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NUMERICAL AND EXPERIMENTAL ANALYSIS OF TRANSONIC WIND TUNNEL WALL INTERFERENCE PROBLEM

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ABSTRACT

The influence of perforated walls of transonic wind tunnels at two-dimensional investigations by employing the Fourier's method for solving Dirichlet's problem formulated for a rectangle of wind tunnel's work section is given in this paper.

To demonstrate the appropriateness of the presented algorithm for calculation of transonic wind tunnel wall interference at two-dimensional investigations, the algorithm has been applied to the aerodynamic experimental results from investigations of NACA 0012 airfoil obtained in trisonic wind tunnel in the Aeronautical Institute - VTI (Yugoslavia).

INTRODUCTION

It is well known that even the best wind tunnels do not provide the flow over a model that would exactly simulate the free air stream condition i.e. to be the same as the flow in free air. Hence, the problem of wind tunnel wall interference accompanies experimental and theoretical investigations when designing a wind tunnel as well as during its operation.

One of the fundamental questions imposed upon us when determining transonic wind tunnel work section wall interference is that one related to its correct experimental and mathematical simulation. Since it is impossible to avoid completely its undesirable influence on measured aerodynamic characteristics (lift, drag, lift-curve slope, pitching moment, flow-field quality etc.), the only practical possibility at our disposal is the combination of experimental data from the characteristics investigations and correct mathematical simulations in the course of calculation of their influence on measured values.

In order to preserve the realistic character of flow at the work section's boundaries, the boundary conditions, which are to be known to solve this type of boundary problem, are experimentally determined by measuring static pressure distribution at the vicinity of work section walls. To solve the formulated problem, the concept of local linearization of external flow far from model and flow over it has been used. This flow has been replaced with singularities of appropriate strengths. The corrections have been interpreted in the way that the pressure distribution measured over airfoil's surface at wind tunnel investigation at undisturbed flow Mach number M_∞, corresponds to the distribution which could be obtained if the airfoil were investigated in a free air (unconstrained by wind tunnel walls) at a Mach number $M_\infty + \Delta M$.

PROBLEM FORMULATION

The basic idea of the method was given in the works of Mokry et al, Chapelier et al, and Paquet with a modification of the part related to modeling of boundary conditions on walls. Instead of using the solution obtained for a infinite segment, the problem is formulated for a rectangle $x_1<x<x_2$, $y_1<y<y_2$ (Fig. 1).

The origin of the co-ordinate system is at the point $0.25\,c$ of airfoil placed within a two-dimensional working section's rectangle, Fig. 1. Assuming that the flow at the rectangle's boundaries G is almost parallel to the undisturbed flow far upstream whereas pressure coefficient has a low value and is therefore subcritical, potential of the disturbance velocity ϕ at vicinity of boundary G satisfies linearized equations

$$\beta^2 \frac{\partial^2 \phi}{\partial x^2}(x,y) + \frac{\partial^2 \phi}{\partial y^2}(x,y) = 0, \quad C_p(x,y) = -2\frac{\partial \phi}{\partial x}(x,y) \qquad (1),(2)$$

In the region of linearized flow (the flow domain between walls and depicted rectangle, Fig. 1) we can use decomposition of potential

$$\phi(x,y) = \phi_w(x,y) + \phi_f(x,y) \qquad (3)$$

where ϕ_w satisfies Eq. (1) in $R \cup G$, and ϕ_f is in infinite region external to G and obeys the far-field conditions. By expanding the complex distributed velocity in the domain of linearized flow in Laurent series, from the main part of the series we obtain the following approximation for ϕ_f

$$\phi_f(x,y) = -\frac{\gamma}{2\pi} atan \frac{\beta y}{x} + \frac{\mu}{2\pi\beta} \frac{x}{x^2 + (\beta y)^2} \qquad (4)$$

where γ is the strength of vortex term equal to the circulation around the boundary G, which is, according to the theory by Joukowsky

