13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

A SPECTRAL MODEL FOR ANISOTROPIC INHOMOGENEOUS TURBULENCE

Hatem TOUIL, Sylvain PARPAIS and Jean Pierre BERTOGLIO

LMFA, UMR CNRS 5509 - Ecole Centrale de Lyon
36 avenue G. de Collongue, 69130 Ecully, France

ABSTRACT

A spectral model for inhomogeneous and anisotropic incompressible turbulence is proposed. This model provides information on the spectra of the Reynolds stress tensor averaged over the wavenumber direction. Results are presented in the case of homogeneous turbulence subjected to a uniform shear and in the case of a fully developed channel flow.

INTRODUCTION

Engineering computations of turbulent flows are generally relying on turbulence models, known as one point-closures, namely the (k, ε) model and the second order or Reynolds stress models $(\overline{u_i u_j}, \varepsilon)$. One of the weaknesses of these models is related to the fact that an information on a turbulent length-scale is required to predict the turbulent energy, and that this information has to be provided by an equation for the dissipation rate ε.

During the last few years, we started developing a new type of model, called S.C.I.T. (for Simplified Closure for Inhomogeneous Turbulence, cf Parpais, Laporta and Bertoglio (1995); Parpais and Bertoglio(1996)) based on a statistical spectral approach, that is to say on a description of turbulence by correlations at two points. It is known that two-point models directly take into account information on different length scales, up to the Kolmogorov scale. Consequently no ε equation is required. Before being applied to real flows, the complex formulations of two-point closures for inhomogeneous turbulence must however be simplified. This is the approach followed when developing the S.C.I.T. model.

Up to now, the basic quantity in the model was the turbulent kinetic energy spectrum $E(K, \vec{X}, t)$ and consequently, although a detailed spectral information was retained, the anisotropic properties of turbulence were only grossly accounted for. The aim of the present paper is to extend the approach to a tensorial description of the turbulent spectrum.

BASIC EQUATION

The basic quantity in the model is the spectral tensor $\varphi_{ij}(K, \vec{X}, t)$. This tensor is defined as the average over a spherical shell of radius K of the full 3D spectral tensor $\Phi_{ij}(\vec{K}, \vec{X}, t)$ obtained by Fourier transforming, with respect to the separation vector $\vec{r}$, the two-point velocity correlation at $\vec{X} - \vec{r}/2$ and $\vec{X} + \vec{r}/2$. The average over the directions of $\vec{K}$ is introduced in order to reduce the computational cost(see Cambon, Jeandel and Mathieu (1981) and Besnard, Harlow, Rauenzahn and Zemach (1990)).

The equation for φ_{ij} reads:

$$\left(\frac{\partial}{\partial t} + \overline{U}_l \frac{\partial}{\partial X_l}\right) \varphi_{ij}(\vec{X}, K, t) = (-2\nu K^2 + \nu \nabla^2)\, \varphi_{ij}(\vec{X}, K, t) + P_{ij}(\vec{X}, K, t) + p_{ij}^{L}(\vec{X}, K, t) + p_{ij}^{LW}(\vec{X}, K, t) + t_{ij}^{L}(\vec{X}, K, t) + t_{ij}(\vec{X}, K, t) + D_{ij}(\vec{X}, K, t)$$

in which the first term of the right hand side is a viscous contribution which can be expressed exactly. The second term P_{ij} is also a term which is not requiring a closure.

This "production" term is

$$P_{ij}(\vec{X}, K, t) = -\frac{\partial \overline{U}_i(\vec{X}, t)}{\partial X_n} \varphi_{nj}(\vec{X}, K, t) - \frac{\partial \overline{U}_j(\vec{X}, t)}{\partial X_n} \varphi_{in}(\vec{X}, K, t)$$

The other terms p_{ij}^{L}, p_{ij}^{LW}, t_{ij}^{L}, t_{ij} and D_{ij} respectively stand for the rapid part of the pressure-strain spectrum, the echo term associated with wall effects, the linear transfer, the non linear transfer and the inhomogeneous transport term.

