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FLOW MODIFICATIONS USING DISTRIBUTED SURFACE SUCTION

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ABSTRACT

Flow modifications induced by distributed surface suction with a fixed/moving pattern have been considered. Very small suction levels with a properly selected pattern can induce very large changes in the flow. The mechanics of flow response can be explained in some generality using linear theory. The available results permit optimal selection of suction that maximizes flow response.

INTRODUCTION

The ability to manipulate a flow field to effect a desired change is of immense technological importance. As defined by Flatt (1961), the term boundary-layer control includes any mechanism or process through which the boundary layer of a fluid flow is caused to behave differently than it would normally were the flow developing naturally along a smooth straight surface. The control device could be passive, requiring no auxiliary power, or active, requiring energy expenditure. The flow could be manipulated to achieve transition delay, separation postponement, lift enhancement, drag reduction, turbulence augmentation, or noise suppression [Gad-al-Hak, 1998].

Passive control strategies involve correct shaping of the moving objects (pressure gradient tailoring, riblets, large eddy breakup LEBU devices, etc.). Active strategies rely either on surface suction or on adjustment of contours of moving bodies. In the latter category, microelectromechanical systems (MEMS) have recently become the focus of active research [Ho and Tai, 1998]. The goal of the present work is to explore the range of flow modifications that can be achieved using surface suction and to determine the optimal form of suction distribution. The optimal distribution is defined here as the distribution that produces largest changes in the flow using smallest possible suction level.

Present work is focused on the channel (Poiseuille) flow which is modified by imposing a distributed surface suction of a very small amplitude. The analysis is carried out in the spectral space, following Floryan (1997), and thus it can deal with an arbitrary suction distribution using Fourier decomposition principles. Both stationary as well as time dependent (in the form of travelling waves) suction distributions are considered.

PROBLEM FORMULATION

Reference flow

Consider plane Poiseuille flow confined between flat rigid walls at $y=\pm 1$ and extending to infinity in the $\hat{x}$-direction. The following velocity and pressure fields describe the fluid motion

$$\overline{V}_0(\hat{x},y)=[u_0(y),0]=[1-y^2,0]\ ,\qquad p_0(\hat{x},y)=-2\hat{x}/Re \tag{1}$$

where the fluid is directed towards the positive $\hat{x}$-axis and the Reynolds number Re is based on the half-channel height and the maximum $\hat{x}$-velocity. This flow is driven by a constant negative pressure gradient.

Flow modified by wall suction

At the upper and lower wall apply suction in the form

$$u(\hat{x},-1,t)=0\ ,\quad v(\hat{x},-1,t)=v_L(\hat{x}-c_{BS}t)\ ,$$
$$u(\hat{x},1,t)=0\ ,\quad v(\hat{x},1,t)=v_U(\hat{x}-c_{BS}t)\ , \tag{2}$$

i.e., the suction is normal to the wall and its spatial pattern moves with a constant speed c_{bs} with respect to the channel. For convenience, we shall introduce a Galilean frame of reference moving with velocity c_{bc} in the $\hat{x}$-direction in which the blowing/suction pattern becomes steady. We shall restrict this analysis to v_L and v_U periodic in $\hat{x}$ with the wavelength $\lambda_x=2\pi/\alpha$, which can be expressed in terms of Fourier series in the form

$$v_L(x)=\sum_{n=-\infty}^{n=\infty}(V_n)_L e^{in\alpha x}\ ,$$
$$v_U(x)=\sum_{n=-\infty}^{n=\infty}(V_n)_U e^{in\alpha x}, \tag{3}$$

where $(V_n)_L=(V_n)_L^*$, $(V_n)_U=(V_n)_U^*$, and $x=\hat{x}-c_{BS}t$ defines the Galilean transformation. In addition we assume that $(V_0)_L=(V_0)_U=0$, i.e. the wall suction carries no net mass flux. The flow in the channel can be represented as

$$\overline{V}(x,y,t)=\overline{V}_0(x,y)+\overline{V}_1(x,y,t)=$$
$$=[u_0(y),0]+[u_1(x,y,t),v_1(x,y,t)] \tag{4}$$
$$p(x,y,t)=p_0(x,y)+p_1(x,y,t),$$