$$\gamma = 0.5 c C_L \qquad (5)$$

For attached viscous flow, the effect of wake is negligible. Hence, Eq. (4) does not include a source term. Its inclusion is justifiable in the case of an airfoil in stalling-flight condition, however, the practicability of the presented theory would have to be totally reconsidered, since it is only in special cases acceptable to assume the flow to be a two-dimensional one. The displacement effect is represented by the doublet term whose strength μ for compressible flow on the surface of airfoil cross section may be represented as

$$\mu = c^2 A + \frac{\kappa+1}{2\pi\beta} M_\infty^{7/4} \left(\frac{cC_L}{4}\right)^2 \left\{\frac{7}{4} + \log\left[M_\infty^{1/2}\left(\frac{t}{c}\right)^{1/3}\frac{|y|}{c}\right]\right\} \quad (6)$$

here t is maximal airfoil thickness.

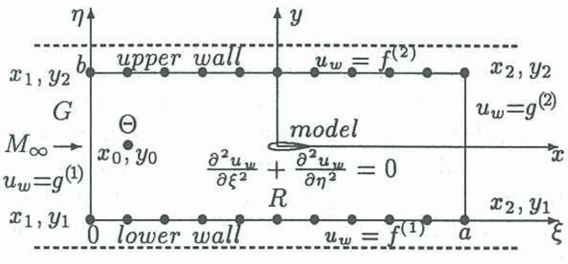

Figure 1: The co-ordinate system for finite-length working section.

SOLUTION OF PROBLEM

For the calculation of ϕ_w, we can utilize the transformation given by Mokry: $\xi = \frac{1}{\beta}(x - x_1)$, $\eta = y - y_1$ which reduce the Eq. (1) into Laplace equation. As for differentiability of harmonic function velocity, the reduced x - component of interference velocity $u(\xi,\eta) = \frac{\partial\phi_w}{\partial\xi}(x,y) = \beta\frac{\partial\phi_w}{\partial x}(x,y)$ satisfies the equation

$$\frac{\partial^2 u}{\partial\xi^2}(\xi,\eta) + \frac{\partial^2 u}{\partial\eta^2}(\xi,\eta) = 0 \quad (7)$$

in domain of rectangle $0 < \xi < a$, $0 < \eta < b$, where $a = \frac{1}{\beta}(x_2 - x_1)$, $b = y_2 - y_1$ are the sides of transformed rectangle. For transformed x - component of interference velocity $u(\xi,\eta)$, we can formulate the following Dirichlet's problem:

$$\frac{\partial^2 u_w}{\partial\xi^2}(\xi,\eta) + \frac{\partial^2 u_w}{\partial\eta^2}(\xi,\eta) = 0, \; 0 < \xi < a, \; 0 < \eta < b \quad (8)$$

$$u_w(\xi,0) = f^{(1)}(\xi) \; 0 < \xi < a, \; u_w(\xi,b) = f^{(2)}(\xi) \; 0 < \xi < a$$
$$u_w(0,\eta) = g^{(1)}(\eta) \; 0 < \eta < b, \; u_w(a,\eta) = g^{(2)}(\eta) \; 0 < \eta < b \quad (9)$$

as illustrated in Fig. 1. Boundary conditions

$$f^{(1)} = -\beta\left[\frac{C_p(x,y_1)}{2} + \frac{\partial\phi_f}{\partial x}(x,y_1)\right], \; f^{(2)} = -\beta\left[\frac{C_p(x,y_2)}{2} + \frac{\partial\phi_f}{\partial x}(x,y_2)\right] \quad (10)$$

are obtained by measuring the static pressure along sides $y = y_1$ and $y = y_2$ and substituting the potential ϕ_f from equation (4). In the case when the pressures at the fore and back boundaries of the working section are not available and when the pressures along the wind tunnel walls are measured far enough upstream and downstream, then the boundary values are obtained by linear interpolation by utilizing pressure values at rectangle's corners

$$g^{(1)} = f^{(1)}(0) + \frac{f^{(2)}(0) - f^{(1)}(0)}{b}\eta, \; g^{(2)} = f^{(1)}(a) + \frac{f^{(2)}(a) - f^{(1)}(a)}{b}\eta \quad (11)$$

