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MIXING, STRETCHING AND CHAOS IN WAVY TAYLOR VORTEX FLOW

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ABSTRACT

Numerical and experimental investigation of the fluid flow, mixing and stretching in wavy Taylor vortex flow is presented which shows that the effective (chaotic) diffusion coefficient is a function of wave state. This result suggests that a universal relationship between dispersion and Reynolds number cannot be found in this regime of cylindrical Taylor-Couette flow. Fluid retention in the cores of the wavy vortices is also predicted, and fluid trapped in vortex cores is only poorly mixed within the core and plays no role in global mixing except via molecular diffusion. Experimental validation of the existence of vortex core regions in wavy vortex flow is presented for the first time. Mean fluid stretching is also calculated, although a quantitative relationship between stretching and effective axial diffusion is not apparent from the results.

INTRODUCTION

Cylindrical Taylor-Couette flow is a flow of fundamental fluid dynamical importance and has been extensively studied since the work of Taylor (1923). Taylor-Couette vessels have been used both as reaction vessels and to quantify the relationship between shear and aggregation by coagulation and flocculation (e.g. Pudjiono and Tavare (1993); Farrow and Swift, (1996)). Pudjiono et al. (1992) describe other applications for cylindrical Taylor-Couette flow, including viscometry, cooling of rotating electrical machinery, dynamic filtration and classification and catalytic chemical reactors.

When only the inner cylinder rotates, a critical Reynolds number, Re_C, exists at which cylindrical Couette flow becomes unstable to axial perturbations. In the resulting axisymmetric Taylor vortex flow, fluid elements are constrained to lie on invariant tori within vortices. Apart from molecular diffusion, each vortex remains disconnected from its neighbours and because of this, the vortices are not efficient mixers.

As the Reynolds number is increased beyond Re_C, a point is reached at which Taylor vortex flow becomes unstable to azimuthal perturbations. The resulting 'Wavy Vortex Flow' (WVF) has been shown to be a far better mixer than Taylor vortex flow due to the presence of Lagrangian chaos in fluid particle trajectories, (Rudman 1998). Coles (1965) observed that for a given value of Re in the wavy vortex flow regime, many different wave states can exist. The wave state has been shown to have a significant effect on mixing (Rudman 1998) and this relationship is examined here using a combined numerical and experimental approach.

EXPERIMENTAL METHOD

A schematic of the Taylor-Couette apparatus used the experiments here is shown in Figure 1. It consists of an anodised aluminium inner cylinder with a radius 5.95 cm and a perspex outer cylinder with a radius of 6.95 cm, giving a ratio of inner to outer radius radius of $\eta = 0.856$. The working height is 37.4 cm and the end walls are fixed to the stationary outer cylinder. The inner cylinder is driven by a reduction motor via a V-belt and cylinder rotation is measured from the motor shaft using a tachometer. The reduction ratio is approximately 217 and allowed an accurate estimate of cylinder rotation rate to be obtained. Two types of flow visualization were used. For velocity pattern identification the working fluid was 2% by volume Kalliroscope in water. For tracer dispersion visualization, an acid-base indicator solution was used.

The vessel was commissioned by visually observing the Reynolds number at which transition from Couette to Taylor vortex flow (TVF) occurred. The theoretical value for $\eta = 0.856$ is 110.8 (Coles 1967) whereas transition was observed to occur at a Reynolds number of 104 ± 2. Drift in the speed controller and temperature effects were ruled out as sources of this discrepancy, which is most likely re-

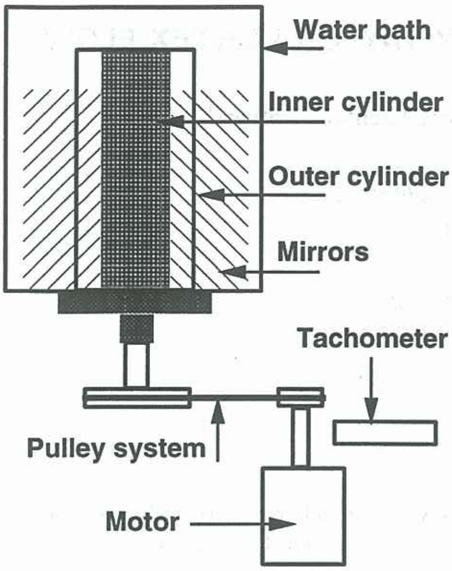

Figure 1: Schematic of the experimental set-up.

lated to imperfections in the rig. The disagreement between theory and experiment does not invalidate any of the qualitative results obtained in this paper.

