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TWO-DIMENSIONAL FLOW OF A VERY VISCOUS NEWTONIAN FLUID AROUND A ROTATING ELLIPTIC CYLINDER

T. TAKIGAWA, K. KATAOKA, H. EMA, T. YOSHIMURA and N. OHMURA

Department of Chemical Science and Engineering
Kobe University, Rokkodai, Nada, Kobe, JAPAN

ABSTRACT

A laminar flow around a rotating elliptic cylinder installed at the center of a circular cylindrical vessel was investigated both theoretically and experimentally for a very viscous Newtonian fluid. The two-dimensional direct numerical simulation was made in a boundary-fitted curvilinear coordinate system rotating at the same angular velocity as the elliptic cylinder. The experimental observation of time-dependent streamlines/streaklines was made by a flow-visualization technique using small resin particles and dyestuff. The theoretical streamlines have successfully described the experimental ones, in which a small three-dimensional effect appears near the both edges of the major axis. A problem of the effect of false centrifugal components resulting from the elliptic CL grid system used is not so significant owing to the very slow creeping flow. It has been found both theoretically and experimentally that the flowfield can be divided into three different zones from the aspect of viscous fluid mixing. The tracer particle or the heteroclinic orbit passing near the wall of the elliptic cylinder showed an interesting trajectory or a streamline, which gives the possibility of effective chaotic mixing if a small three-dimensional perturbation is superimposed on the streamlines.

INTRODUCTION

The purpose of this work is to obtain both theoretically and experimentally a fundamental knowledge of fluid mechanics concerning how to effectively mix very viscous fluids without wasteful energy consumption. Our interest has been focused on the effect of chaotic behavior in time-dependent creeping flows (Aref, H., 1984,Ottino, J.M. *et al.*, 1989, 1992, Lamberto, D.J. *et al.*, 1996). An elliptic cylinder was adopted as a simplified impeller model and the ellipticity, i.e. the ratio of the shorter to longer diameter was varied as a parameter in the numerical simulation. A direct numerical simulation was firstly made to obtain two-dimensional time-dependent velocities and streamlines as a first approximation. A flow-visualization technique was utilized to obtain trajectories of tracer particles as the time-dependent streamlines and streaklines of tracer dyestuff as the mixing patterns.

THEORETICAL APPROACH

For simplicity, a very viscous Newtonian fluid was assumed for two-dimensional laminar flow around a rotating elliptic cylinder. Figure 1 shows the flowfield to be solved and the coordinate system. In a sense similar to paddle-type agitators, the Reynolds number for this flow system can be defined as $Re = a^2 \omega \rho / \mu$

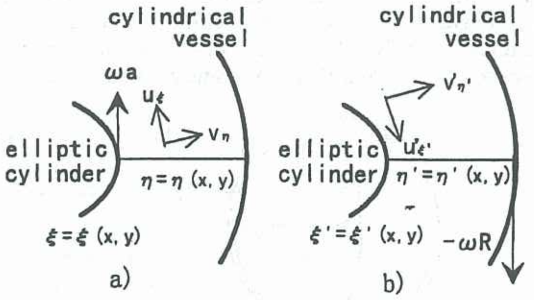

Figure 1 : Geometry of flowfield and the coordinate system. a) stationary coordinate system, b) coordinate system rotating at the same angular velocity as the elliptic cylinder.

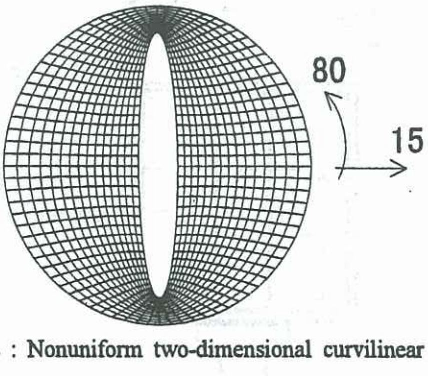

Figure 2 : Nonuniform two-dimensional curvilinear grid system.