The boundary values problem given by Eqs. (8) and (9) can be solved by Fourier method of separation of variables. Solution, adapted for the application of Fourier transform is

$$u(\xi,\eta) = f(\xi,\eta) + g(\xi,\eta) \quad (12)$$

where

$$f(\xi,\eta) = \sum_{k=1}^{\frac{m}{2}-1} sin\mu_k\xi \left[A_k^{(1)}\frac{sinh\mu_k(b-\eta)}{sinh\mu_k b} + A_k^{(2)}\frac{sinh\mu_k\eta}{sinh\mu_k b}\right],$$
$$g(\xi,\eta) = \sum_{k=1}^{\frac{m}{2}-1} sin\nu_k\eta \left[B_k^{(1)}\frac{sinh\nu_k(a-\xi)}{sinh\nu_k a} + B_k^{(2)}\frac{sinh\nu_k\xi}{sinh\nu_k a}\right] \quad (13)$$

that is, the reduced component of wall interference

$$u_w(\xi,\eta) = \sum_{k=1}^{\infty} sin\mu_k\xi \left[A_k^{(1)}\frac{sinh\mu_k(b-\eta)}{sinh\mu_k b} + A_k^{(2)}\frac{sinh\mu_k\eta}{sinh\mu_k b}\right]$$
$$+ \sum_{k=1}^{\infty} sin\nu_k\eta \left[B_k^{(1)}\frac{sinh\nu_k(a-\xi)}{sinh\nu_k a} + B_k^{(2)}\frac{sinh\nu_k\xi}{sinh\nu_k a}\right] \quad (14)$$

where the eigenvalues are: $\mu_k = \frac{k\pi}{a}$ and $\nu_k = \frac{k\pi}{b}$.

The series coefficients are described with integrals:

$$A_k^{(\,)} = \frac{2}{a}\int_0^a f^{(\,)}(\xi) sin\mu_k\xi d\xi, \quad B_k^{(\,)} = \frac{2}{b}\int_0^b g^{(\,)}(\eta) sin\nu_k\eta d\eta \quad (15)$$

where empty space within parentheses should have the index (1) or (2) depending what boundary is involved, lower (1) or upper (2), fore (1) or back (2). The above solution is uniformly convergent inside a rectangle, with exception of vicinity of corners where they should be modified. For models placed in the center of work section, the Eq. (14) could be approximated by truncation of series. The coefficients $A_k^{()}$ can be approximated according to the rule of rectangle

$$A_k^{(1)} = \frac{2}{m}\sum_{j=0}^{m-1} f^{(1)}\left(a\frac{2j+1}{m}\right) sin\frac{2\pi jk}{m}, \; A_k^{(2)} = \frac{2}{m}\sum_{j=0}^{m-1} f^{(2)}\left(a\frac{2j+1}{m}\right) sin\frac{2\pi jk}{m} \quad (16)$$

If m is chosen to be an integer exponent of 2 (2^1, 2^2, 2^3,...), the above given sum can be solved very efficiently by employing Fast Fourier Transform algorithm for $k = 1, 2, 3, ..., m/2-1$. Consequently, the upper limit of the first series (14) might be $m/2-1$. Such an approach for Fourier coefficient calculation has shown a fairly good convergence of the solution, what can be seen from my references. Derivation of coefficients $B_k^{()}$ is always simpler by using Eq. (11) since in that case a closed form of integral is obtained

$$B_k^{(1)} = \frac{2}{k\pi}[f^{(1)}(0) - (-1)^k f^{(2)}(0)], \; B_k^{(2)} = \frac{2}{k\pi}[f^{(1)}(a) - (-1)^k f^{(2)}(a)] \quad (17)$$

