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AN INTEGRAL APPROACH TO THE TURBULENT BOUNDARY LAYER STRUCTURES

Atsushige TANAKA
1st Research Center, TRDI, JDA, Meguro-ku, Tokyo, JAPAN

Makoto YAMAMOTO
Department of Mechanical Engineering, Science University of Tokyo, Shinjuku-ku, Tokyo, JAPAN

Joel H.FERZIGER
Department of Mechanical Engineering, Stanford University, Stanford, California, USA

INTRODUCTION

Direct Simulations (DNS) has been used to investigate the kinematics of turbulent boundary layers, and to demonstrate spatio-temporal relationships between various structures in turbulent boundary layers, population and three dimensional character of the types of vortical structure as reported in Refs.1 and 2. These turbulence structures interact and change their shapes with time. Although DNS yields accurate data, the physics of the turbulence cannot be understood through simple observation of the data. This is also true of experiments. To deepen understanding of turbulence, it is necessary to analyze the data.

In the present paper, the expression of Navier-Stokes equations is changed by adding complementary variables and by using the convolutions of kernel functions and these complementary variables.

As the results, equations in integral forms corresponding to N-S equations are derived. These equations do not include the derivatives of pressure and velocity. The kernel functions are selected as Gaussian, Bessel and delta functions, which may have the interesting functional features in the analysis of turbulent boundary layer.

CONCEPT OF THIS APPROACH

Considering MAC method, generally used to obtain the solutions for incompressible Navier-Stokes equations, the solution procedure can be expressed as a block diagram depicted in Fig.1-1. It is obvious that the diagram consists of the solution processes of linear partial equations for pressure p and velocity u, and some complementary feedback processes (; complementary equations for complementary variables g and f corresponding to p and u respectively), which explicitly represent the nonlinearity of Navier-Stokes equations. Therefore, if pressure and velocity can be expressed in integral forms from the•Poisson equation for pressure and heat equations for velocity, we can obtain the simultaneous equations for pressure, velocity and complementary variables in integral forms for Navier-Stokes equations. For the purpose, we utilize the results on multi-dimensional linear partial differential equations in Ref.3 to investigate the integral equations for pressure and velocity. These linear partial differential equations are rewritten as a sum of integrals using Fourier and Laplace transforms fundamentally, and the integrals are given as convolutions of known kernel functions and complementary variables. Complementary equations contain the derivatives of p and $\tilde{u}$, which can also be rewritten as the convolutions of partial derivatives of the kernel functions and variables. Finally, equations in integral forms for N-S equations are introduced with none of partial derivatives of pressure p and velocity $\tilde{u}$.

INTEGRAL EXPRESSION FOR PRESSURE

Applying spanwise Laplace transform and the inverse transform under the spanwise periodic boundary conditions to the Poisson equation for pressure p, the 3 dimensional (below D) Poisson equation can be rewritten as a sum of infinite series of 2D Poisson equations as shown in Ref.3. One 2D Poisson equation can be rewritten in a integral form, using 2D Laplace transform in spanwise direction (z) under the convergent conditions as described in Ref.3. We named the integral as $p\alpha$. Remaining equations can be rewritten in integral forms by Fourier transform in streamwise direction (x) and Laplace one in the direction normal to a wall (y) under the same convergent conditions, respectively (see Ref,3). The sum of these integrals is referred to $p\varsigma$. The boundary position in x is determined at x=-L, where L is a large value comparing the boundary layer thickness. This is simply because it is convenient for setting the region of Laplace transform and to that of Fourier transform. Selecting the boundary position x=-L, Laplace operator u in x is replaced with u+L. Then pressure $p\alpha$ and $p\varsigma$ are given as shown in Eqn.(1-1).