A two-dimensional, time-dependent Navier-Stokes equation was numerically solved in an elliptic boundary-fitted curvilinear coordinate system. In order to remove the difficulty resulting from the moving boundary condition given on the wall of the rotating elliptic cylinder, the governing equation was given in the coordinate system rotating at the same angular velocity as the elliptic cylinder:

$$\frac{\partial V_1}{\partial t}+((V_1-V_2)\cdot\nabla)(V_1-V_2)$$
$$=-\nabla p+\mathrm{Re}^{-1}\nabla^2(V_1-V_2)$$

where V_1 and V_2 are, respectively, the rotating velocity of the coordinate system and the relative velocity. No-slip boundary condition was given on both the elliptic cylinder and the cylindrical vessel. The initial condition that the elliptic cylinder is accelerated stepwisely from rest to a specified rotation was applied.

Nonuniform curvilinear grid shown in Figure 2 was adopted with the aid of the Poisson equation.

The first-order time integration scheme was used with central finite difference method for spatial discretization. The Poisson equation for pressure was solved by means of the point SOR method based on the concept of the MAC method (Harlow, F.H. and Welch, J.H., 1965). The numbers of the grid points corresponding to the coordinate system (η', ξ') were 15 and 80, respectively. After a steady-state velocity distribution in the coordinate system (η', ξ') was obtained, the streamlines were calculated. In addition to the angular velocity, ω, of the elliptic cylinder, the shorter diameter, b, is changed as the geometrical parameter with the longer diameter, a, kept constant.

It can be considered that this numerical simulation might have a problem of false centrifugal effect (Takizawa, A., *et al.*, 1992) coming from the deviation from the cylindrical coordinate system. Since this effect is too difficult to estimate directly, it will be discussed in comparison between the theoretical and experimental streamlines.

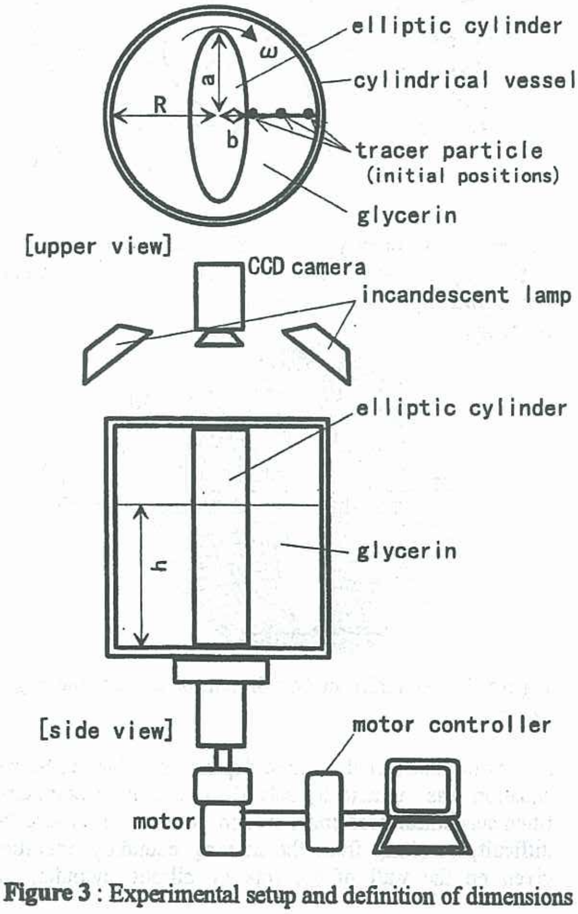

Figure 3 : Experimental setup and definition of dimensions

EXPERIMENTAL APPROACH

Figure 3 shows an experimental setup for observing time-dependent streamlines/streaklines. The dimensions defined within the figure were given by the inside radius of the cylindrical vessel R = 0.090 m, the longer diameter (major axis) a = 0.075 m, the shorter diameter (minor axis) b = 0.0115 m, the liquid height h = 0.09 m. The test elliptic cylinder has an ellipticity of α = b/a = 0.153.

The elliptic cylinder was put at the center of the cylindrical vessel so that those centerlines could be coincident with each other. The upper view picture of Figure 3 also shows the initial positions of three tracer particles for the flow-visualization experiment. These tracer particles are placed on the same horizontal plane along the extension of the minor axis at three different positions: (1) near the wall of the elliptic cylinder, (2) near the wall of the cylindrical vessel, and (3) midway from the elliptic cylinder to the cylindrical vessel. The trajectories of the three tracer particles observed by a CCD camera were analyzed by an image processor of computer.

For the first phase of the investigation, glycerin (viscosity μ = 0.7 Pa s, density ρ = 1248 kg/m^3) was used as a very viscous Newtonian fluid. Very small droplets of the glycerin colored by dyestuff (Thymol Blue) were aligned like a straight extension line of the minor axis of the elliptic cylinder as the initial line for observation of streaklines.

