13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

HYDRODYNAMIC HEAVE DAMPING OF A VERTICAL CYLINDER

Longbin TAO, Liang CHENG and Krish THIAGARAJAN*

Department of Civil Engineering
The University of Western Australia, Nedlands, Western Australia, AUSTRALIA
*Center for Oil & Gas Engineering

ABSTRACT

Nonlinear viscous damping forces of a vertical circular cylinder, subject to a forced oscillation, are calculated by directly solving Navier-Stokes equations. The problem is formulated axisymmetrically and discretized using finite-difference method in a curvilinear coordinate system. The pressure Poisson formulation is employed to march the solution in time-space and a third-order upwind scheme is used to approximate the convection terms in the momentum equations. A surface piercing circular cylinder subjected to a harmonic oscillation is simulated for the Keulegan-Carpenter number ($KC = 2\pi a/D$) less than 1.0 and the frequency parameter ($\beta = D^2 f/\nu$) up to 3.98×10^6. Factors affecting the accuracy of the simulation, such as the mesh size and time step, are investigated. It is found that the damping force is very sensitive to the variation of the time-step, while the total force converges very quickly to a stable value as the time-step is refined. The favorable comparison of the numerical results with the experimental results indicates the feasibility of the present method.

INTRODUCTION

The Tension Leg Platform (TLP) is a relatively new kind of offshore structure which provides an important key to the development of oil and gas resources from deep waters. It is normally designed so that the resonant frequencies of vertical-plane motions are well above the dominant frequencies of the ocean wave spectrum. This is in order to prevent the excessive resonant motions induced by the first-order wave forces in these directions. However, nonlinear wave effects may still cause resonant response of a TLP at high frequency vertical oscillations, i.e. heave, pitch and roll, the so called springing vibration, which is typically of high frequency and small amplitude. This may cause significant fatigue stresses in the tethers. Therefore, the accurate prediction of the viscous damping force is of great importance for TLP design. To predict the viscous damping of the resonant oscillations of the TLP column in the vertical mode, simplification has to be made by considering the vertical-plane motions separately in order to prevent dealing with the complex response problem of motions with multiple degrees of freedom. Simplified analyses based on experimental investigations were performed by considering the heave motion only by Huse(1990), Chakrabati and Hanna(1991) and Thiagarajan and Troesch(1994). Considerable insights about the flow and force characteristics have been achieved. It was found that the damping force is typically very small in magnitude in comparison with the dominating inertia force. This makes the measurement of the damping force extremely difficult for small amplitude oscillations. Therefore attention has been paid to the numerical modelling of the flow in recent years. Yeung and Ananthakrishan(1992) conducted a two-dimensional simulation of a vertical cylinder in the heave motion using the finite difference method. A certain range of frequency and amplitude of heave motion was investigated, and it was found that non-linear effects were evident in the force characteristics. Huse and Utnes(1994) carried out a numerical simulation of a single column of a TLP experiencing high frequency and small amplitude oscillations using the Galerkin finite-element method. The numerical results of the hydrodynamic and damping forces compared quite well with the experimental results.

Numerical investigation of the hydrodynamic springing damping of a vertical cylinder has been performed in the present paper. The axisymmetric viscous flow problem corresponding to heave oscillation of a semi-submerged vertical cylinder is solved using finite-difference method in a curvilinear coordinate. The pressure Poisson method is employed to solve the Navier-Stokes equations and a third-order upwind scheme, proposed by Kawamura and Kuwahara(1984), is used to approximate the convection terms. A surface piercing vertical cylinder experiencing the resonant heave oscillations are investigated. The flow structure with details of the vortex generation around the oscillating cylinder and force characteristics will be investigated for the KC number less than 1.0 and β up to 3.98×10^6. Numerical results on hydrodynamic and damping forces will also be compared with previously published experimental results.