where $\overline{V}_1$ and p_1 are the velocity and pressure modifications due to the presence of the wall suction. The flow field can be described in terms of streamfunction Ψ_1 defined as $u_1=\partial_y\Psi_1$ and $v_1=-\partial_x\Psi_1$. The field equation for Ψ_1 assumes the following form

$$\left((u_0-c_{BS})\partial_x+\partial_y\Psi\partial_x-\partial_x\Psi\partial_y\right)\Delta\Psi-$$
$$-D^2u_0\partial_x\Psi=Re^{-1}\Delta^2\Psi, \tag{5}$$

where D=d/dy and Δ is the Laplace operator. Equation (5) is subject to boundary conditions

$$\Psi_y(x,-1)=0 \quad , \quad \Psi_x(x,-1)=v_L(x) \ ,$$
$$\Psi_y(x,1)=0 \quad , \quad \Psi_x(x,1)=v_U(x) \ . \tag{6}$$

Since v_L and v_U are periodic, the streamfunction Ψ can be represented as

$$\Psi(x,y)=\sum_{n=-\infty}^{n=+\infty}\Phi_n(y)e^{in\alpha x} \tag{7}$$

where $\Phi_n=\Phi^*_{-n}$. The functions Φ_n, n≥0, in (7) are governed by a nonlinear system of ordinary differential equations in the form [Floryan, 1997]

$$\left[D_n^2-in\alpha Re[(u_0-c_{BS})D_n-D^2u_0]\right]\Phi_n- \tag{8}$$
$$i\alpha Re\sum_{k=-\infty}^{k=+\infty}[kD\Phi_{n-k}D_k\Phi_k-(n-k)\Phi_{n-k}D_kD\Phi_k]=0,$$

where $D_n=D^2-n^2\alpha^2$. The equation (8) has been obtained by substituting (7) into (5) and separating Fourier components. The corresponding boundary conditions have the form

$$D\Phi_n(-1)=D\Phi_n(1)=0 \quad , \quad n\geq 0,$$
$$\Phi_n(-1)=\frac{i(V_n)_L}{n\alpha} \ , \Phi_n(1)=\frac{i(V_n)_U}{n\alpha} \ , \quad n\geq 1. \tag{9}$$

The system is closed by specifying two additional conditions. The first condition is selected, without loss of generality, by assuming that $\Phi_0(-1)=0$. The second condition can be selected by either specifying mass flux or pressure gradient [Floryan, 1997]. Results presented in this paper have been obtained for the specified mass flux, which was assumed to be the same as in the Poiseuille flow without wall suction, i.e., 4/3. The corresponding boundary condition has the form $\Phi_0(1)=0$. The resulting problem has been solved by truncating (8) at a finite number of terms, expressing Φ_n in terms of expansions based on Chebyshev polynomials and constructing algebraic nonlinear system by imposing orthogonality condition on residua of (8), i.e., using the Chebyshev-tau technique. Details of the numerical solution are omitted from this presentation.

MECHANICS OF FLOW RESPONSE

The mechanics of flow response to wall suction can be explained by considering v_L and v_U in the form of a single Fourier harmonic, i.e.,

$$v_L(x)=\tfrac{1}{2}S_Le^{i\alpha x}+CC \ , v_U(x)=\tfrac{1}{2}S_Ue^{i\alpha x}+CC \ , \tag{10}$$

where CC denotes complex conjugate, and neglecting nonlinear interactions in (8). It can be shown that, up to a certain level of suction amplitudes S_L and S_U, the nonlinear interactions affect only the magnitude of the flow response and do not change its qualitative character. The linear model problem has the following form

$$(A-c_{BS}B)\Phi_1=0 \ , \ D\Phi_1(\pm1)=0,$$
$$\Phi_1(-1)=\tfrac{i}{2\alpha}S_L \ , \Phi_1(-1)=\tfrac{i}{2\alpha}S_L \ , \tag{11}$$

where

$$A=(D^2-\alpha^2)^2-i\alpha Re\left[u_0(D^2-\alpha^2)-D^2u_0\right],$$
$$B=-i\alpha Re(D^2-\alpha^2) \ .$$