RESULTS AND DISCUSSION

Figures 4(a) and 5(a) show instantaneous theoretical streamlines for two different ellipticities in the stationary coordinate system (i.e. observed from the circular cylindrical vessel at rest). A comparison between these figures suggests that the low-velocity region near the wall of the cylindrical vessel is expanded as α is decreased. The symmetry of the streamline patterns with respect to the major axis is broken when α goes below a certain critical value. It can be conjectured that an asymmetric streamline pattern appearing near both edges of the major axis has a possibility of effective chaotic mixing if the direction of rotation is alternated. The time-dependent streamlines can be crossed each other in phase with the alternating motion. The chaotic mixing means the alternating effect due to elongation and folding.

Figures 4(b) and 5(b) show stationary theoretical streamlines in the coordinate system rotating at the same angular velocity as the elliptic cylinder. There appears a large vortical flow zone midway from the edge of the minor axis to the wall of the cylindrical vessel. This zone is not well mixed and expanded as ω is decreased.

Each theoretical streamline passing near the edges of the major axis of the elliptic cylinder suggests a heteroclinic orbit having a hyperbolic fixed point at each edge of the major axis and an elliptic fixed point at the center of each vortical flow zone. It can be considered that effective chaotic mixing occurs around the hyperbolic points of the heteroclinic orbit if a small three-dimensional perturbation is superimposed on the orbit.

Figures 6(a) and 7(a) show instantaneous theoretical streamlines expressed in the stationary coordinate system for two different angular velocities. Figures 6(b) and 7(b) show the corresponding streamlines expressed in the rotating coordinate system. The symmetry of the streamline patterns with respect to the major axis is broken when the rotation speed of the elliptic cylinder is increased.

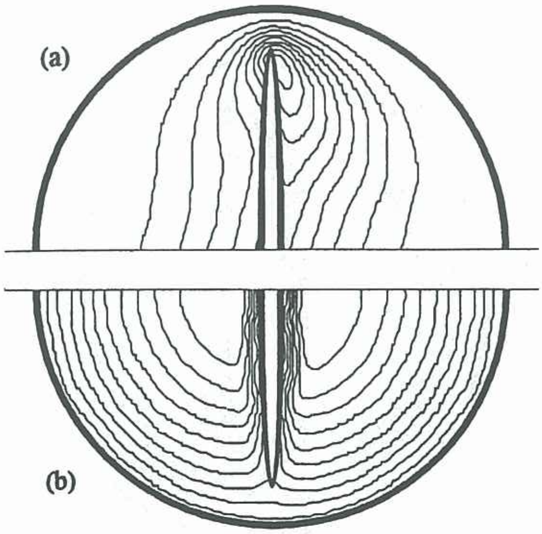

Figure 4 : Theoretical streamlines obtained for $\omega = 0.382$ 1/s, $\alpha = 0.067$, (a) in the stationary coordinate system, (b) in the rotating coordinate system.

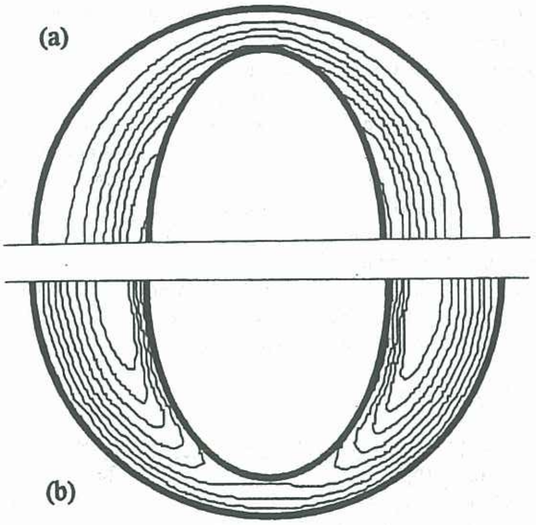

Figure 5 : Theoretical streamlines obtained for $\omega = 0.382$ 1/s, $\alpha = 0.667$, (a) in the stationary coordinate system, (b) in the rotating coordinate system.