In this study we consider the wavy vortex flow (WVF) regime in which many different wave states may exist for a fixed Reynolds number. The wave state is defined using the number of waves in the azimuthal direction (m) and the number of vortex pairs in the axial direction (n). Note that there must always be an even number of vortices, $2n$ in a vessel with end walls fixed to the same cylinder. Here, a single Reynolds number of 715 is studied (based on the velocity of the inner cylinder and gap between cylinders).

Two different wave states are considered: WS1 in which $(m, n) = (4, 15)$; and WS2 $(m, n) = (5, 17)$. While more than one wave state can exist for any Re, once established a given wave state is stable. Importantly, accessible wave states can be reliably established by fixing the way in which the desired Re is approached. To reproducibly obtain the two wave states studied here, the procedures were:

WS1, $(4, 15)$ Start at $Re = 1690$ and decrease the speed setting in 12 equal increments until the final rotation rate ($Re = 715$) was reached.

WS2, $(5, 17)$ Start at $Re = 92$ and increase the speed setting in 15 equal increments until the final rotation rate ($Re = 715$) was reached.

In addition to Kalliroscope visualisation, an acid-base chemical reaction was used to study local and global mixing patterns in the flow and to compare to the results of simulation. A solution of Nitrazine Yellow in NaOH was used which changes colour from dark blue at pH 7.2 to bright yellow at pH below 6. The titration procedure was:

1. Fill the vessel with 1450 mL of Nitrazine Yellow/NaOH solution (pH 8.5), leaving space for the addition of the acid neutralising solution.
2. Obtain the desired wave state.
3. Add 20ml of HCl (pH 2.4) which, when well mixed with the solution already in the vessel, is enough to change the entire volume of solution to a pH of 5.1 and thus from blue to yellow.

NUMERICAL METHOD

In the numerical study, the time-dependent solution of the fluid flow field in the experimental apparatus is obtained in a stationary coordinate frame using the conservative finite-difference method detailed in Rudman (1998). Only one wavelength in the axial and azimuthal directions are simulated and doubly-periodic boundary conditions are used (see Figure 2). The solution is integrated forward in time until the flow becomes time periodic. In the wavy vortex regime, the flow field is steady when viewed in a coordinate frame that rotates with the azimuthal wave. This steady fluid flow solution is used to integrate 10,000 fluid trajectories forward in time in a coordinate frame that rotates with the azimuthal wave using the fourth-order Runge-Kutta method discussed in Rudman (1998).

Validation of the numerical method was undertaken by comparing wave speeds for WVF against the experimental wave speeds measured by Coles (1965) and those predicted numerically by King et al. (1984) Discrepancies were well within the experimental scat-

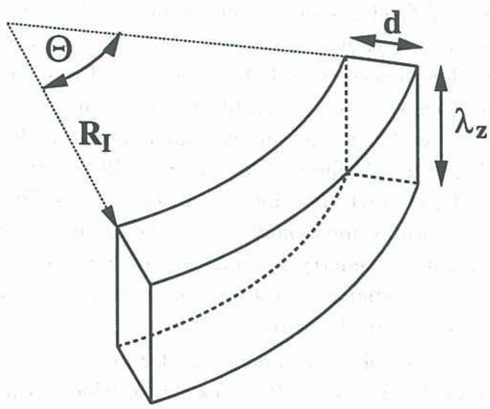

Figure 2: Computational domain for simulation. The boundary conditions are periodic in the axial and azimuthal directions.

ter and differed from the numerical results by less than 0.5%. The fluid trajectory code was validated by comparing results to those presented in Broomhead and Ryrie (1988) and showed excellent agreement.

RESULTS

In Taylor vortex flow, the velocity field is axisymmetric and fluid elements traverse the surfaces of invariant tori. Mixing occurs only as a result of molecular diffusion. Once the symmetry of Taylor vortex flow is broken, global transport and chaotic mixing result (Broomhead and Ryrie 1988). The velocity field on one $r - z$ slice is shown in Figure 3a for WS1 and in Figure 3d for WS2. In contrast to axisymmetric Taylor vortex flow, the velocity field changes significantly in the azimuthal direction (not shown) and the relative size of each vortex shows a shift-and-reflect symmetry in the vertical direction. The ability of the velocity field to advect fluid between vortices in wavy vortex flow is clearly seen in the plots.

