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AXISYMMETRIC CONTRACTION OF FULLY DEVELOPED PIPE FLOW

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ABSTRACT

An experimental investigation has been performed to determine the behaviour of fully developed turbulent pipe flow, when exposed to an axisymmetric contraction producing constant strain rate. In the early stage of the contraction, the straining causes a severe reduction of the streamwise velocity component, and a similar gain in the cross-stream components, without changing the total kinetic energy. In the later part a slight increase in kinetic energy occurs, which is due to secondary production terms. The redistribution of energy among the three normal Reynolds stress components is found to occur within the large scale motion.

INTRODUCTION

A renowned phenomenon of turbulence is that a mean flow acceleration reduces the streamwise normal stress more than the two lateral. This property has been extensively applied to e.g. wind tunnel design, to reduce the turbulence level and make it more isotropic. Comte-Bellot & Corrsin (1966) reported isotropisation of grid turbulence after a 1.27 area ratio contraction. Recent experiments by Sjögren (1997) downstream of a 9:1 area wind tunnel contraction, produced a state of axisymmetric and near two-dimensional turbulence. Obviously this implies strong anisotropy, due to the dominating cross-stream fluctuations. Tennekes & Lumley (1972) presented a visual picture of how streamwise vortex stretching increases the vortex angular velocity, thus suppressing streamwise fluctuations in favour of cross-stream fluctuations. The phenomenon is clearly connected to the influences of redistribution (pressure-strain) and secondary production terms, caused by the streamwise acceleration. The present study addresses the effect of axisymmetric contraction of an initially fully developed pipe flow. Similar previous studies are not known to the authors. Changes to the Reynolds stress tensor are reported and related to some of the terms in the Reynolds stress transport equations.

EXPERIMENTAL DETAILS

The contraction, shown in Fig.1, was mounted at the end of a 80 diameter fetch of smooth straight pipe, where the flow was found to be fully developed. The pipe diameter was $D = 186\,\mathrm{mm}$ and the bulk velocity $U_o = 2.83\,\mathrm{m/s}$, giving $Re = \frac{DU_o}{\nu} = 35.000$. Except at the initial transition from the pipe, the contraction was designed to produce a constant streamwise strain. Figure 2 shows that the streamwise acceleration, calculated from the measured static wall pressure, was near constant beyond $x = 170\,\mathrm{mm}$. For this constant strain region, $\partial U/\partial x = 40\,1/\mathrm{s}$. The contraction rate is defined as the area ratio $\xi = A(0)/A(x)$. Maximum contraction at $x = 600\,\mathrm{mm}$ was $\xi = 8$.

Velocity fluctuations were measured by means of hot-wire anemometry. Cross-wire probes were used to cover all non-zero elements of the Reynolds stress tensor ($\overline{u_i u_j}$). Probes were manufactured from $2.5\,\mu\mathrm{m}$ diameter Pt-10% Rhodium wire, with a length to diameter ratio of $l_w/d_w = 180$.

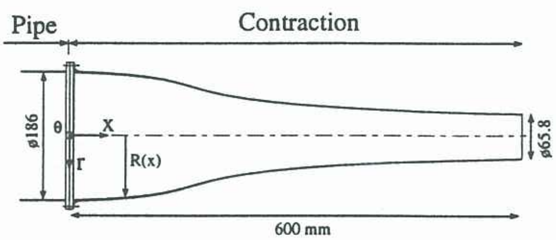

Figure 1: Contraction design.

RESULTS

Experimental results are presented for fully developed pipe flow ($\xi = 1$), and for the locations where $\xi = 4, 6$ and 8.

