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EXPERIMENTS AND SIMULATIONS OF TRANSITION IN CYLINDRICAL PIPE FLOW

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ABSTRACT

Transition of a cylindrical pipe flow can be considered as one of the classic problems in fluid mechanics going back to the famous experiments carried out by Osborne Reynolds in 1883. In principle it is still considered to be an unsolved problem although the main characteristics of transition in pipe flow are familiar to every student of fluid mechanics. A review is presented here on some recent developments in experiments, numerical simulations and theory for this interesting and challenging problem.

INTRODUCTION

Fully developed laminar flow in a cylindrical pipe is given by the well-known parabolic Hagen-Poiseuille profile. At low Reynolds numbers this flow is found to be stable to all disturbance irrespective of their shape or magnitude. When a critical value of the Reynolds number is exceeded the flow can become unstable to disturbances of finite magnitude resulting in turbulence. The lowest Reynolds number at which disturbances can grow, is called Re_{crit}^{min}. Its value is generally put near 2000 with the definition of the Reynolds number based on the pipe diameter and the mean velocity. For instance, the recent experiments of Darbyshire and Mullin (1995) state $Re_{crit}^{min} = 1760$.

Experiment	Re_{crit}^{max}	$Re_{99\%}$
Reynolds 1883	13 000	1 000
Ekman 1909	45 500	1 000
Reshotko 1958	23 000	14 300
Leite 1959	20 000	12 500
Pfenniger 1961	100 000	8 000
Fox *et al.* 1968	5 000	4 000
Wygnanski & Champagne 1973	45 000	8 900
Barker & Gile 1981	250 000	1 100
Darbyshire & Mullin 1995	17 000	3 400
Draad *et al.* 1998	63 400	14 300

Table 1: Observed values for Re_{crit}^{max} and $Re_{99\%}$ for various experiments

Beyond Re_{crit}^{min} the flow is found to be unstable to finite amplitude disturbances. Already Reynolds himself noted that when introduction of flow disturbances, e.g. at the pipe entrance, is avoided, the transition can be delayed till Reynolds numbers much larger than 2000. Let us call the upper bound of the critical Reynolds number: Re_{crit}^{max}

The behaviour of transition in cylindrical pipe flow sketched above has been confirmed by many other experimentalists since Reynolds. A not necessarily complete list of critical Reynolds numbers that have been reported in the literature, is given in table 1. In this table we also give for each experiment the value of the $Re_{99\%}$ which is defined as the largest Reynolds number for which the velocity profile at the end of the pipe facility is within 1 % of the fully developed Hagen-Poiseuille profile. We see that in all cases $Re_{max} > Re_{99\%}$. This leads us to conclude that all these observations refer to a developing pipe flow and that the transition is probably caused by disturbances introduced at the pipe entrance. The implication is that these observations are not necessarily relevant for transition of fully developed pipe flow. So from an experimental point of view the value of Re_{crit}^{max} for fully developed pipe flow remains unknown.

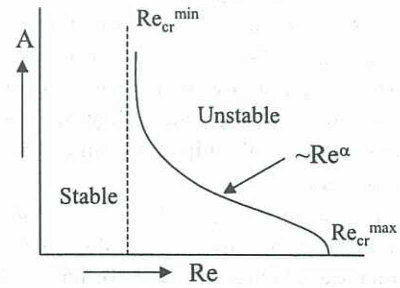

Figure 1: Sketch of the critical disturbance amplitude versus Reynolds number for a subcritical bifurcation.

In mathematical terms transition in cylindrical pipe flow can be denoted as a subcritical bifurcation. In its most simple form the subcritical bifurcation is illustrated in figure 1 where we shown the disturbance amplitude A as a function of the Reynolds number

Re. Left of the dashed line which denotes $Re_{\text{crit}}^{\text{min}}$, the flow is by definition stable for all amplitudes of the perturbation. This means that whatever large perturbation is imposed on the flow, it always returns to its laminar state, i.e. $A \equiv 0$.

Right from the dashed line we find two regions separated by the solid line Below this line the perturbation is stable and above unstable. Unstable means here that the perturbation grows and eventually the flow becomes turbulent. When the Reynolds number increases the line separating the two region approaches the $A = 0$ axis which means that smaller perturbation are sufficient to cause transition. This behaviour is sometimes approximated by a power law as indicated in figure 1:

$$f(Re) \simeq Re^{\alpha}. \qquad (1)$$

The separation line may intersect with the $A = 0$ axis. This value can perhaps be interpreted as the $Re_{\text{crit}}^{\text{max}}$ which we already have introduced above. Above this value the flow is unstable for all perturbations even infinitesimal small ones. The latter means that the initial instability process can then be described by linear theory. It is, however, generally accepted, although not yet formally proven, that laminar pipe flow is linearly stable for all Reynolds numbers. In other words, theoretically, $Re_{\text{crit}}^{\text{max}}$ is infinity. This is consistent with the fact noted above that the value of $Re_{\text{crit}}^{\text{max}}$ has not been determined yet experimentally. The consequence is that the instability of laminar pipe flow can not be described in terms of linear stability theory and we have to refer directly to non-linear theory.