WALL INTERFERENCE CORRECTIONS

By differentiating the adiabatic relation between velocity and Mach number, we may obtain the correction of Mach number

$$\Delta M = \left(1 + \frac{\kappa-1}{2}M^2\right) M \frac{\partial\phi_w}{\partial x}(0,0) \quad (18)$$

where, we get velocity correction for model position $x = y = 0$:

$$u_w(0,0) = \frac{1}{\beta}u_w\left(-\frac{x_1}{\beta}, -y_1\right) = \frac{\partial\phi_w}{\partial x}(0,0) \quad (19)$$

and correction of angle of attack

$$\Delta\alpha = \frac{\partial\phi_w}{\partial y}(0,0) = v_w(0,0) \qquad (20)$$

For correction of the attack angle, by combining the previous equations we obtain:

$$\frac{\partial\phi_w}{\partial y}(0,0) - \frac{\partial\phi_w}{\partial y}(x_0,y_0) = v_w\left(-\frac{x_1}{\beta}, -y_1\right) - v_w\left(\frac{x_0-x_1}{\beta}, y_0-y_1\right) \qquad (21)$$

where v_w is a function of conjugated velocity

$$v_w(\xi,\eta) = \int \frac{\partial u}{\partial \eta}(\xi,\eta)d\xi \qquad (22)$$

which can be obtained from Eqs. (12) and (13).

If we now select the reference point (x_0, y_0), to be inside the linearized flow (Fig. 1), then from Eq. (3) we have:

$$\frac{\partial\phi_w}{\partial y}(x_0,y_0) = \Theta(x_0,y_0) - \frac{\partial\phi_f}{\partial y}(x_0,y_0) \qquad (23)$$

where $\Theta(x_0,y_0) = \frac{\partial\phi}{\partial y}(x_0,y_0)$ represents the angle of flow (in radians) at the reference point.

It follows:

$$\Delta\alpha = v_w\left(-\frac{x_1}{\beta}, -y_1\right) - v_w\left(\frac{x_0-x_1}{\beta}, y_0-y_1\right) + \Theta(x_0,y_0) - \frac{\partial\phi_f}{\partial y}(x_0,y_0) \qquad (24)$$

The first two terms on the right-hand side of Eq. (24) are derived from Eq. (22) and the last term from Eq. (4). In this equation, the angle of flow Θ is the only unknown term, which is to be obtained by measurements (measurement of flow deflection, laser velocity-meter etc.) or to be calculated in some way. We can choose the reference point to be $x_0 = x_1$ and $y_0 = 0$ with initial angularity of flow in that point: $\Theta(x_1, 0) = 0$ or, what is more accurate, by assuming that the initial flow angle is equal to the known value of flow angle from empty wind tunnel calibration.

RESULTS OF CALCULATIONS

The computation of wind tunnel wall interference with pressure measurements at wall boundary has been conducted according to the correction procedure given in this paper. According to this procedure, the corrections of Mach number and flow attack angle were calculated for experimentally determined pressure coefficients distribution along the airfoil. As noted, the problem formulated by Eq. (8) and boundary conditions (9) is one of Dirichlet's type formulated for a rectangle (Fig. 1). By employing Fourier method for solution of such a type of problem with experimentally defined boundary conditions (10) and (11), the problem has been solved on Faculty of Mechanical Engineering's VAX 750 computer. The model of linearized free-air flow was represented through doublet and vortex terms as described by Eq. (4).

During the investigation, the pressure distribution in 80 points along the upper and lower side of model NACA 0012 were measured (Fig. 2). Based on the measurements, aerodynamic coefficients were defined. Also, the pressure distribution in 46 points along the upper and lower wall of working section were measured and values of coefficient of pressure evaluated. These pressure coefficients, along the upper and lower wall at angle of attack of 2.0° at Mach number of 0.8, are shown in Fig. 3. Relative coordinates of points along the x-axis, where the static pressure is measured are also given in Fig. 3, with the relative coordinate along the y-axis being $y/c = \pm 2.9528$.