It is convenient for analysis purposes to convert the above inhomogeneous boundary value problem into an equivalent problem with an inhomogeneous differential equation and homogeneous boundary conditions. A substitution in the form $\Phi_1=\phi+P$ leads to the following problem

$$(A-c_{BS}B)\phi=r \ , \ \phi(\pm1)=D\phi(\pm1)=0, \tag{12}$$

where $r\equiv -(A-c_{BS}B)\,P$ and P is a conveniently selected, smooth function that satisfies the inhomogeneous boundary conditions in (11).

The main objective of this analysis is determination of c_{BS} that would maximize the flow response for a given α. The magnitude of the flow response is determined by properties of the operator $K_c^{-1}:=(A-c_{BS}B)^{-1}$ and properties of the forcing function r. In general, $\|\phi\|\leq\|K_c^{-1}\|\|r\|$. It is known that $\|K_c^{-1}\|=\sigma_0$ where σ_0 is the leading singular value of the operator K_c^{-1}. When K_c^{-1} is normal then $\sigma_0=|\lambda_0|$, where λ_0 is the largest (in absolute value) eigenvalue of K_c^{-1}; otherwise $\sigma_0\geq|\lambda_0|$. It is thus possible to determine the largest possible response of the flow to any given external forcing as well as the actual response resulting from application of wall suction. The methodology used for evaluation of the relevant norms is discussed below.

The right-hand side function r represents an element of the Hilbert space $L^2_\omega[-1,1]$, i.e. the space of square integrable functions with the weight $\omega(y)=(1-y^2)^{-1/2}$ in the domain [-1,1]. The solution of (12) is sought in the linear subspace *D* of functions satisfying homogeneous boundary conditions in (12), and with fourth derivative in $L^2_\omega[-1,1]$. Two orthonormal sets of basis functions are introduced, one in $L^2_\omega[-1,1]$ and one in *D*. The first set consists of the normalized Chebyshev polynomials $t_k(y)=\sqrt{\frac{\delta}{\pi}}T_K(y)$, where δ=2 for k=0 and δ=1 for k>0. The second set $\{q_k\}$ is obtained through orthonormalization of the following polynomials

$$Q_k=T_{k+4}-\tfrac{2(k+2)}{k+1}T_{k+2}+\tfrac{k+3}{k+1}T_k$$

where the orthonormalization is carried out with respect to the inner product $\langle f,g\rangle_{2,\omega}=\int_{-1}^{1}f(y)g(y)\omega(y)dy$. One can define the matrix representation of the operator K_c as

$$(\mathbf{K}_c)_{ij}=\langle K_cq_j,t_i\rangle_{2,\omega} \ .$$

The forcing function r and the solution φ can be approximated by the following expansions

$$r\approx\sum_{k=0}^{N}\langle r,t_k\rangle_{2,\omega}t_k\equiv\sum_{k=0}^{N}R_kt_k \ , \ \phi\approx\sum_{k=0}^{N}F_kq_k \ .$$

Substitution of the above expressions into (12) and projection on t_k for k=0,..,N yields the algebraic linear problem $\mathbf{K_cF=R}$. The inverse matrix $\mathbf{K}_c^{-1}$ is a finite dimensional approximation of the inverse operator K_c^{-1}.

It can be shown that the norm of K_c^{-1} for certain pairs (α,c_{BS}) can exceed significantly its value for (α,0). For example, for Re=5000 and α=1 the numerical calculations show that the norm of K_c^{-1} for $c_{BS}\approx0.27$ is two orders of magnitude bigger than for $c_{BS}=0$. The reason for this amplification effect can be explained physically by noting the existence of the Tollmien-Schlichting (TS) wave with the wave number around unity and the wave speed close to 0.27. For Reynolds numbers lower but close to the critical value $Re_{cr} \cong 5772.22$, this TS wave is weakly damped. The suction wave applied at the wall is in near resonant conditions with this TS wave, the operator K_c is "nearly" singular and the norm of its inverse assumes large values. This physical argument can be stated with full mathematical rigor in terms of the relation between the distance of the regular value of the linear operator from its spectrum, and the norm of its inverse.

