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ON THE USE OF VARIOUS CONTOUR SHAPES FOR EVALUATING CIRCULATION FROM PIV DATA

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ABSTRACT

Estimating circulation from discrete velocity information, e.g. from two-dimensional Particle Image Velocimetry (PIV), commonly involves using a rectangular contour to evaluate circulation of a vortex in the flow. It is shown in this paper that a non-regular octagonal contour can be deployed to accomplish the same goal without significant additional computational effort. The geometry of the octagon is optimized such that the difference in area between the octagon and enclosed circle is a global minimum. This is shown to happen when the ratio of the lengths of adjacent sides of the octagon is 0.744. Benchmark tests for estimating circulation of a forced vortex showed that while the error with using a rectangular contour was as high as 27%, this error reduced to less than 5% with the optimal octagonal contour.

Particle Image Velocimetry technique is applied to visualize flow due to a circular disk oscillating normal to its plane, in an otherwise quiescent fluid (water). The disk is 0.15 m in diameter oscillating at amplitude of 0.63 cm, and a period of 5.0 sec. A 5-cm wide laser sheet is used to visualize a cross-section of the vortex rings formed by the oscillating edge of the disk. To evaluate the circulation of the vortex rings, standard rectangular contour and the optimal octagonal contour are used. Results show a remarkable correlation between circulation evaluated using the octagonal contour and the average tangential velocity in the flow. The rectangular contour was found to overestimate the vortex circulation by 20 – 30%.

INTRODUCTION

Particle Image Velocimetry is a standard experimental technique to obtain simultaneous measurements of velocity at several points in a flow. Comprehensive reviews on the subject are provided by e.g. Adrian 1991, and Willert and Gharib, (1991). The concept of single exposure digital PIV (Willert and Gharib, 1991) is as follows: If a flow of interest is seeded, and the seed particles are assumed to follow the flow closely, then the velocity field is estimated by knowing the positions of the particles at two successive instants of time. The seeded flow is illuminated at two time instants and recorded on successive video frames. A small portion of the first image is correlated with the corresponding region in the second frame. The interrogation region typically contains several particle images, and the corresponding particles are assumed to displace uniformly. The position of the correlation maximum with respect to the origin gives the displacement of the group of particles in the small region of the flow. Knowing the magnification of the camera and the time interval between the records, the local velocity of the flow can be estimated. This process is repeated over the entire frame to obtain the velocity field. The double exposure method used here is conceptually similar to the single exposure technique, with the main difference being that images from two exposures are recorded on a single video frame. To avoid directional ambiguity (Adrian, 1991), an image shift is introduced such that the second image is recorded on an adjacent region to the first one. Thus successive regions on the same frame are interrogated to obtain the velocity information.

Once the spatial distribution of velocity is known, one can obtain the vorticity in the flow by a numerical differentiation technique. If regions of concentrated vorticity are observed, we can obtain the circulation (Γ) around the region, or part thereof, by a numerical integration over a closed contour, based on the definition

$$\Gamma = \oint \vec{v} \cdot d\vec{l} \qquad (1)$$

Once such evaluation has been conducted, the resulting circulation value may be used as the strength of a discrete vortex to model the flow in a computer simulation.

For a potential flow with point vortex singularities, the integral in Eq. (1) will be path-independent, i.e. the choice of the contour shape will not be important, as long as these singularities are included within the contour. However, in a real flow, the vorticity is diffused in and around the region of concentration. Hence a contour has to be carefully chosen to evaluate the circulation in a region.

Due to the circular nature of the streamlines around a vortex, we can expect that a contour which closely approximates a circular contour to be most effective. However, since the velocity grid is rectangular, a rectangular contour is a natural choice for integration of Eq. (1), see e.g. Abrahamson and Lonnes (1995) and Weigand and Gharib (1997). This also simplifies the calculation to summation of u-velocities along the horizontal parts of the contour and the v-velocities along the vertical parts, with the appropriate signs. Since the area enclosed by the rectangular contour is 27% higher than the area of the enclosed circle, a larger amount of vorticity is accounted for in the evaluation, thus leading to a significant overestimate of the circulation value. One could instead avoid using an approximate contour shape and resort to evaluating circulation over a circular contour (e.g. Weigand and Gharib, 1997). However, this procedure will involve numerical interpolation of the rectangular grid of velocity data to points along a circle, which introduces an additional level of complexity to the post-processing software, along with introducing additional errors due to interpolation. To obtain an accurate estimate of circulation in a flow, it thus becomes necessary to identify a contour:

- that continues to enable calculating circulation from spatially discrete velocity data, without additional interpolation of data.
- whose area closely approximates that of the enclosed circle, i.e. the difference in areas is a minimum.