In numerical calculations, effect of selection of integer exponent n ($m = 2^n$) on the solution convergence has been tested. The calculation of needed sum in Fourier coefficient calculation, given in (16) and (17) as well as determination of eigenvalues has been performed using Fast Fourier Transformation, with $k = 2^l - 1$, where the integer exponent took values $l = 1, 2, ..., n-1$, and summation was done for $j = 0, 1, ..., m-1$.

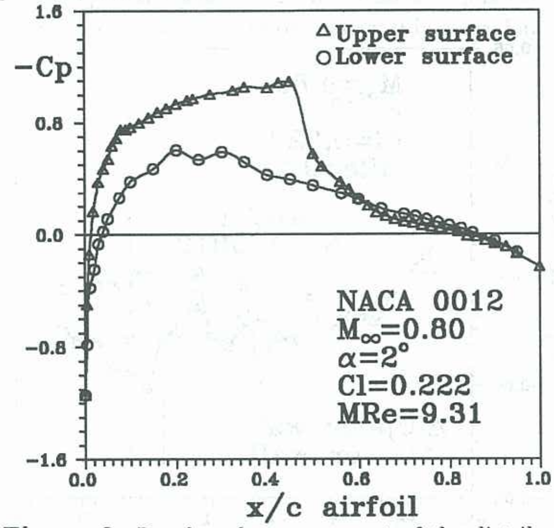

Figure 2: Results of measurement of the distribution of the static pressure along the upper and lower side of airfoil at angle of attack of 2.0° at $M = 0.8$.

For the reference position, the point with coordinates $x_0/c = -2.7559$ and $y_0 = 0$ (inside the control rectangle, Fig. 1) at which the flow angle was measured during calibration testing, was selected. This angle was from $-0.2°$ to $-0.25°$ for Mach numbers $M = 0.7 - 1.0$ (the calculation was performed with an angle of $-0.2°$). The others principal parameters for that investigations were: the airfoil NACA 0012 with a cord $c = 0.254\,m$, a reference Mach number $M = 0.802$, the angle of attack $\alpha = 2°$, the lift coefficient C_L=0.222, the chord to tunnel height ratio $c/h = 1/6$ and a Reynolds number of 9.31×10^6. The solutions for the Mach number and angle of attack correction, for the test case are given in Table 1.

The obtained calculation results of the transonic wind tunnel wall interference performed by the present numerical method show a completely satisfactory agreement together with some calculation results from others authors (Fig. 4).

CONCLUSION

Due to large absolute values of required corrections of angle of attack, it follows that all measurements which are related to the determination of angle of attack, that is, to the evaluation of lift-curve slope, have no significance for practical application without the suggested wind tunnel corrections.

To conclude, it can be said that the boundary value problem described by Eqs. (8) and (9) can also be solved numerically, for example, by employing: panel methods, finite difference method or finite element method. It would be more appropriate to solve the problem by using the mentioned methods when it is required to calculate the velocity interference in the whole flow region and not only the flow correction in the model investigated position. Thus, the numerical methods are more useful for more complex geometry of working section or for a combination of pressure boundary conditions and normal velocity component.

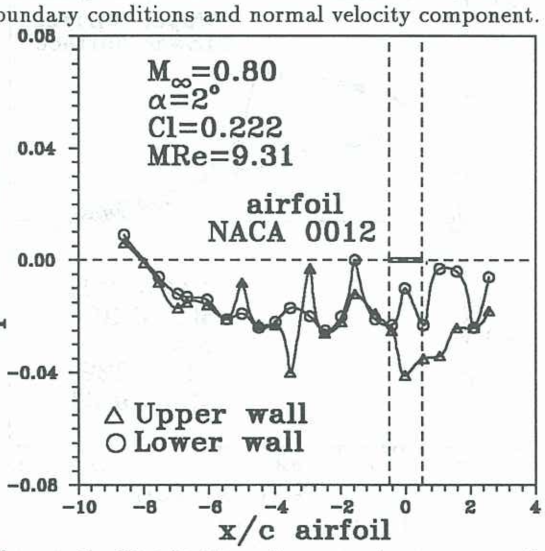

Figure 3: Distribution of measured pressure coefficients along the upper and lower wall of working section at angle of attack of 2.0° and $M = 0.8$.