Reynolds Stresses

Figures 3, 4 and 5 show the measured normal stresses, $\overline{u_i^2}$, as a function of the distance from the wall, scaled with the local radius (R_ξ) and the pipe bulk velocity (U_o). This displays the total change from one position to the next. (The normal stresses at $\xi = 1$, scaled with the friction velocity, $\overline{u_i^+}$, are inserted in

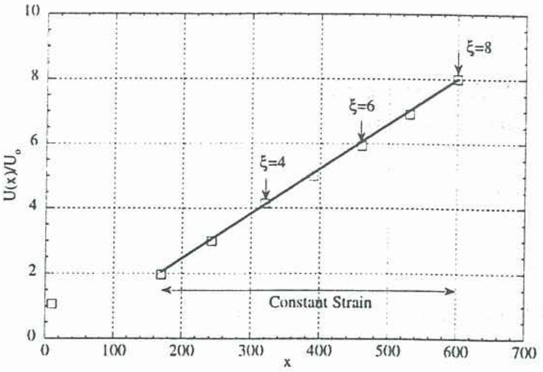

Figure 2: Flow acceleration, calculated from measured static wall pressure; $U(x)/U_o$, □.

Fig.3.). The results are presented on a logarithmic scaling on both axes, to properly display the variations. The streamwise component is very quickly affected by the strain, from being the dominant stress at $\xi = 1$(Fig.3), to becoming the smallest in the outer region at $\xi = 4$(Fig.4), where it has been reduced by almost an order of magnitude. Significant streamwise fluctuations are restricted to the region very near the wall at $\xi = 4$. The core region is quickly dominated by the two cross-stream fluctuations, which have become approximately equal. Figure 6 shows the total turbulent kinetic energy, 2k=$[\overline{u_x^2}+\overline{u_r^2}+\overline{u_\theta^2}]$, which at $\xi = 4$ has changed only marginally. This indicates that the loss of energy in the streamwise component has been compensated by a gain in the two cross-stream components. The shear stress (Fig. 7), is virtually unaffected in the core region. Thus the correlation between u_x and u_r is conserved, despite the strong redistribution between the velocity components. With further contraction to $\xi = 8$(Fig.5), $\overline{u_x^2}$ continues to decrease in the core region, though at a lower rate, and increases in the near wall region, as the production rate here increases downstream. The increase in total kinetic energy downstream indicates that the two cross-stream components increases more than the attenuation of the streamwise component. The core region develops towards a situation of strong anisotropy, where the cross-stream components are dominant and approximately equal. At $\xi = 8$ the shear stress is considerably reduced for $y/R_\xi > 0.1$, compared to the other stations. This indicates a sudden loss in correlation between u_x and u_r. This state corresponds to a situation of axisymmetry, described by Lee and Reynolds (1985) as "disc-like" turbulence. Figure 8 shows the elements of the anisotropy tensor:

$$b_{ij} = \frac{\overline{u_i u_j}}{2k} - \frac{1}{3}\delta_{ij}, \qquad (1)$$

along the contraction center line. The normal stresses clearly approach a state of two-component-axisymmetric turbulence.

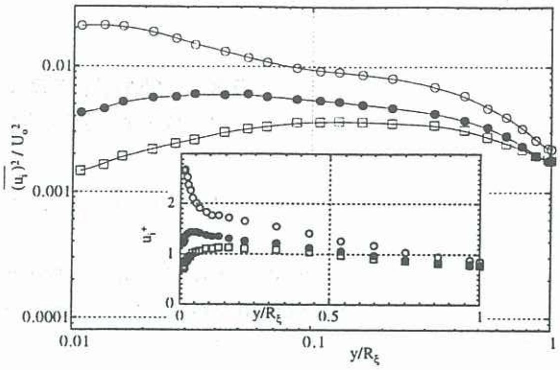

Figure 3: Contraction rate, $\xi = 1$; $\overline{u_x^2}/U_o^2$, ∘; $\overline{u_r^2}/U_o^2$, □; $\overline{u_\theta^2}/U_o^2$, •.