After the initial instability the flow develops further into turbulence by means of an intermittent process in which patches of turbulent flow occur that are separated by regions of laminar flow. This behaviour has been first documented by Wygnanski and Champagne (1973) who distinguish two types of turbulent structures. Below a value of $Re = 2700$ they observe so-called puffs and above this value they find slugs. These turbulent patches grow in size while they are transported along the pipe until they overtake each other. At that instant the pipe flow can be considered as fully turbulent.

In this review we will present some of our recent work on transition of laminar pipe flow in which we will consider the phenomena that we have discussed above. First, we will give some attention to experimental data and we finish with a discussion of numerical simulations and their theoretical implications.

EXPERIMENTS

To study transition of a cylindrical pipe flow, we have designed and constructed a recirculating pipe flow facility with a length of 32m and a diameter of 0.04m. A schematic view of the setup is shown in figure 2 and for a detailed description of the facility we refer to Draad *et al.* (1998). The results that we obtained for $Re_{\text{crit}}^{\text{max}}$, have been mentioned in Table 1.

In this pipe we introduce a disturbance in the form a periodic blowing and suction through a narrow slit in the pipe wall. The angular distribution of this disturbance has a wave number equal to 1 so that for one half of the pipe the perturbation velocity is positive (blowing) whereas for the other half the velocity is negative (suction). As a results there is no mass flux entering the pipe. The blowing and suction alternates periodically with a frequency ω. The frequency and the amplitude of the disturbance can be varied. For further details regarding this disturbance we refer again to Draad *et al.* (1998).

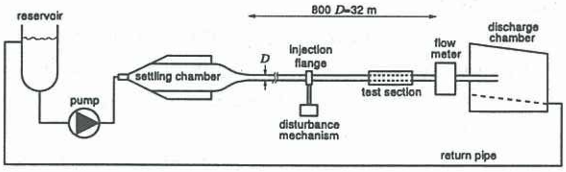

Figure 2: Sketch of the pipe facility.

Some distance behind the disturbance mechanism the pressure is measured. The first measuring point is positioned at 1m downstream of the disturbance mechanism and the second pressure point at 2.5m further. The pressure difference between these two points is now used as an indication whether the flow is laminar or turbulent.

An example of the observed pressure difference as a function of the frequency of the disturbance is illustrated in figure 3 for the case of $Re = 35\,000$. We must note here that a change of frequency includes also a variation of the disturbance amplitude so that the results shown in figure 3 can not be interpreted solely in terms of a dependence on the disturbance frequency. We find that for low values of the frequency, i.e. small values of the disturbance, the flow is laminar. At a critical frequency the pressure difference changes from the laminar to turbulent value indicating transition. We observe that this transition is rather abrupt. Only a small change in frequency, either forward of backward (no hysteresis was found), is needed to change from flow regime.

After the flow has become turbulent one would perhaps suppose that the flow would remain so irrespective of further change in disturbance conditions. This could perhaps be called the conventional picture for which one expects a single and unique value for the critical disturbance. The results of figure 3, however, show another behaviour. When the frequency increases the flow returns to laminar and becomes turbulent again at a second critical value. The background of this surprising multiple transition behaviour which by the way only occurs for large

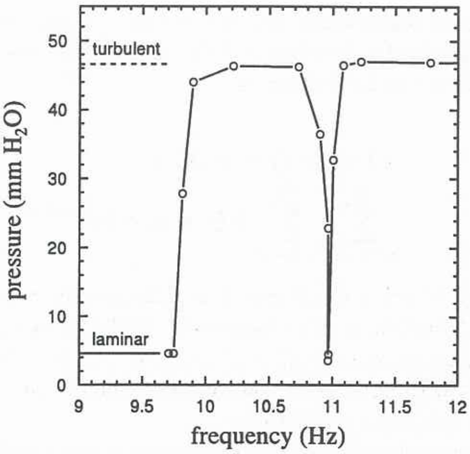

Figure 3: Observation of the pressure drop as a function of the frequency for a Reynolds number $Re = 35\,000$. The values for the pressure drop under fully developed laminar and turbulent conditions are indicated.

Reynolds numbers, remains unclear and must be left for further study.