$p\alpha$ involves a complex function of unknown complementary variable g as g(x-•,y+i(•+L),z) (see Fig.1-1), where i is an imaginary number and • is a integration variable. If g is given locally from several point data with polynomial approximation as Eqn.(1-2), g(x-•, y+i(•+L), z) becomes Eqn.(1-3). Eqn.(1-3) consists of terms of (y+i(•+L))^{m} , where m is an integral number specified arbitrarily. These terms mean the revolutionary changes of a complex number y+*i*(•+L) in a complex plane. And the real part of the sum of these terms expresses an irregular value, if • is selected as an arbitrary real value. As a integration variable• is an arbitrary real value in the range [0,x] in this case, Eqn.(1-3) provides fluctuating values. Hence, p_α has the possibility to create some fluctuation in x-y plane. Spanwise periodic boundary condition restricts such a fluctuation in x-y plane only. However, similar fluctuation may occur in y-z and x-z planes, because the periodic boundary conditions of these fluctuations are difficult to be defined strictly. Although DNS uses the spanwise periodic condition due to the limited storage of computers, this condition is not exact mathematically. Based on this consideration, we adopt spanwise quasi-periodic boundary condition. This means periodic conditions for pressure except for fluctuating pressure in y-z and x-z panes, and free boundary conditions for fluctuating pressure in y-z and x-z planes. Under the quasi-periodic boundary conditions, two types of fluctuating pressure p_β in x-z plane and p_γ in y-z plane should be added into Eqn.(1-1).

INTEGRAL EXPRESSION FOR VELOCITY

To obtain the integral expression for velocity, we need to obtain kernel functions on heat equation for velocity vector $\tilde{u}$. The inverse Fourier transform in x and usual inverse Laplace transforms in y, z and time t are employed. Easily, the integrals u_f , u_a , u_{la} and u_t are obtained as shown in Eqn.(1-1), where f , a_x , a_{1x} and u_{ot} means a complementary vector, the boundary velocity, the boundary acceleration and the initial velocity distribution, respectively.

INTEGRAL EXPRESSION FOR INCOMPRESSIVE NAVIER-STOKES EQUATIONS

Finally, the equations in integral forms for p and u described above are obtained as a sum of convolutions of known kernel functions and unknown variables p and $\tilde{u}$. Complementary equations for g and f contain the partial derivatives of the above p and $\tilde{u}$. But these derivatives can be rewritten as the partial derivatives of the convolutions, and become the convolutions of partial derivatives of known kernel functions and unknown variables.

Then simultaneous equations in integral forms corresponding to incompressible N-S equations can be obtained as Eqn.(2-1), which does not include the partial derivatives of variables $\tilde{u}$, p , f and g, and needs only to calculate many integrals. These integrals can be divided into three types. The first type has a kernel function represented by R_{fr} in Eqn.(1-1), which expresses the characteristics as Gaussian distribution. The second one has a kernel function for the propagation of the pressure, and it rapidly decreases with distance, expressed as a sum of Modified Bessel functions of second kind denoted as H_{fr} in Eqn. (1-1). The third one expresses as a delta function, which corresponds to a transform from a real function g(x,y,z) to a complex function as $g(x-\phi, y, z+i(\phi+L))$, $g(x-\phi, y+i(\phi+L), z)$ or $g(x, y-\phi, z+i(\phi+L))$.

Using variables $\tilde{u}$, p , f and g, boundary conditions a_x, a_{1x} and a_x , initial condition u_{ot} , and the above three types of kernel functions, the analytical flow chart of the simultaneous equations for the incompressible N-S equations is expressed as shown in Fig.2-1. Fig.2-1 shows a hierarchy of variables, measurable and physical variables in the upper part, complementary and boundary ones in the middle, and functional ones in the lower part. Arrows among these variables mean the corresponding transforms. Dashed lines correspond to the process to generate fluctuating pressure. Mixed length dash lines show smoothing process with Gaussian type of kernel functions. After the reputation of the arrows among these variables, convergent solution may be obtained.

Since kernel functions can be thought to play some roles of spatio-temporal filters on pressure p and velocity $\tilde{u}$, the estimation of these functions would be connected to the observation process for DNS or experiments. Therefore, it is useful and important to investigate the kernel functions referring to the various structures visualized in DNS results.