Figures 8(a) and (b) show a comparison of theoretical streamlines obtained in the rotating coordinate system with the corresponding experimental streaklines under the same condition. The streaklines imply an instantaneous distribution of colored tracer droplets whereas each streamline indicates a trajectroty of tracer particle of interest. Those streamlines have successfully described the tendency of the streaklines. Despite the first approximation, the numerical simulation succeeded in predicting the shape and size of the vortical flow zone. It can be noticed from the experimental streaklines that there appears a folding motion around the two hyperbolic fixed points. This implies a three-dimensional chaotic behavior that cannot be predicted by the two-dimensional numerical simulation. This can be an important fundamental knowledge useful for the next phase of this investigation.

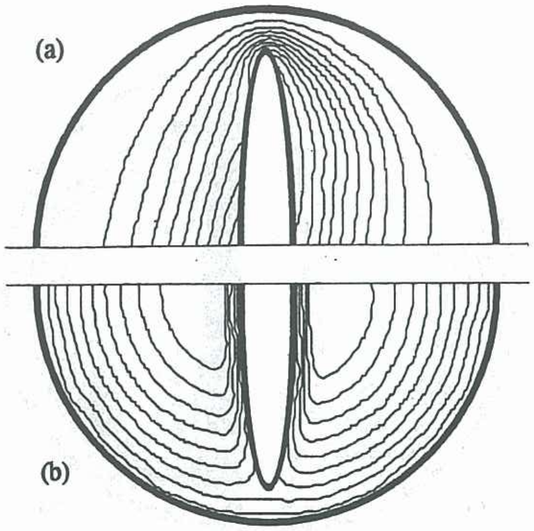

Figure 6 : Theoretical streamlines obtained for $\omega = 0.382$ 1/s, $\alpha = 0.153$, (a) in the stationary coordinate system, (b) in the rotating coordinate system.

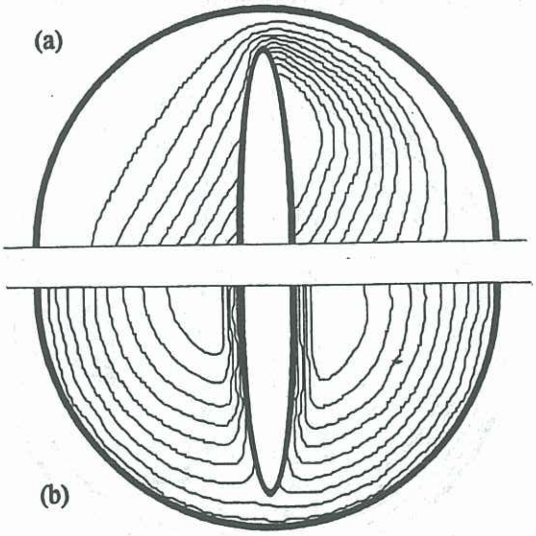

Figure 7 : Theoretical streamlines obtained for $\omega = 1.528$ 1/s, $\alpha = 0.153$, (a) in the stationary coordinate system, (b) in the rotating coordinate system.

Figure 9(a) and (b) show a comparison of the same theoretical streamlines with experimental trajectories of the tracer particles obtained by flow-visualization. As can be seen from the experimental streamlines, there should exist three distinct zones with different mixing characteristics.

The tracer particle passing near and along the wall of the elliptic cylinder shows a complicated trajectory, which can be interpreted as the three-dimensional chaotic behavior coming from the unavoidable experimental perturbation /disturbance (Solomon, T.H. and Gollub, J.P.,1988). This suggests a method for effective chaotic mixing. It can be conjectured that the false centrifugal components (Takizawa, A., *et al.*, 1992) coming from the deviation from the cylindrical coordinate system does not cause a significant influence on the result of the numerical simulation.

As shown schematically in Figure 10, the theoretical streamlines indicate a heteroclinic orbit having a hyperbolic

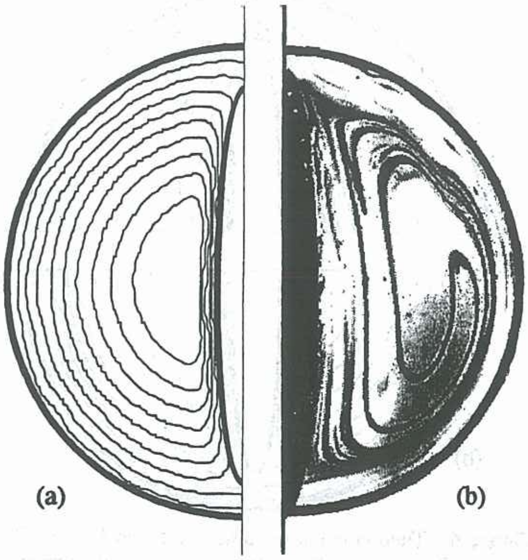

Figure 8 : Comparison of theoretical streamlines with experimental streaklines in the rotating coordinate system: $\omega = 0.382$ 1/s, $\alpha = 0.153$. (a) theory, (b) experiment.