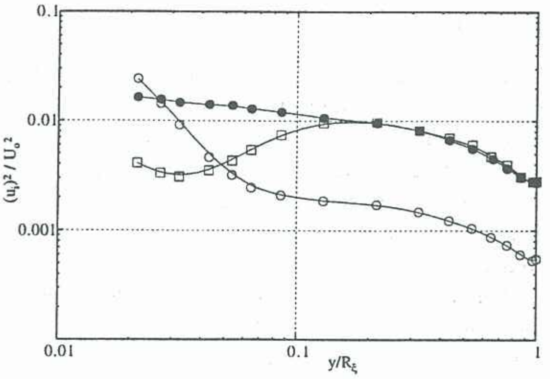

Figure 4: Contraction rate, $\xi = 4$; $\overline{u_x^2}/U_o^2$, ∘; $\overline{u_r^2}/U_o^2$, □; $\overline{u_\theta^2}/U_o^2$, •.

Production of turbulence

The production terms for $\overline{u_x^2}$ are:

$$P_{xx} = -\frac{\partial U_x}{\partial r}\overline{u_r u_x} + \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right)\overline{u_x^2} \qquad (2)$$

The first term on the right hand side is everywhere positive and increases the amount of $\overline{u_x^2}$. This term will always be dominant in the near wall region, with presence of turbulent shear stress and a strong mean velocity gradient. The remaining terms are negative and will tend to suppress $\overline{u_x^2}$. These terms are believed to be partly responsible for the abrupt reduction in the streamwise component. On the pipe center line the first term vanish by definition, and only the negative production term will be present.

For $\overline{u_r^2}$ the production terms are:

$$P_{rr} = -\frac{\partial U_r}{\partial r}\overline{u_r^2} - \frac{\partial U_r}{\partial x}\overline{u_r u_x} \qquad (3)$$

The first term on the right hand side, which is caused by a non-zero gradient of the wall normal mean velocity, becomes one of the main contributors to the increase in turbulent kinetic energy in the contraction. This term is always positive and proportional to $\overline{u_r^2}$ itself. The last term is small away from the wall, due to the vanishing shear stress, and the fact that U_r tends towards zero at the center line. The

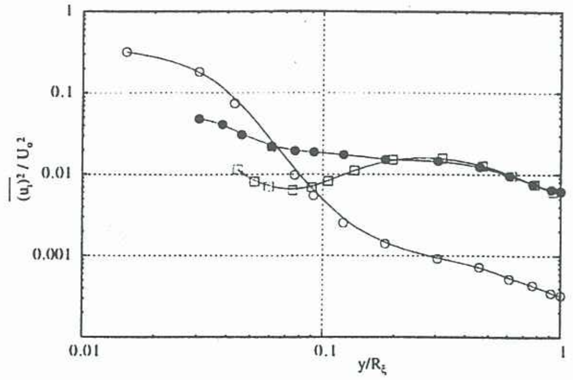

Figure 5: Contraction rate, $\xi = 8$; $\overline{u_x^2}/U_o^2, \circ$; $\overline{u_r^2}/U_o^2, \square$; $\overline{u_\theta^2}/U_o^2, \bullet$.

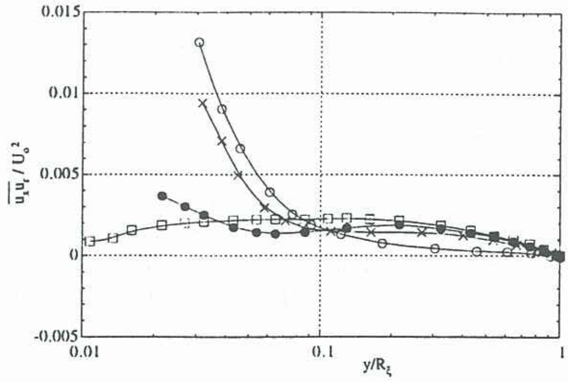

Figure 7: Turbulent shear stress; $\overline{u_x u_r}/U_o^2; \xi = 1, \square; \xi = 4, \bullet; \xi = 6, \times; \xi = 8, \circ$.