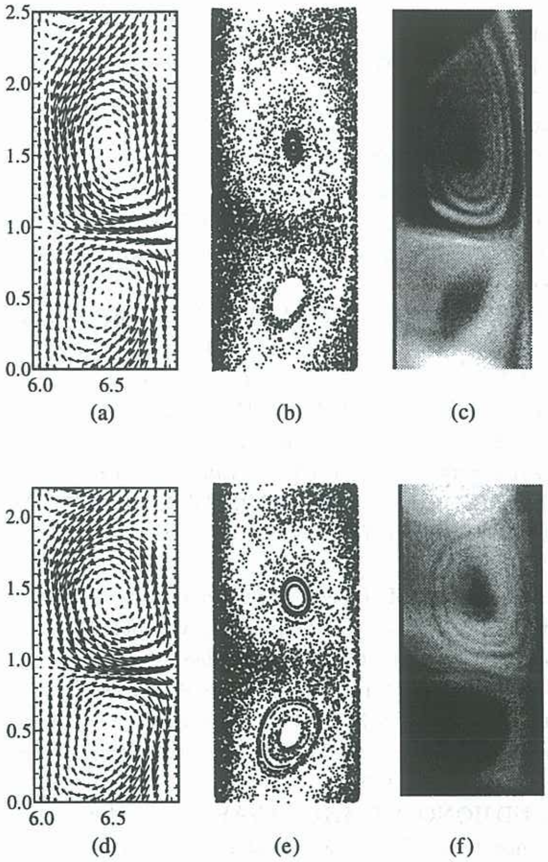

Figure 3: Results for WS1 (top) and WS2 (bottom) (a,d) Predicted velocity field, (b,e) Poincaré section and (c,f) experimental visualisation on one $r - z$ slice.

DISCUSSION

Evidence of chaos in these flows is seen in the Poincaré sections in Figure 3b for WS1 and Figure 3e for WS2. These sections are obtained with respect to a coordinate frame that rotates with the azimuthal wave and clearly show 'vortex core' regions in which fluid is trapped. Outside the core regions is a well-mixed region in which particle trajectories are chaotic. The presence of non-mixing vortex cores so far from the transition to wavy flow was numerically predicted in Rudman (1998) but has not been experimentally verified previously. The acid-base indicator system described above was used to visualise this flow and the experimentally observed vortex cores are shown in Figure 3c for WS1 and Figure 3f for WS2.

One estimate of mixing can be defined using an effective axial diffusion coefficient, D_z, as done by Broomhead and Ryrie (1988). This diffusion coefficient is a measure of chaotic transport in the flow. In the wavy vortex regime the wave state of the flow has a significant impact on the predicted diffusion. For WS1 $D_z = 1.18 \times 10^{-2}$ and for WS2 $D_z = 8.47 \times 10^{-3}$—a factor of 1.4 different. Given the often large range of wave states attainable in wavy vortex flow, the factor of 1.4 is likely to be a lower bound on the range of D_z. Although some studies have found that the effective diffusion in Taylor-Couette flow is a power law function of the Reynolds number (Moore and Cooney 1995), the results shown here suggest that such correlations are not valid in the wavy vortex regime at least and that neglect of wave state is not justified in this case.

The volume of the vortex core region in each flow was estimated by following fluid trajectories for a dimensionless time of 10,000 - those that has not moved axially more than one quarter of a domain height from their initial positions were considered trapped. Using this criterion, the vortex core size for WS1 was 7.6% and for WS2 was 15.0%. Although the different core sizes goes part of the way to explaining the different effective diffusion coefficients in WS1 and WS2, it is insufficient to explain all of it. The full explanation must be more fundamentally related to the chaotic dynamics of the fluid flow.

