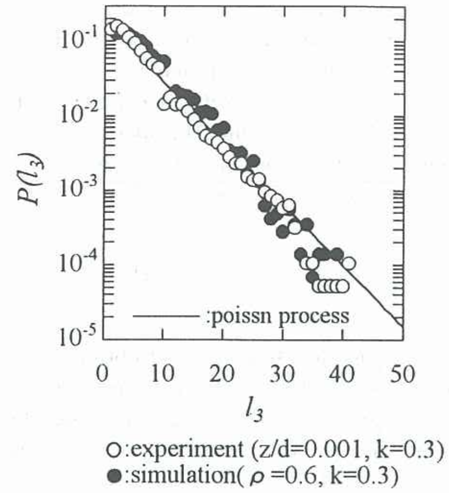

Figure 7 Probability density function of the gap size, ℓ_3.

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