CLOSURE ASSUMPTIONS

The rapid part of the pressure-strain term p^L_{ij} is modeled by extending to a spectral formulation the procedure usually adopted in the case of Reynolds stress models. The closed form is (see also Clark and Zemach (1995)):

$$
\begin{aligned}
&p^L_{ij}(\vec{X},K,t) = \\
&c_b\left(\frac{\partial \overline{U}_i}{\partial X_n}\varphi_{nj}(\vec{X},K,t)+\frac{\partial \overline{U}_j}{\partial X_n}\varphi_{in}(\vec{X},K,t)\right) \\
&-\tfrac{2}{3}c_b\delta_{ij}\frac{\partial \overline{U}_m}{\partial X_n}\varphi_{mn}(\vec{X},K,t) \\
&+(8c_b-6)\left(\frac{\partial \overline{U}_n}{\partial X_i}\varphi_{nj}(\vec{X},K,t)+\frac{\partial \overline{U}_n}{\partial X_j}\varphi_{in}(\vec{X},K,t)\right) \\
&-\tfrac{2}{3}c_b\delta_{ij}\frac{\partial \overline{U}_m}{\partial X_n}\varphi_{mn}(\vec{X},K,t) \\
&+(\tfrac{11}{5}-3c_b)\left(\frac{\partial \overline{U}_i}{\partial X_j}+\frac{\partial \overline{U}_j}{\partial X_i}\right)\varphi_{nn}(\vec{X},K,t)
\end{aligned}
$$

with $c_b = 0.76$.

The non linear term t_{ij} is the term accounting for the energy transfer from the large scales to the small dissipative scales, as well as for the non linear part of the pressure strain term. Its trace, t_{ii}, is modeled using the Eddy Damped Quasi Normal Markovian theory (see Orszag (1970)). As for its deviatoric part, associated with the return to isotropy effect, it is also modeled using an EDQNM type of formulation, in which only the local triadic interactions are retained (the non local interactions being considered as only associated to a "sweeping" effect which is not contributing to the return to isotropy).

The inhomogeneous term D_{ij}, corresponding to the transport by triple correlations (and pressure velocity correlations) in Reynolds stress models is expressed via a diffusive form (see Besnard, Harlow, Rauenzahn and Zemach (1990)):

$$D_{ij}(\vec{X},K,t)=\frac{\partial}{\partial X_l}\left(\nu_{ts}(\vec{X},t)\frac{\partial \varphi_{ij}}{\partial X_l}(\vec{X},K,t)\right)$$

the eddy viscosity ν_{ts} being defined as :

$$\nu_{ts}(\vec{X},t)=\int_0^{\infty}\frac{E(\vec{X},K,t)\,dK}{A_2\sqrt{K^3E(\vec{X},K,t)}+A_s\Delta^*(\vec{X},t)}$$

in which Δ^* is defined as :

$$\Delta^*(\vec{X},t)=\sqrt{S^*_{ij}(\vec{X},t)S^*_{ij}(\vec{X},t)+\Omega^*_{ij}(\vec{X},t)\Omega^*_{ij}(\vec{X},t)}$$

with

$$S^*_{ij}=\frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial X_j}+\frac{\partial \overline{U}_j}{\partial X_i}\right);$$

$$\Omega^*_{ij}=\frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial X_j}-\frac{\partial \overline{U}_j}{\partial X_i}\right)$$

and

$$E(\vec{X},K,t)=\frac{1}{2}\varphi_{ii}(\vec{X},K,t)$$

A_2 and A_s are two constants whose values are identical to the ones used in the previous version of the model (see Parpais and al (1996)).

The linear transfer term t^L_{ij} is modeled as a flux to the small scales, whose characteristic time is built on the mean velocity gradient.