The details of the mechanism of the amplification near the resonance as well as in other regions of the (α, c_{BS}) plane can be conveniently analyzed using the Singular Value Decomposition (SVD) technique applied to $\mathbf{K}_c^{-1}$. The singular values of $\mathbf{K}_c^{-1}$ can be arranged into a decreasing order $\sigma_0 > .. > \sigma_N$, with σ_0 being equal to the norm $\| \mathbf{K}_c^{-1} \|_2$ which, in turn, approximates the norm of K_c^{-1}. SVD gives also two sets of orthonormal (in $\mathbf{R}^{N+1}$) vectors $\{\mathbf{V}_k\}$ and $\{\mathbf{U}_k\}$, such that $\mathbf{K}_c^{-1}\mathbf{V}_k = \sigma_k \mathbf{U}_k$, k=0,..,N. Each pair of the singular vectors $\{\mathbf{V}_k, \mathbf{U}_k\}$ determines the pair of the singular functions of the operator K_c^{-1} according to the formulas

$$\nu_k(y) \approx \sum_{j=0}^{N} (\mathbf{V}_k)_j t_j(y) \ , \ \upsilon_k(y) \approx \sum_{j=0}^{N} (\mathbf{U}_k)_j q_j(y) \ ,$$

where $K_c^{-1}\nu_k = \upsilon_k$. The solution of the boundary problem (12) can be expressed in terms of the singular values and the singular functions as follows

$$\phi(y) \approx \sum_{k=0}^{N} \sigma_k \langle r, \nu_k \rangle_{2,\omega} \upsilon_k(y) \ , \qquad (13)$$

which is equivalent to

$$F_j \approx \sum_{k=0}^{N} \sigma_k \langle \mathbf{R}, \mathbf{V}_k \rangle_{\mathbf{R}^{N+1}} (\mathbf{U}_k)_j \ .$$

Since the operator K_c^{-1} involves only an even order of differentiation and since the Poiseuille velocity profile u_0 is symmetric, the singular functions $\{\nu_k, \upsilon_k\}$ have the same type of symmetry, i.e., they are both either symmetric or asymmetric. In all cases studied, the pair $\{\nu_0, \upsilon_0\}$ was found to be symmetric while the pair $\{\nu_1, \upsilon_1\}$ was found to be asymmetric. Because of linearity of (12), all features of the solution can be described by considering symmetric $S_L = S_U = S$ and asymmetric $S_L = -S_U = S$ forms of the wall suction separately. The reader may note that the type of symmetry of the forcing function r is the same as the symmetry of wall suction. The numerical calculations show that the first term of the solution strongly dominates the rest of terms in both cases of symmetry, i.e.,

$$\phi \approx \sigma_0 \langle r, \nu_0 \rangle_{2,\omega} \upsilon_0 \ \text{(symmetric suction)},$$
$$\phi \approx \sigma_1 \langle r, \nu_1 \rangle_{2,\omega} \upsilon_1 \ \text{(asymmetric suction)}. \qquad (14)$$

The omitted terms form less than 1% of the complete solution almost in the entire region of (α, c_{BS}) plane considered in this study. This finding allows for a relatively simple analysis of the flow response.

The magnitude of the flow response under various suction conditions is well illustrated by relating it to flow response for $c_{BS}=0$. The relative amplification A is defined as

$$A = \frac{\|\phi(c_{BS})\|_{2,\omega}}{\|\phi(0)\|_{2,\omega}} \ .$$

It can be shown using (14) that

$$A \cong \frac{\sigma_L(c_{BS})}{\sigma_L(0)} \cdot \frac{\|r(c_{BS})\|_{2,\omega}}{\|r(0)\|_{2,\omega}} \cdot \frac{|p_L(c_{BS})|}{|p_L(0)|} \ , \qquad (15)$$

where the lower index L indicates the leading term (L=0 for the symmetric case and L=1 for the asymmetric case) and $p_L = \langle r, \nu_L \rangle_{2,\omega} / \|r\|_{2,\omega}$.