The model usually used to describe axisymmetric Taylor-Couette flow when it is considered as a reaction vessel is that of a series of well mixed reactors (Kataoka 1975). Quite clearly this model is invalid once the symmetry of Taylor vortex flow has been broken, and the picture more closely resembles the model of Campero and Vigil (1997) in which the flow is broken into well mixed and non-mixed regions. The predicted volume of the vortex cores here is considerably lower than those used in the reactor model of Campero and Vigil in which the core volume was a parameter found by least squares fitting to experimen-

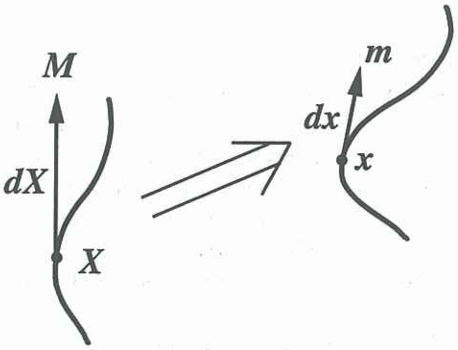

Figure 4: Stretching of an infinitesimal vector $\boldsymbol{dX}$ with unit direction $\boldsymbol{M}$.

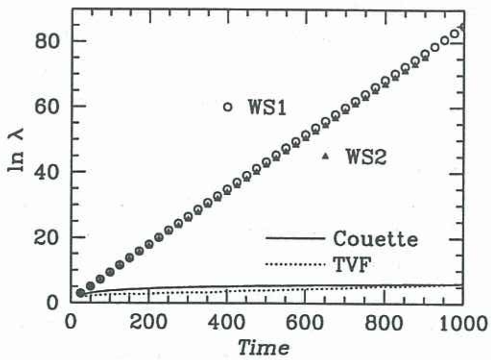

Figure 5: Log mean total stretching versus time for WS1, WS2, TVF and Couette flow.

tal data. However, the titration experiments suggest there is a larger area around the vortex cores that mixes quite slowly. This slow mixing region is approximately 1/3 of the domain in WS1. This type of region is common when invariant KAM surfaces break up, such as happens after the transition from Taylor to wavy vortex flow. The area of the slow mixing regions are closer to the core size values deduced in Campero and Vigil, and it may be that these regions are important in understanding and predicting the performance of Taylor-Couette reaction vessels.

An alternative way to quantify mixing is suggested by Ottino (1989) (Chapters 2 and 4) in his discussion of fluid stretching. Consider an infinitesimal vector $\boldsymbol{dX}$ at $\boldsymbol{X}$ with a unit normal direction given by $\boldsymbol{M}$ (see Figure 4). After some small elapsed time the vector has been advected and stretched and is now represented by an infinitesimal vector $\boldsymbol{dx}$ at $\boldsymbol{x}$ with unit normal direction $\boldsymbol{m}$. The length stretch, λ, of the infinitesimal vector is defined to be the limit as $\boldsymbol{dX} \to 0$ of $\boldsymbol{dx}/\boldsymbol{dX}$. The equations that describe the the evolution with time of λ and the unit direction vector $\boldsymbol{m}$ in a fluid flow are

$$\frac{D(\ln\lambda)}{Dt} = (\boldsymbol{m}^T \cdot \boldsymbol{S} \cdot \boldsymbol{m}) \qquad (1)$$

$$\frac{D\boldsymbol{m}}{Dt} = \boldsymbol{m} \cdot \nabla \boldsymbol{U} - (\boldsymbol{m}^T \cdot \boldsymbol{S} \cdot \boldsymbol{m})\boldsymbol{m}, \qquad (2)$$

where $\boldsymbol{S}$ is rate of strain tensor.

The mean total stretching experienced by 1,000 fluid particles as a function of time are shown in Figure 5 for WS1 and WS2, as well as for axisymmetric Taylor vortex flow and Couette flow. The mean stretching for Couette and TVF increase at a similar rate to WVF for very small times but rapidly flatten out, increasing at best linearly in time. In contrast, the mean stretching for both wavy flows is exponential in time indicating the degree to which mixing is superior in this flow regime. Surprisingly, the total mean stretching for these two flows is almost identical and thus cannot explain the differences obtained in effective axial diffusion coefficients. A similar result has been found in an unreported study of other wavy vortex flows in which higher D_z have been found for flows with less mean stretching. Although there is a clear qualitative relationship between stretching and mixing performance, it is not clear how stretching can be used to formulate quantitative relationships.

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