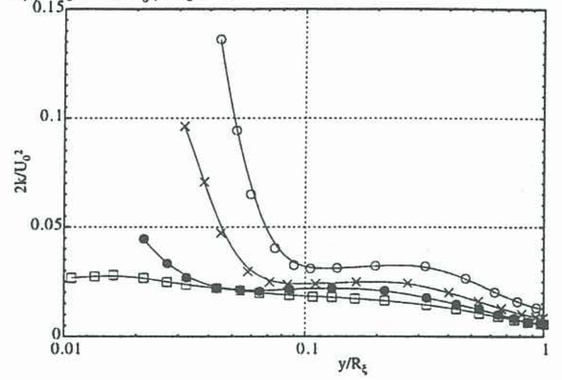

Figure 6: Turbulent kinetic energy; $2k/U_o^2; \xi = 1, \square; \xi = 4, \bullet; \xi = 6, \times; \xi = 8, \circ$.

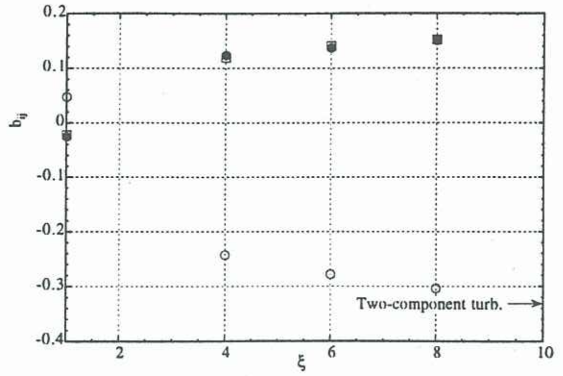

Figure 8: Reynols stress anisotropy at $r = 0$; $b_{xx}, \circ; b_{rr}, \bullet; b_{\theta\theta}, \square$.

wall normal mean velocity (U_r) must take zero value on the symmetry line, but the derivative is non-zero.

The only direct production term for $\overline{u_\theta^2}$ takes the simple form:

$$P_{\theta\theta} = -\frac{U_r}{r}\overline{u_\theta^2} \tag{4}$$

Since U_r is everywhere negative, this term is positive and contributes to a positive input of energy. A series expansion of U_r near the symmetry line yields that $\frac{U_r(r)}{r} \rightarrow \frac{\partial U_r}{\partial r}$, such that $P_{\theta\theta} \rightarrow P_{rr}$. On the center line of the fully developed pipe flow, the Reynolds stresses are axisymmetric and near isotropic, which implies $P_{xx} = -2P_{\theta\theta} = -2P_{rr}$, so the total production here becomes zero. This is reflected in the very small change in total kinetic energy between $\xi = 1$ and $\xi = 4$. As $\overline{u_x^2}$ vanishes, the total turbulence production is positive everywhere, which causes the increase in kinetic energy in the later part of the contraction.

The total production of turbulent kinetic energy becomes:

$$\begin{aligned} P_k &= -\left(\frac{\partial U_x}{\partial r} + \frac{\partial U_r}{\partial x}\right)\overline{u_r u_x} \\ &+ \left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right)\overline{u_x^2} - \frac{\partial U_r}{\partial r}\overline{u_r^2} - \frac{U_r}{r}\overline{u_\theta^2} \end{aligned} \tag{5}$$

If an isotropic turbulence model is applied to the problem, all terms in the second line vanish. This clearly indicates the restrictions of e.g. a standard k-ε model, since the total production will be considerably underestimated.

The production terms obviously favour the two cross-stream components, and will tend to suppress the streamwise component. It is clear however that this is not a sufficient explanation for the observed redistribution. It appears plausible to argue that the effect of the dissipation rate, diffusion and advection are of minor importance in this context. Since the dissipative scales are believed to be near isotropic, it is not likely that this should cause large scale anisotropy. When integrating the transport equations with respect to r, the radial diffusion and advection must vanish. The center line streamwise diffusion in the equations for $\overline{u_r^2}$ and $\overline{u_\theta^2}$ is found to be small compared to the production terms. The only remaining term, which has the capability of redistributing energy, is then the pressure-strain term. It is commonly stated that this term has the property of making turbulence more isotropic, suggesting that the energy flow is always directed towards the least energetic components. This means that, for the latter part of the contraction, the pressure-strain terms should cause an energy transfer from the two cross-stream components to the streamwise component. However, this increase in $\overline{u_x^2}$ will be suppressed by the last term in Eq. 3.