Since the full solution to (12) consists of the sum $\Phi_1 = \phi + P$, distributions of $A(\alpha, c_{BS})$ for $\|\phi\|_{2,\omega}$ and for $\|\Phi_1\|_{2,\omega}$ have similar patterns only when ϕ dominates this sum. In the antisymmetric case, the absolute value of $\|\phi\|_{2,\omega}$ is small in comparison with $\|P\|_{2,\omega}$ and the amplification effect is almost entirely masked by P. As a result, the magnitude of the solution is practically insensitive to variations of c_{BS}. The situation is quite different in the symmetric case, as illustrated in the case of flow with Re=5000. The resonance with the principal TS wave causes a dramatic increase in the norm of K_c^{-1}. The ratio $\sigma_1(c_{BS})/\sigma_1(0)$ in (15) calculated near the resonance peak is about 150. The other two factors in (15) are close to unity and their product is about 1.07, giving the total amplification A of about 160 (see Fig.1). The second region of high amplification (although with much smaller A than in the vicinity of the resonance) has the form of a narrow strip centered approximately around $\alpha \approx 1.7$ with c_{BS} ranging from 0.3 up to 1.0 and giving the extreme values of A about 50 in the bottom part of the strip. In contrast to the resonance interaction described earlier, all three factors in (15) are significant. Near the local extremum the first factor assumes values in the range 6÷7 and the second one assumes values about 1.4÷1.5. The most spectacular effect occurs due to the presence of the third factor ("alignment" of r and ν_0), which reaches values of about 5 at the local extremum of A and 10 in its immediate neighborhood. This factor as well as the norm of the forcing function (which increases monotonically with c_{BS}) extend the region of large amplification to larger values of c_{BS}, and eventually produce a foot-shaped area shown in Fig.1. Examination of the Orr-Sommerfeld spectra reveals that existence of this part of the amplification region can be explained in terms of weak resonant interactions with a group of symmetric TS waves. Since all these waves have damping much stronger than the principal TS wave, the total amplification effect is weaker. The amplified solution ϕ is comparable in norm with the function P and thus the upper part of the amplification region of Φ_1 is partially masked by P. Since the function P is a constant function $P = iS/(2\alpha)$ and since the component u_1 of the velocity field is determined only by the derivative $D\phi$, the masking affect of P for $D\Phi_1$ is eliminated and the map of the amplification A for $\|D\Phi_1\|_{2,\omega}$ (Fig.2) reproduces the characteristic foot-shaped pattern of the isolines.

The singular function υ_0, calculated for (α, c_{BS}) near the main resonance peak, has the shape very similar to the shape of the principal TS wave (Fig.3). This means that, in resonant conditions, the modification of the velocity field has the form of the sum of the (essentially) TS wave and P. Since P is small compared to ϕ, the streamfunction modification is everywhere nearly similar to the TS wave, except very close to the wall where P dominates.

This observation is in full agreement with Laurien and Kleiser (1986). One might expect, therefore, that the function r generating the solution of (12) with the highest possible magnitude would be very close, if not identical, to the TS wave. This is, however, not true. The "optimal" right-hand side function is the singular function v_0, which is quite different from the TS wave (Fig.3). One should note that since the absolute value of the projection $\langle\zeta_{TS},v_1\rangle_{2,\omega}$ is about 0.79, the use of the TS wave as the forcing function r gives nevertheless about 79% of the possible maximum amplification.

The pattern of the amplification A changes with decreasing Reynolds number. Since damping of the principal TS wave increases, the magnitude of the main resonance is significantly reduced and this part of the foot-shaped amplification area disappears for low enough Re (see Fig.4). The large amplification found away from the main resonance (see Fig.1) changes differently. The variation of the amplification for the function r and the "alignment" $p_1=\langle r,v_1\rangle_{2,\omega}/\|r\|_{2,\omega}$ with Re is very weak. While the norm of the inverse K_c^{-1} decreases with decreasing Re, this reduction is much less dramatic than in the vicinity of the main resonance. The combined effect preserves the fairly high amplification at the heel of the foot-shaped high amplifications area, as shown in Fig.4 for Re=1000. One should note that TS waves other then the principal one are responsible for this amplification.