WALL EFFECT

The presence of a wall is modeled in a twofold way. A spectral cut-off at large scale is first introduced to take into account the scale limitation introduced by the wall (see Parpais and Bertoglio (1996)). Secondly, as in the case of one-point closures, an echo term p^{LW}_{ij}, is introduced to reproduce the effect of the wall on the anisotropy (see Daly and Harlow (1970), Shir (1973) or Gibson and Launder (1978)). This term is expressed assuming that it is a linear function of the deviatoric part of the production term

$$\overline{P}_{ij}=P_{ij}-\tfrac{2}{3}P_{ll}\delta_{ij}$$

and of the outward wall normal vector $\vec{N}$. One gets

$$
\begin{aligned}
p^{LW}_{ij}(\vec{X},K,t)= \;&(\,\alpha_1 N_l\overline{P}_{lm}N_m(\delta_{ij}-3N_iN_j)\\
&+\alpha_2\overline{P}_{ij}\,)f(\delta^w)
\end{aligned}
$$

in which f is a classical damping function depending on the distance to the wall δ^w.

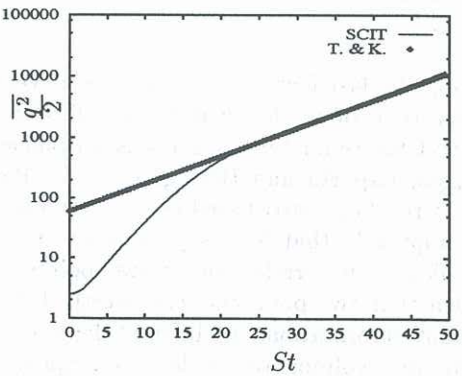

Figure 1: exponential evolution of $\frac{q^2}{2}$ compared to the asymptotic law proposed by Tavoularis and Karnik (exp(0.1*St) with S=shear rate - t=time);

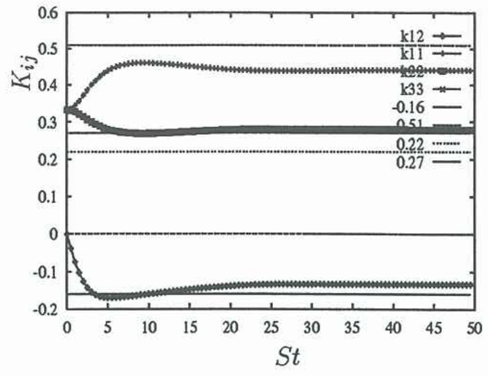

Figure 2: evolution of K_{ij} compared to Tavoularis and Karnik data;

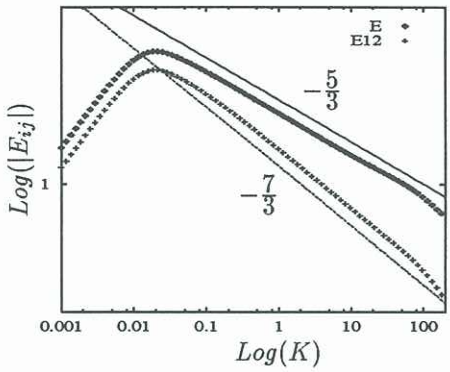

Figure 3: E and φ_{12} spectrum; SCIT calculation in the case of Homogeneous Shear.

RESULTS

The model was first applied to homogeneous turbulence subjected to a uniform shear, and compared to the experimental data of Tavoularis and Karnik (1989). The results show an exponential increase of all the turbulent quantities as well as a self-similar behavior of the large scales.

Figure 1 shows the time evolution of the turbulent kinetic energy. It can be observed that, after a transient period, the turbulent energy follows an exponential law with a numerical coefficient close to the one that was deduced from the experimental data by Karnik and Tavoularis (1989). Figure 2 shows that the non dimensional Reynolds stress tensor $\left(K_{ij} = \dfrac{\overline{u_i u_j}}{q^2/2}\right)$ reaches an asymptotic state. Again the agreement with the numerical values proposed by Karnik and Tavoularis (1989) is satisfactory ($K_{11} = 0.51$, $K_{22} = 0.22$, $K_{33} = 0.27$ and $K_{12} = -0.16$). Finally, the energy spectrum $E(K)$ and the Reynolds stress spectrum $E_{12}(K)$ are shown in figure 3.

In the case of a fully developed channel flow, the mean velocity profile obtained with the present model is compared with DNS results (data from Moin, personal communication) in figure 4. The agreement is satisfactory. The Reynolds stress and the turbulent kinetic energy are given in figure 5. The $\overline{uv}$ component appears to be fairly well predicted. As for the turbulent energy, the agreement is less satisfactory in the near wall region where there is probably still room for improvements in the model formulation.