OCTAGONAL CONTOURS

Apart from rectangles, one may investigate other polygonal shapes for the purpose of evaluating circulation from a discrete equally spaced grid of velocity data. Within the family of polygons, the octagon emerges as the sensible choice, since the inclined sides can be drawn by connecting grid points that are diagonally across. This feature enables this type of contour to be incorporated easily into a post-processing program, without the need for interpolation of velocity data. To identify an octagon whose area is closest to the enclosed circle, we need to allow for adjacent sides of the octagon to be of different lengths. Let us consider a circle of diameter $2a$, enclosed by an octagon whose sides alternate in lengths between $2b$ and $\sqrt{2}(a-b)$. Figure 1. The difference in areas between the two shapes is:

$$\Delta A = (4-\pi)a^2 - 2(a-b)^2 \qquad (2)$$

This is the objective function that has to be minimized in terms of the parameters (a and b) of the problem. The global minimum is obtained from the intuitive result of setting ΔA=0, which gives

$$b = 0.3449a \qquad (3)$$

Using this result, it can be seen that the ratio of the lengths of adjacent sides of the octagon is 0.744.

For the purposes of validation of the new contour, the velocity field due to a forced vortex (see e.g. Massey, 1983) was simulated. This field is given by the following equations:

$$v = -\frac{z}{2\pi}$$
$$w = \frac{y}{2\pi} \qquad (4)$$
$$\Gamma = r^2$$

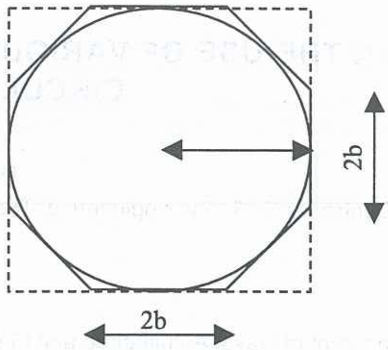

Figure 1: Various contours approximating a circle

where v and w are the velocity components along the y (horizontal) and z (vertical) directions respectively.

The data created using this velocity field was evaluated on an evenly spaced rectangular grid. The circulation was then calculated using rectangular and the optimal octagon contours. The magnitude of error in calculation as a function of distance from the core of the vortex (r/s; s – grid spacing) is plotted in Figure 2.

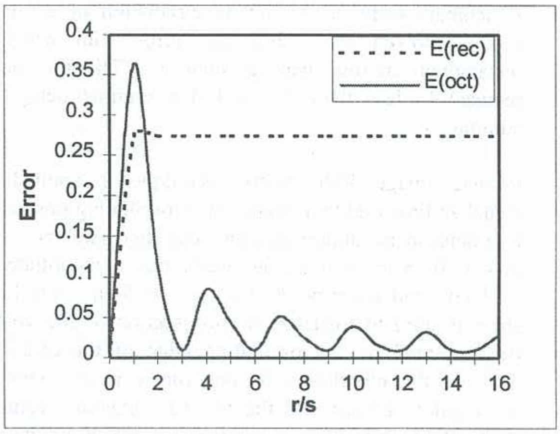

Figure 2: Percentage Error in circulation values using rectangular and octagonal contours.

It is noted that the error in using rectangular contours is as high as 27%, beyond a radius of one grid spacing, while that of the octagon drops to below 11% beyond r/s = 2, and continues to decrease with increasing r. The high error in using rectangular contour is a clear manifestation of including a higher area in the calculation. The error magnitude is approximately equal to the area difference of 27% because vorticity is a constant for a forced vortex. Similar calculations performed for other rotational fields showed different error magnitudes for the rectangle, though always significantly higher than the octagonal contour. The octagonal contour thus clearly proved superior for all radii above 1. In terms of practical application, since r/s=1 is close to the core of the vortex, where velocity magnitude is very small, it can be argued that an octagonal contour is a better substitute to the conventional rectangular contour.