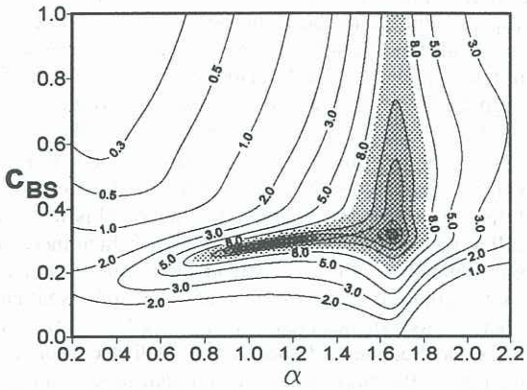

Figure 1. Distribution of amplification A for ϕ as a function of α and c_{BS} for Re=5000 and symmetric suction $S_U=S_L=1$.

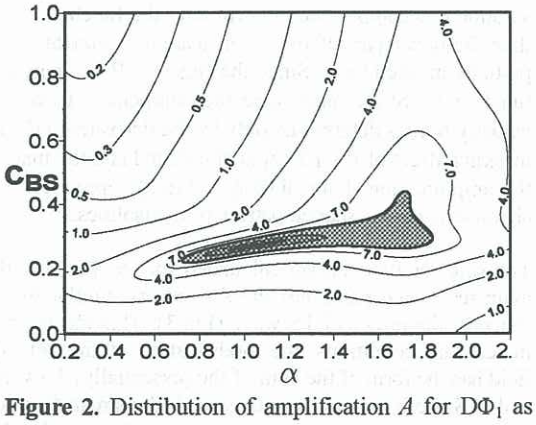

Figure 2. Distribution of amplification A for $D\Phi_1$ as a function of α and c_{BS}. Other conditions as in Fig.2.

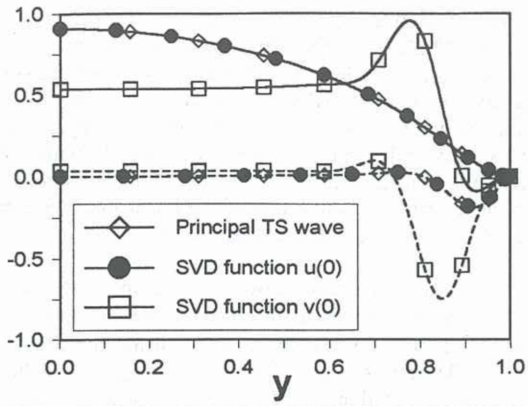

Figure 3. Distribution of amplitude of principal TS wave and SVD functions v_0 and υ_0. Dash lines represent imaginary parts. Imaginary parts of the TS wave and υ_0 are multiplied by 10 for presentation purposes.

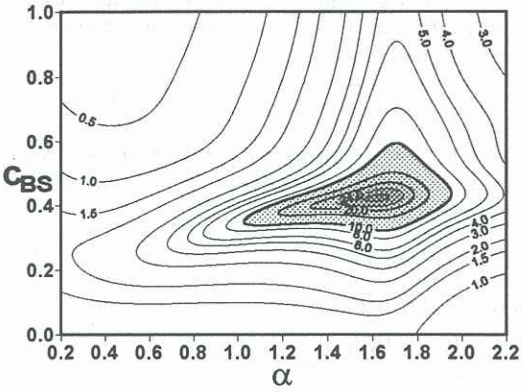

Figure 4. Distribution of amplification A for $D\Phi_1$ for Re=1000 and symmetric suction $S_U=S_L=1$.

CONCLUSION

Flow modifications induced by surface suction in the form of travelling waves have been considered. It has been shown that small suction can induce flow changes $O(10^2)$ larger than the suction amplitude by properly tuning suction waves. The form of the external forcing that maximizes the flow response has also been determined.

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