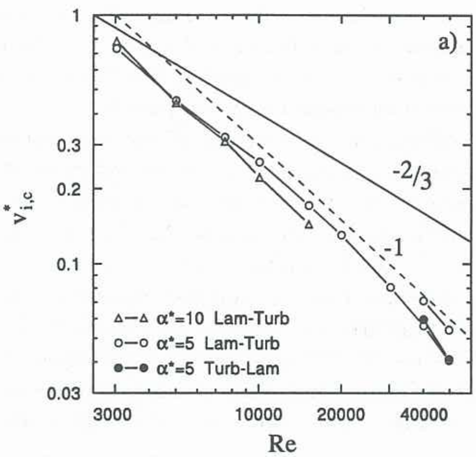

Figure 4: The critical amplitude of the disturbance velocity as a function of the Reynolds number for small wave numbers (upper figure) and for large wave number (lower figure)

In the introduction we have introduced the scaling law for the critical disturbance given by (1). The critical disturbance is defined as the first amplitude of the periodic blowing and suction starting from zero at which the flow changes from laminar to turbulent. We distinguish between small and large wave numbers of the disturbance. Here, the wave number which is indicated by α^*, is defined as the frequency divided by the mean velocity. We find that for the low wave numbers the exponent α is equal to $-2/3$ and for large wave number equal to -1. The latter case is illustrated in figure 4. This suggests that the mechanism of transition changes as the wave number of the disturbance varies.

NUMERICAL SIMULATION

We now consider the results obtained for the transition of cylindrical pipe flow obtained by means of a numerical simulation. For this we have developed a spectral element code formulated in cylindrical coordinates. Spectral techniques have favourable properties with respect to numerical accuracy which are needed in this case where already small inaccuracies in the phase of the simulated disturbance can lead to large errors in the final results. The elements consist of annular rings parallel to the pipe axis. In each element the radial dependence is described in terms of an expansion in Chebyshev polynomials. Fourier components are used for the axial direction. For further details regarding the code we refer to Shan *et al.* (1998).

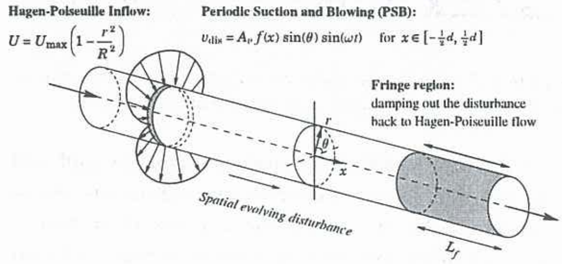

Figure 5: Computational domain used in the numerical simulations

In the numerical model we introduce a disturbance which is similar to the one used in the experiments, i.e. periodic blowing and suction through a slit in the pipe wall. The angular wave number of the disturbance is again chosen equal to one. The development of this disturbance is then computed as a function of the downstream distance. The numerical model can be run in two versions. In the first version periodicity is assumed as boundary condition in the axial direction which the most convenient boundary condition in view of the Fourier components used in this direction. This boundary condition implies that a flow structure which leaves the domain at one side, reenters at the other side. In the second version we have applied at the end of the computational domain a so-called fringe region which allows a flow disturbance to leave the computational domain with only negligible reflection. This allows for a continuous forcing of the space-developing disturbances. The characteristics of the computational domain for the second version are illustrated in figure 5.

With the first version of the code we have performed a simulation of a puff and slug structure. These are created by imposing the periodic blowing and suction disturbance for a limited period of time. Thereafter, the suction and blowing is turned off and the created flow disturbance is left to develop in either a puff or slug. A simulation of a puff structure has been carried out for a Reynolds number equal to $Re = 2200$ and a slug has been simulated for $Re = 5000$. For the numerical resolution we have used $128 \times 53 \times 16$ collocation points in the axial, radial and tangential direction for the case of a puff and $256 \times 53 \times 64$ point for the case of the slug. In both cases the computational domain is a pipe section with a length of $16\pi D$. The computations are continued until the leading edge of the flow structure approaches its trailing edge. At that point the computations are terminated.

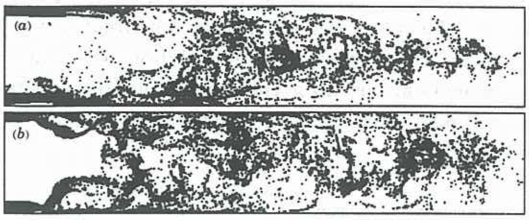

Figure 6: Numerical flow visualization of the instantaneous structure of a puff (top) and a slug (bottom)

In figure 6 we show a visualization of the puff and slug structure as obtained from our simulation results some time after the end of the imposed disturbance. The main difference between both structures is found in the leading-edge region and this agrees with experimental observations. Our computations show also that the leading and trailing edge move with a constant speed. The values obtained for the puff and slug are given in table 2 and they are found to agree reasonably well with experimental values.

	Puff	Slug
C_{LE}	1.56	1.69
C_{TE}	0.73	0.52

Table 2: The velocity of the leading edge C_{LE} and the trailing edge C_{TE} of a puff and slug scaled in terms of the mean velocity.