NUMERICAL MODEL

Basic Equations

The prediction of hydrodynamic damping of heave resonant motions of a vertical cylinder can be carried out by solving the incompressible Navier-Stokes equations. To simplify the analysis, the problem is idealized to an axisymmetric flow around a vertical cylinder in heave oscillations. In this paper, the governing equations and all physical quantities are presented in a nondimensional form, with fundamental variables being density of fluid ϱ , acceleration of gravity g, and radius of the cylinder R. The coordinate system is fixed to the bottom open boundary with positive axis pointing upward. The governing equations in cylindrical polar coordinate system (r, y) read:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$\begin{aligned}&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+v\frac{\partial u}{\partial y}\\ =\ & -\frac{\partial p}{\partial r}+\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)\end{aligned}$$

$$\begin{aligned}&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+v\frac{\partial v}{\partial y}\\ =\ & -\frac{\partial p}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (3)\end{aligned}$$

here (u, v) denote radial and axial components of velocity respectively, t time, and p the dynamic pressure. The Reynolds number is defined as $R_e = \frac{R\sqrt{Rg}}{\nu}$, where ν is the coefficient of kinematic viscosity of the fluid. The Poisson equation for pressure can be derived by taking divergence of the momentum equations (2) and (3), and is not presented here due to the page limitation.

Boundary Conditions

Since the cylinder is forced to oscillate sinusoidally along longitudinal axis as

$$Y(t) = a\sin(\omega t) \quad \text{for} \quad t > 0, \qquad (4)$$

the velocity boundary conditions on the cylinder surface are given by

$$u = 0 \quad \text{and} \quad v = a\omega\cos(\omega t), \qquad (5)$$

where a, ω are the amplitude and angular frequency of the oscillation respectively. The Neumann boundary condition for the pressure on the cylinder surface can be obtained by substituting (5) into the momentum equations. On the axis of symmetry ($r = 0$), the radial velocity is set to zero, and zero normal derivative is applied for axial velocity and pressure. The zero normal derivative of the axial velocity and the zero radial velocity are applied at the far bottom open boundary. Because of the very small amplitude and high frequency oscillation, the free surface is approximated as an horizontal surface. At the right open boundary($r \gg R$) the axial velocity and the normal derivative of the radial velocity are set to zero . The governing equations together with the given boundary conditions are solved as an initial boundary value problem. It is assumed that the flow induced by a TLP column in heave motion starts from a quiescent state, i.e., velocity is zero at time t=0.

Hydrodynamic Force Calculation

The hydrodynamic heave force acting on the cylinder can be calculated by integrating the vertical component of the stress vector along the cylinder surface. In order to estimate the viscous and pressure term contributions, the three components of the total hydrodynamic heave force, i.e. the viscous shear-stress component F_s, the normal viscous-stress component F_n, and the dynamic pressure component F_p are calculated separately. The damping force can be estimated from the phase shift of the total hydrodynamic heave force. Because of the inertially dominant resonant oscillation, the inertia force can be approximated by a harmonic function with the same frequency and amplitude as the total force but a zero phase shift. The damping force is then found by subtracting this inertia force from the total hydrodynamic force.

RESULT AND DISCUSSION

Mesh Sensitivity Study

Firstly, the test cases with different mesh were conducted at $KC = 0.01$ and $\beta = 3.98\times10^6$. The mesh is constructed in such a way that the node points are concentrated near the cylinder surface and streched out graduately. Calculation results indicate that a 68×83 mesh is normally fine enough to result in a mesh-independant result. A series of numerical tests with different time-steps and grid spacings were then conducted in order to obtain a solution which does not depend on the grid spacings and time-steps used. Figure 2 shows the variation of the hydrodynamic and damping forces with the normalized minimum grid size, $\min(\delta x/a)$, where δx is the minimum mesh size near the cylinder surface. It is shown that the minimum grid spacing near the cylinder edge should be in the range of $\min(\delta x/a) \leq 0.33$ to result in resonable estimate of hydrodynamic and damping forces. Another interesting finding is that the hydrodynamic force is not as sensitive as drag force to the value of time step, as shown in Figure 3. This suggests that a smaller value of the time step is required to ensure the accuracy of damping force. It can be seen that $\delta t = T/500$ is sufficient for an accurate prediction of hydrodynamic force, while $\delta t = T/5000$ is required for an accurate damping force estimation. It is thus concluded that a mesh of (68×83) nodes should be sufficient under the conditions that $\min(\delta x/a) \leq 0.33$ and the time step $(\delta t) \leq T/5000$.