EXPERIMENTAL METHOD

The flow of interest is the axisymmetric flow generated by a circular disk oscillating parallel to its axis in an otherwise quiescent fluid (water). The disk is 0.15 m in diameter and 2.54 mm thickness at the center, tapering off to 0.1 mm at the edge. The disk was attached to a motor-flywheel mechanism by means of a slender rod, and oscillated normal to its plane in a tank of water. The water was seeded with titanium dioxide particles of average diameter 2.79 μm, with an average image density (Adrian, 1991) of 10-20.

The experimental setup used is similar to a standard PIV setup, and is briefly described below. For further details, please refer to Thiagarajan (1993). A 4-watt Argon-ion laser was used as the illumination source. An area of 3.7 x 2.7 cm^2 adjacent to the edge of the disk was examined for flow patterns. The laser beam was fanned out to a 5-cm wide sheet, to uniformly illuminate the region of interest, and was pulsed at the desired rate using an acousto-optic modulator. The image acquisition system consisted of a Pulnix CCD video camera with a standard framing rate of 33 Hz, and a resolution of 512 x 492 pixels. A mirror attached to a galvanometer was placed between the camera lens and the CCD array, to provide an artificial image shift between successive light pulses for double exposure. Both the acousto-optic modulator and the galvanometer were synchronized in time. The signal to the modulator consisted of two pulses spaced 5 msec apart, and of 2 msec duration each. The galvanometer was given a step function input to change position within the 5 msec interval, thus providing the required shift.

Images acquired by the camera were written onto an optical disk and stored for later analysis. A cross-correlation routine was used to analyze the peak shift within a 32 x 32 pixel interrogation window.

RESULTS AND DISCUSSION

The flow chosen for the present discussion was obtained by oscillating the disk at amplitude of 0.63 cm and a period of 5.0 sec. The field of view was focussed towards a small area in the vicinity of the disk edge so as to visualize the vortex roll-up that occurs as the disk is oscillated. Figure 3 shows the vector plot at a time instant using the PIV technique. The edge of the disk is located at (y,z) = (1.8, 2.0) cm in Figure 3. The disk as shown is at the top dead center of its oscillation cycle. The figure shows the cross section of the vortex ring that has formed during the previous half cycle.

One can get an idea of the tangential velocity (v_θ) distribution along the vortex core by taking longitudinal and vertical cuts across the core. The velocity distributions along these cuts at four positions (0, 90, 180 and 270 deg) with the origin located at the vortex center point are shown in Figure 4. Some features of the vortex are fairly obvious. Due to the presence of the disk above, the vortex is compressed vertically. This results in the

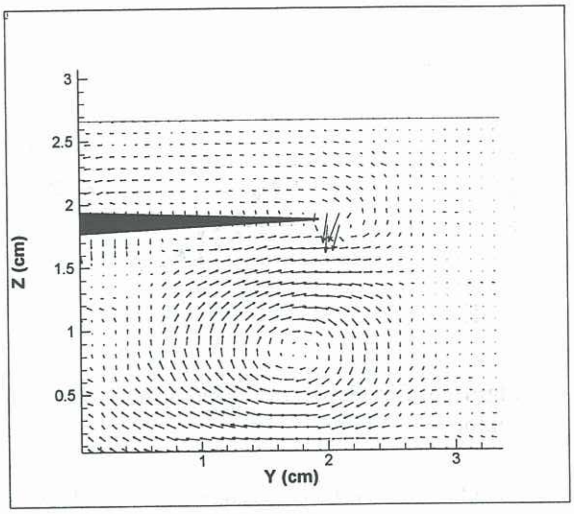

Figure 3: Velocity vectors in the flow due to an oscillating disk

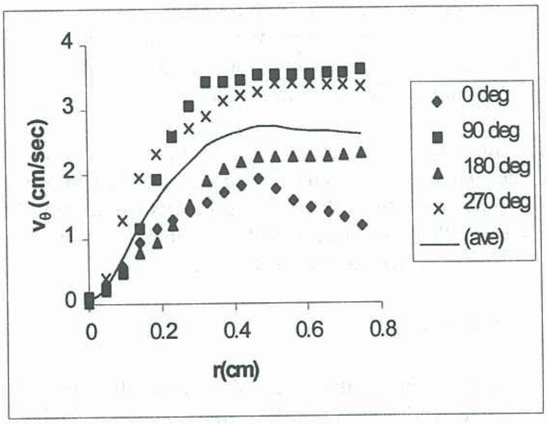

Figure 4: Tangential velocity distribution along the vortex core.

the tangential velocities along the vertical cut to be higher than in the longitudinal cuts, as can be seen in Figure 4. Also evident is the effect of dissipation of vorticity in the outward longitudinal (0 deg), where the vortex interacts with the ambient flow, thus losing strength. This effect is not so pronounced for the vertical cut, but still can be seen in the 270 deg curve. Upon observing the vector plot in Figure 3, one can reasonably conclude that the tangential velocity distributed over a circular contour around the center point of the vortex is bracketed by the curves of Figure 4. This implies that one can obtain a reasonable value for the average tangential velocity at a radius by taking the average of the four curves as shown in Fig. 4.