	Measured	Corrections	Corrected (Free Air)
M	0.802		0.8077
α	2.0		1.3545
C_z	0.222		0.2185
ΔM		0.0057	
$\Delta\alpha$		-0.6455	

Table 1: The solutions for the Mach number and angle of attack correction for the test case.

The employed approach has certain advantages over some others, say, over the Lo's method with measurement of flow angle in disturbed flow region in the vicinity of wind tunnel walls or over the method by Capelier, Chevallier and Bouniol who have solved a problem formulated for an infinite flow region, or over the Paquet's method formulated and solved for a semi-infinite flow region. Also, this conclusion can be related to the other group of problems, where the cross-flow characteristics through wind tunnel walls are first experimentally determined. Unfortunately, this is to be made for every Mach number, for every form of model and even for every angle of attack in order to determine, as precise as possible, the boundary conditions, and then, by using an appropriate method (finite differences, panel method or Fourier method) to solve the wind tunnel wall interference. The last method is quit appropriate for solution of the wall interference problem for three-dimensional models.

In order to levelage the wind tunnel's role with the ones that the computer analysis and free air investigations have, that is, in order to be supplied with the satisfactory aerodynamic data obtained from wind tunnel investigations for airplanes design applications, it is necessary to supplement the wind tunnel data with calculation of wind tunnel wall corrections as their integral and inseparable part. Relying on this approach that is, through development of a reliable method of correction of wind tunnel wall interference calculation, we fill the gap that exists due to non-availability of a faithful turbulent mathematical model. Through a computer analysis the approach does deliver true aerodynamic characteristics needed for airplane design.

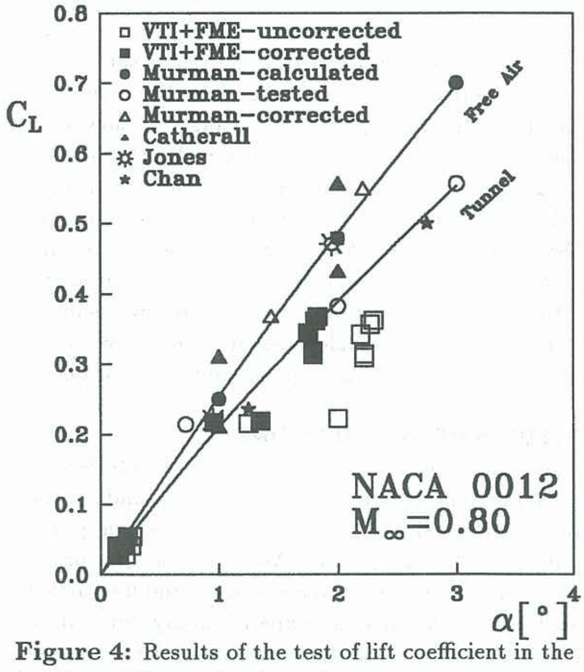

Figure 4: Results of the test of lift coefficient in the function of the angle of attack.

REFERENCES

MOKRY, M. et al, "Two-dimensional Wind Tunnel Wall Interference", *AGARDograph*, **281**, 1983.

CHAPELIER C. et al, "Nouvelle Methode de Correction des Effets de Porois en Courant Plan", *La Recherche Aerospatiale*, Jan.-Feb., 1-11, 1978.

PAQUET J.P., "Perturbations Induites par les Porois d'une Soufflerie Methodes Integrales", *These Doc. Ing.*, Universite de Lille, juin, 1979.

RAŠUO, B., "Fourier's Method for Solving Dirichlet's Problem With an Application for Two-dimensional Transonic Wind Tunnel Wall Interference", *Proc. of the 2nd Australian Eng. Math. Conf.*, Sydney, 335-342, 1996.