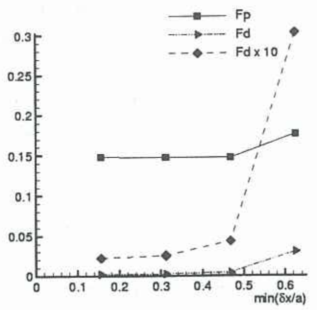

Figure 2: Force amplitudes vs. min. grid spacing.

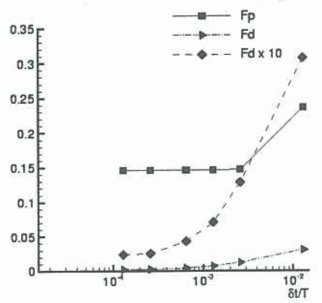

Figure 3: Force amplitudes vs. time step.

Flow Structure

Flow structures around the oscillating cylinder rim are characterized by the shed vorticity in the flow. Figure 4 shows the vortex generation around the cylinder corresponding to a typical case of small amplitude of oscillation $(KC = 0.01, \beta = 3.98 \times 10^6)$. Figure 4a $(t = 0)$ shows that when the cylinder starts to move up from the equilibrium position, the flow is into the void generated by the ascending movement of the cylinder, and the incipient formation of the wake eddies near the cylinder rim can be observed. Wake vortices are fully developed when the cylinder reaches the top position where the instantaneous moving speed of the cylinder is zero, as shown in Figure 4b $(t = 1/4T)$. Figure 4c shows that the vortex formed earlier during the ascent nearly dissolves when the cylinder moves downwards and reaches the maximum speed at $t = 1/2T$, a major vortex forms in the shear layer generated along the side wall near the bottom edge of the cylinder. When the cylinder stops at the lowest position and cylinder starts accelerating in the other direction $(t = 3/4T)$, single vortex develops at the side near cylinder rim, as can be seen in Figure 4d. The secondary vortices can be clearly observed at the instantaneous time when the cylinder stops and changes the direction of oscillation. (see Figure 4b, 4d)

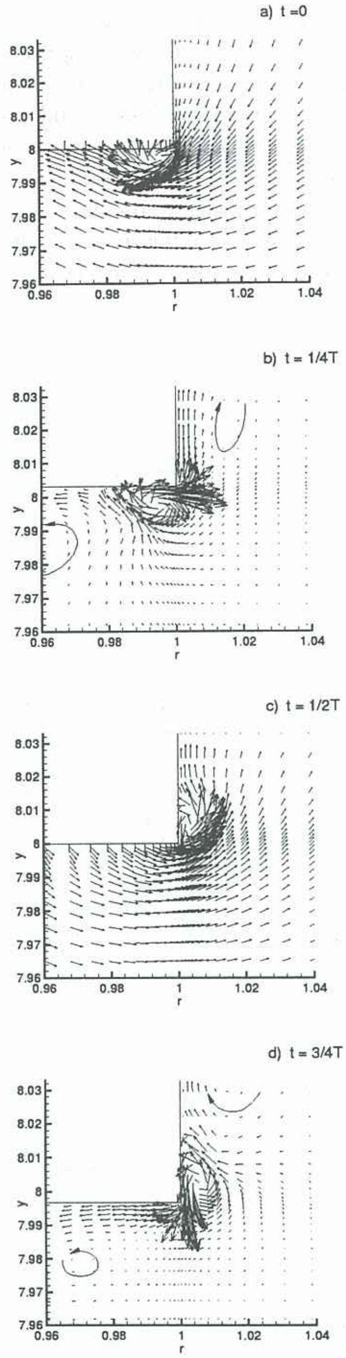

Figure 4: Vortex generation near the cylinder rim.

Hydrodynamic Force Comparisons

In order to demonstrate the different sources of the hydrodynamic force for a vertical cylinder experiencing heave motion, the three contributions of the hydrodynamic heave forces have been computed. Figure 5 shows that the shear-stress component F_s is only about 0.45%, while the normal viscous-stress component F_n is only about 0.00144% of the pressure drag $F_p(F_s, F_n, F_p$ are nondimensional forces) for the present case with $KC = 0.01$ and $\beta = 3.98 \times 10^6$. Figure 6 shows that the hydrodynamic force obtained from the present numerical solution agrees very well with the results from the experiments conducted by Huse et al(1989, 1990), while the damping force is slightly higher than the experimental results. It is also illustrated that the vertical cylinder in high-frequency resonant heave oscillation is a lightly damped system. The damping force is only 1.7% of the total hydrodynamic force in this typical case.