The circulation around the vortex center point in Figure 3, as a function of core radius was calculated using both rectangular and optimal octagonal contours, and is plotted in Figure 5. Consistent with the convention, the circulation is negative for the clockwise vortex. The figure shows that while the trends using the two contours

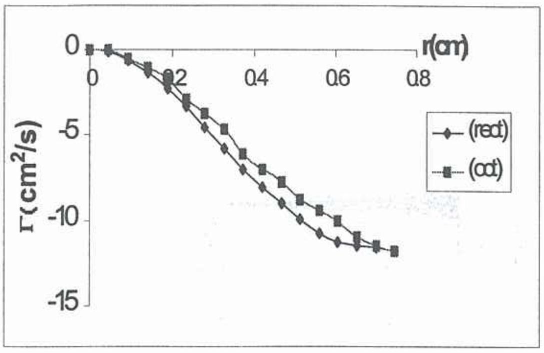

Figure 5: Circulation evaluated using different contours

are similar, the rectangular contour results in values that are about 20-30% higher compared to the octagonal contour values. As explained previously, this is due to the rectangular contour including a larger area than the enclosed octagon or circle. At higher radius, both contours appear to tend towards the same result as can be expected. At very large radius, when the entire vortex is enclosed at the center, the contour shape becomes immaterial, and evaluation of the integral of Eq. 1 is independent of changes in contour.

Following on the definition of circulation from Eq. 1, if the tangential velocity is weakly dependent on the angular coordinate (θ), and is primarily a function of the radius, then one can rewrite the integral in terms of azimuthally averaged v_θ, as follows:

$$\Gamma = 2\pi r \bar{v}_\theta \qquad (5)$$

This then implies that $\Gamma/2\pi r$ should closely approximate the averaged v_θ. Figure 6 shows $\Gamma/2\pi r$ evaluated using the two different contours, plotted along with averaged v_θ obtained from Figure 4. This figure shows clearly that while the octagonal contour closely follows the average tangential velocity curve, the rectangular contour over-predicts Γ over most of the vortex core. This figure shows conclusively that the optimal octagonal contour is to be preferred over the rectangular contour when there is a radial variation in vorticity, and when it is required to evaluate circulation more accurately.

CONCLUSIONS

An optimal octagonal contour is proposed for efficient and accurate evaluation of circulation from discrete velocity data. Benchmark tests showed that the octagonal contour had an error of less than 5%, compared to the rectangular contour, which had a steady error of 12%. The two contours are applied to a real flow of vortex roll-up due to an oscillating disk. The octagonal contour again showed good correlation with averaged tangential velocity, while the rectangular contour over-estimated the circulation in the flow by as much as 30%. It is suggested that the optimal octagonal contour proposed in this paper can be incorporated into a standard circulation evaluation program without need for

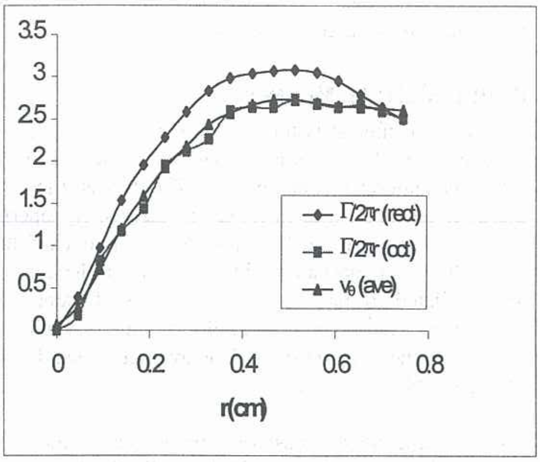

Figure 6: Comparison of circulation distribution with averaged tangential velocity distribution.

data interpolation, and accurate evaluations of circulation be achieved.

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