FLUCTUATING PRESSURE TERMS

Here the fluctuating characteristics of pressure is investigated, based on the mathematical expression of g. As shown in Figs.1-1 and 2-1, g consists of two terms. One is $\mathrm{trace}(\partial\tilde{u}/\partial x \cdot \partial\tilde{u}/\partial x)$ and the other is $(-D/Dt+1/\mathrm{Re})\nabla\tilde{u}$. The latter may be treated to make smaller effect on the fluctuation than the former because of keeping a conservation equation $\nabla\tilde{u}=0$. Expressing the former as g_t , the transform of g_t with a delta type of kernel function becomes as Eqn.(3-1). Eqn.(3-1) is expressed as a sum of • integrals of the products of velocity gradients. One of the components, for example, becomes as Eqn.(3-2), which consists of four products of exponential and sinusoidal functions. The other components can be similar as Eqn.(3-2), and the integral of g_t is expressed finally as a sum of 152 integrals. These •-integrals can be solved mathematically, and become integrals of which variables are f, a_x, a_{1x} and u_{ot} . From the similar procedures, fluctuating pressure p_β and p_γ can be expressed respectively as a sum of about one hundred of integrals. These sums would result in fluctuating phenomena. From Eqn.(1-1), it is apparent that vectors ∇p_α , ∇p_β and ∇p_γ are zero components in spanwise z, in streamwise x and in perpendicular to a wall y. Then these fluctuating pressures are thought to have their own constant coordinates, and have the possibility to relate to vortical structures of pressure in turbulent boundary layer visualized in Ref.1 as these vortical structures have their specified directions of the core. Then it seems to be important to deepen the study of the relations among these fluctuating pressures referring to DNS results. These will be our future work.

CONCLUSION

The integral expression for incompressible N-S equations were derived analytically, by reconstructing N-S equations with linear partial differential equations and complementary equations. The integral equations

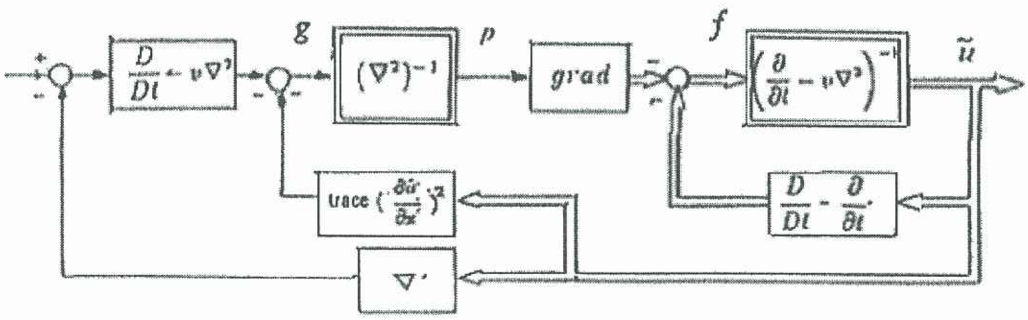

Fig.1-1. Block diagram for Navier-Stokes equations

for linear partial differential equations are obtained, employing multi-dimensional Fourier-Laplace transforms under spanwise quasi-periodic boundary condition and convergent restrictions. Using the solutions as kernel functions, complementary equations can be expressed in integral forms.

It was classified that there are three kinds of kernel functions; Gaussian, Bessel and delta functions. It was shown that these analytical expressions would give some insights to investigate turbulence structures within boundary layers.

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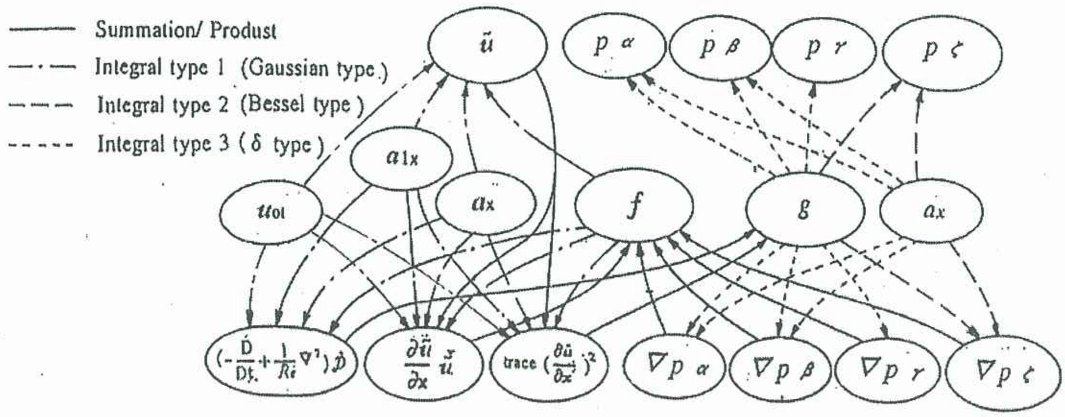

Fig.2-1. Calculating flows of Navier-Stokes equations in integral form