Turbulence spectra

The streamwise and radial power spectra are presented in Fig. 9 for the center line at contraction rates $\xi = 4$ and 8. Note that the spectra are shifted two decades for $\xi = 4$ and four decades for $\xi = 8$, and have been scaled with the pipe diameter (D) and bulk velocity (U_o). The spectra are compared to the fully developed pipe flow($\xi = 1$), and relate to the normal stresses as:

$$\frac{\overline{u_i^2}}{U_o^2} = \int_o^\infty \frac{E_{ii}(Dk_x)}{DU_o^2}\, d(Dk_x), \qquad (6)$$

where $k_x = \frac{2\pi}{U_x} f$. Thus the spectra represent the total energy distribution. For comparison at the high wavenumber range, the isotropic spectrum

$$E_{rr}^{iso}(k_x) = \frac{1}{2}\left[E_{xx}(k_x) - k_x \frac{\partial E_{xx}(k_x)}{\partial k_x}\right] \qquad (7)$$

is included. In fully developed pipe flow the energy containing scales are dominated by the streamwise fluctuations, but the radial spectra intercept the streamwise spectrum at $Dk_x \simeq 10$, and remains dominant in the high wavenumber range. This is consistent with the hypothesis of local isotropy. Due to the low Reynolds number, no pronounced inertial range is present. At $\xi = 4$, a severe large scale reduction in $E_{xx}(k_x)$ has occurred, while the small scales are related in a similar way as in the fully developed pipe flow. The radial spectrum is larger than the streamwise spectrum for all wavenumbers. Lindborg (1996) suggested that redistribution, due to pressure-strain, occurred mainly at large scales. Turbulent production is described by interaction between the mean flow and turbulence, and must therefore also be restricted to the large scales. If the interaction between eddies of same size are local in wavenumber space, this suggests that the large scale straining should have a decreasing influence with increasing wavenumber. This is supported by the present results, where the small scales are found to be less affected by the straining than the large scale events. Contracting further to $\xi = 8$ contributes mainly to an increase in the cross-stream spectra at large scales. Only minor changes are observed in the streamwise spectrum for the largest scales.

CONCLUSION

A rapid redistribution of energy between the normal components was found as the flow accelerated. In the initial stage, the level of turbulent kinetic energy remained constant in the core region, which is caused by a negative production term in the streamwise direction, closely balanced by a corresponding positive

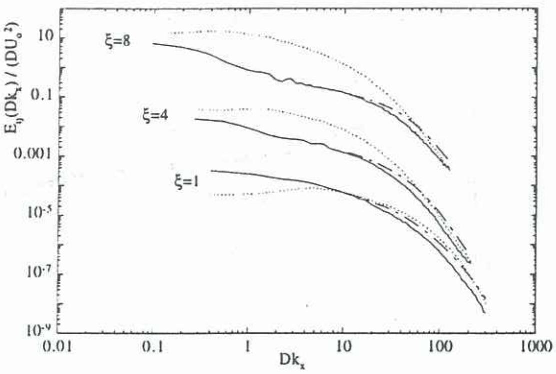

Figure 9: Turbulence spectrua at the center line; $E_{xx}(Dk_x)$ (solid) ; $E_{rr}(Dk_x)$ (dotted); $E_{rr}^{iso}(Dk_x)$ (dashed-dot)

production term for the cross-stream components. In the downstream part of the contraction, the streamwise component almost vanish in the bulk part of the cross section, and the total production was everywhere positive. This results in a gradual increase in the turbulent kinetic energy. The flow develops towards a situation of two-component-axisymmetric turbulence, dominated by cross-stream fluctuations. The streamwise normal stress is dominant only in a narrow layer near the wall. By examining the turbulence spectra, the inter-component transfer of energy is found to be restricted to the large energy containing scales. The rapid change in the Reynolds normal stresses is believed to be controlled mainly by secondary production and pressure-strain.

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