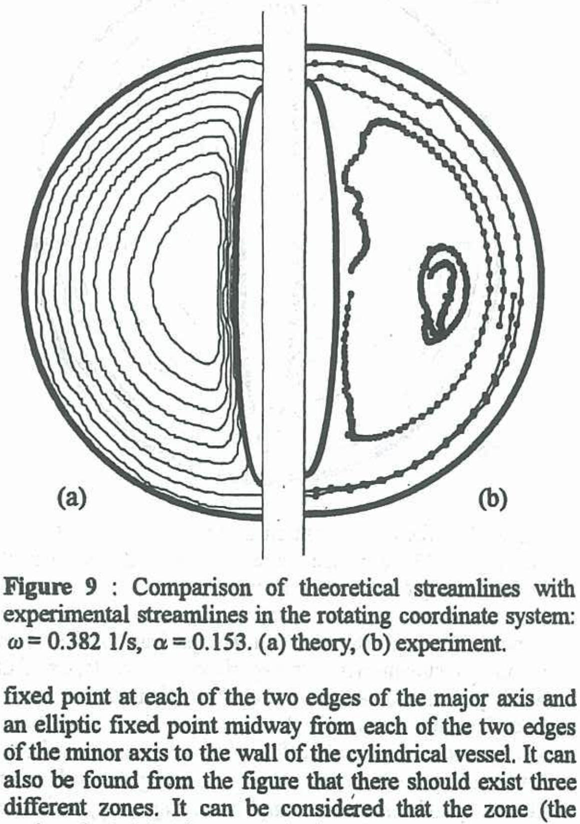

Figure 9 : Comparison of theoretical streamlines with experimental streamlines in the rotating coordinate system: $\omega = 0.382$ 1/s, $\alpha = 0.153$. (a) theory, (b) experiment.

fixed point at each of the two edges of the major axis and an elliptic fixed point midway from each of the two edges of the minor axis to the wall of the cylindrical vessel. It can also be found from the figure that there should exist three different zones. It can be considered that the zone (the region shaded in Figure 10) having the heteroclinic orbit is more sensitive to a superimposed perturbation and more advantageous to chaotic mixing than the remaining two zones.

CONCLUSION

1) The two-dimensional direct numerical simulation has successfully described the experimental results, in which a small three-dimensional effect appears near the both edges of the major axis owing to an unavoidable small experimental disturbance. This three-dimensional behavior suggests a method for effective chaotic mixing.

2) The existence of three distinct zones has been found both theoretically and experimentally. It can be expected that the zone which has a tracer particle or a heteroclinic orbit passing near and along the wall of the elliptic cylinder may become unstable if a small three-dimensional perturbation is added and that the instability may lead to three-dimensional chaotic mixing.

3) A problem of the effect of false centrifugal components resulting from the elliptic CL grid system used is not so significant in very slow creeping flows of this kind.

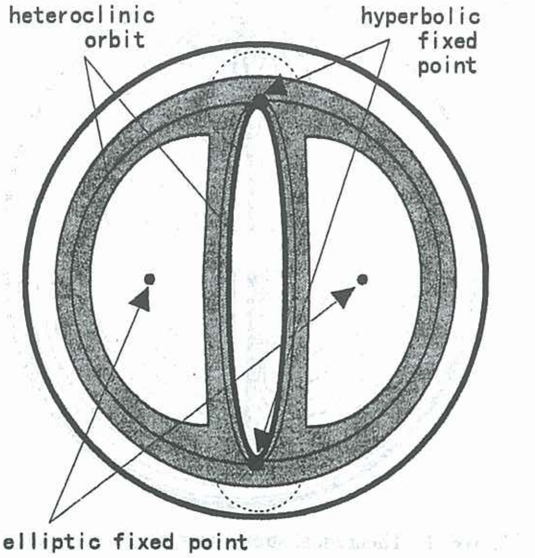

Figure 10 : Schematic picture of theoretical streamlines indicating a heteroclinic orbit.

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