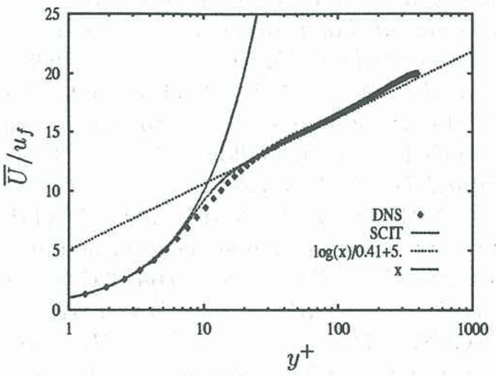

Figure 4: mean velocity profile; present model compared with DNS results.

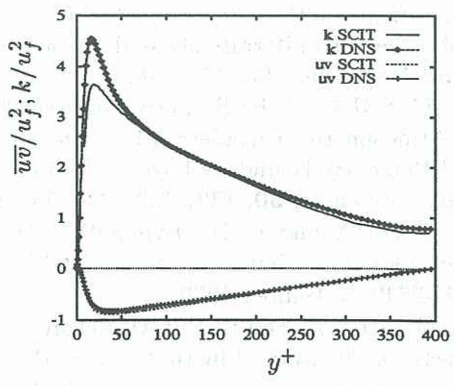

Figure 5: Kinetic energy k and Reynolds tensor stress $\overline{uv}$; present model compared with DNS results.

CONCLUSION

The spectral model for anisotropic and inhomogeneous turbulence presented in the paper has been validated in the case of a uniform shear flow. The asymptotic results of the model are in agreement with the data deduced from experiments by Karnik and Tavoularis. For inhomogeneous turbulence, the agreement with DNS data in the case of a fully developed channel flow appears to be satisfactory.

The model is now implemented in a finite element 2D Navier Stokes solver, and will be applied to more complex geometries in the near future.

REFERENCES

S. PARPAIS, A. LAPORTA, and J. P. BERTOGLIO, "An E.D.Q.N.M. model for inhomogeneous turbulence applied to diffusive turbulence generated by an oscillating grid", *Twelfth Australasian Fluid Mechanics Conference - University of Sydney,* 759-762, December 1995.

S. PARPAIS and J. P. BERTOGLIO, "A spectral closure for inhomogeneous turbulence applied to turbulent confined flow", *Kluwer Academic, editor,ETC VI,* July 1996.

C. CAMBON, D. JEANDEL, and J. MATHIEU, "Spectral modeling of homogeneous non isotropic turbulence", *J. Fluid Mech.*, **104**, 247-262, 1981.

D.C. BESNARD, F.H. HARLOW, R.M. RAUENZAHN, and C. ZEMACH, "Spectral transport model for inhomogeneous turbulence", *Technical Report Report LA-11821-MS,* Los Alamos National Laboratory, 1990.

T. CLARK and C. ZEMACH, "A spectral model applied to homogeneous turbulence", *Phys. of Fluids,***7(7)**, 1674-1694, 1995.

S. ORSZAG, "Analytical theories of turbulence", *J. Fluid Mech.,* **41(2)**, 363-386, 1970.

S. TAVOULARIS and U. KARNIK, "Further experiments on the evolution of turbulent stresses and scales in uniformly sheared turbulence", *J. Fluid Mech.,***204**, 457-478, 1989.

C. C. SHIR, "A Preliminary Numerical Study of Atmospheric Turbulent Flows in the Idealized Planetary Boundary Layer", *J. of the Atmospheric Sciences,* **30**, 1327-1339, October 1973.

B. J. DALY and F. H. HARLOW, "Transport Equations in Turbulence", *Phys. of Fluids,* **13**, 2634-2649, November 1970.

M. M. GIBSON and B. E. LAUNDER, "Ground Effects on Pressure Fluctuations in the Atmospheric Boundary Layer", *J. Fluid Mech.,* **86**, 491-511, 1978.