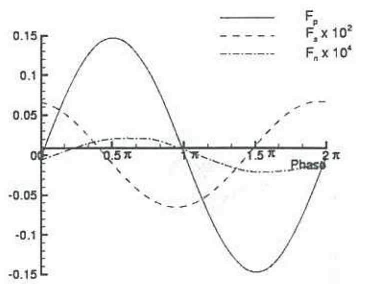

Figure 5: Viscous-stress and pressure contributions to the hydrodynamic heave force of a vertical cylinder.

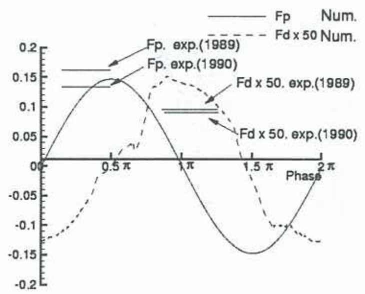

Figure 6: Damping force from pressure induced contribution($KC = 0.01$, $\beta = 3.98 \times 10^6$).

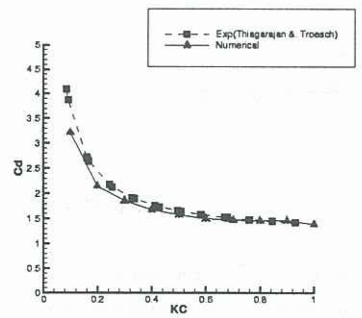

Figure 7: Drag coefficient versus KC, $\beta = 89236$.

The experiments performed by Thiagarajan and Troesch(1994) are chosen as another test case for the present numerical simulation. Figure 7 gives the variations of drag coefficient, computed by using Fourier-Averaged approach, with KC number. It is seen that the numerical solutions agree with the experimental results very well, for the range of KC number $(0.1 - 1.0)$ investigated. It is seen that the drag coefficient responses non-linearly in this range of KC numbers, and Cd decreases as KC increases.

CONCLUSIONS

Nonlinear axisymmetric viscous problem corresponding to high frequency heave oscillation of a vertical cylinder is solved successfully by using a finite-difference method in a curvilinear coordinate system. Flow structures around the oscillating cylinder rim have been simulated successfully. It is found that a mesh of (68×83) nodes should be sufficient under the conditions that $\min(\delta x/a) \leq 0.33$ and the time step $(\delta t) \leq T/5000$ for the hydrodynamic and damping forces estimation. The hydrodynamic and damping forces from the numerical solution agree with previously published experimental results very well.

REFERENCES

Sarpkaya, T., and Isaacson, M., "Mechanics of Wave Forces on Offshore Structures", van Nostrand Reinhold, New York, 1981.

Faltinsen, O. M., "Sea Load on Ships and Offshore Structures", Cambridge University Press, London, U.K., 1990.

Thiagarajan, K.P., and Troesch, A.W., "Hydrodynamic heave damping estimation and scaling for tension leg platforms", *Journal of Offshore Mechanics and Arctic Engineering,* vol. 116, no. 2, pp70-76, 1994.

Ananthakrishnan, P., "Three-Dimensional Wave-Body Interactions in a Viscous Fluid: Heave Oscillation of a Submerged Vertical Cylinder", *Proceedings of the Seventh(1997) International Offshore and Polar Engineering Conference,* vol. III, pp672-679, 1997.

Utnes, T., Moe, G., and Eidsvik, K. J., "Dynamic separation over bluff bodies Predictions based upon Reynolds equations", *Journal of Hydraulic Research,* vol. 33, No. 2, pp219-242, 1995.

Huse, E., and Utnes, T., "Springing Damping of Tension Leg Platforms", *Proceedings 26th Annual OTC,* Paper 7446, pp259-267, 1994.

Chakrabarti, S.K., and Hanna, S.Y., "High-Frequency Hydrodynamic Damping of a TLP Leg", *Proceedings 10th International Offshore Mechanics and Arctic Engineering,* Vol. 1-A, pp147-151, 1991.

Kawamura, T., and Kuwahara, K., "Computation of High Reynolds Number Flow around a Circular Cylinder with Surface Roughness", *AIAA Paper No. 84-0340,* pp1-11, 1984.