We have also computed the statistics of velocity fluctuations inside the slug. The rms values of the three velocity components are shown in figure 7. In the same figure we also show the rms values of a DNS of a fully developed turbulent pipe flow at $Re = 5300$. On the average the results for the slug flow agree excellently with those obtained for the turbulent pipe flow with perhaps as exception the axial velocity fluctuations which are somewhat lower for the slug flow. The interpretation of such good agreement is that the flow conditions inside a slug are already quite close to fully developed turbulence.

THEORY

In this section we return to the initial response of the flow to the imposed disturbance with the purpose of investigating the mechanism by which this disturbance can grow. In view of the time periodicity of the imposed disturbance and the natural 2π periodicity of the angular coordinate θ the disturbance velocity vector $\boldsymbol{v}$ can be written as

$$\boldsymbol{v}(r,\theta,x,t) = \boldsymbol{V}(r) + \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \hat{\boldsymbol{v}}(r,x;n,m)\, e^{i(m\theta-n\omega t)}$$

where $\boldsymbol{V}(r)$ is the Hagen-Poiseuille profile on which the disturbances are superposed. In the following we shall use the notation (m,n) to indicate a disturbance mode with an angular wavenumber equal to m and a time frequency equal to $n\omega$.

By substitution of this expansion in the equations of motion we can derive a set of equations for the amplitudes $\hat{\boldsymbol{v}}$. To make this set of equations amenable to analytical treatment we have to make the assumption that the amplitude of the imposed disturbance velocity, A_v, with respect to the mean velocity is small. In that case we can expand the disturbance amplitude $\hat{\boldsymbol{v}}$ in terms of the eigenfunctions of the linear perturbation problem with amplitudes that depend only on the axial coordinate x. The problem of a disturbance which develops as a function of x, can then be written in terms of a set of coupled, non-linear ordinary differential equations for the amplitude.

A subsequent consequence of the assumption of small amplitude A_v is that the development of the disturbance can be approximated in terms of only a few (m,n) modes. The first are the $(\pm1,\pm1)$ modes which we call the fundamental modes because they have the same time and angular dependence as the imposed disturbance. The second are the $(0,0)$, the $(\pm2,0)$, the $(0,\pm2)$ and the $(\pm2,\pm2)$ modes which all result from a non-linear self-interaction of the fundamental modes and which we call the harmonics.

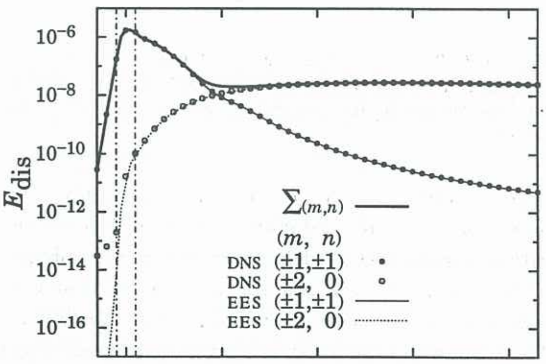

Figure 8: Spatial evolution of the total disturbance energy $E_{dis}(x)$ and the disturbance energy for the dominant modes $(m,n) = (\pm1,\pm1)$ and $(\pm2,0)$. Two vertical dashed-dotted lines delimit the periodic blowing and suction region.

Given the boundary conditions in the form of con-

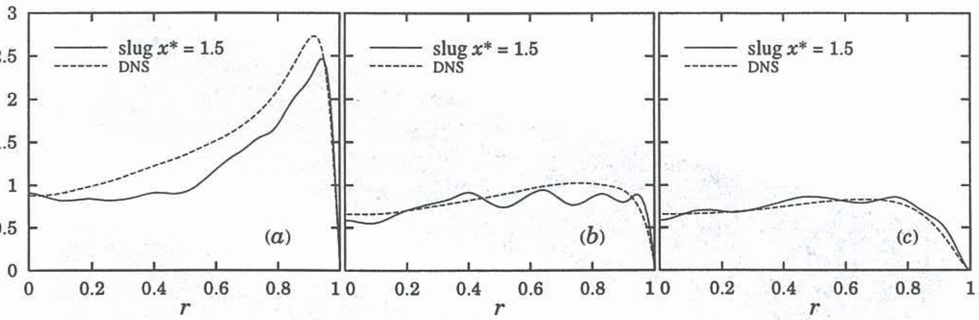

Figure 7: Radial distribution of the rms of the velocity fluctuations: axial component (left), azimuthal component (centre) and radial component (right)

tinuous periodic blowing and suction through a slit in the pipe wall, we can determine the development of the fundamental and harmonic modes as the solution of a set of linear differential equations. With help of this solution we can compute various parameters. One of these is the disturbance energy which for mode (m, n) is denoted as $E_{dis}(x; m, n)$. The results based on this expansion in eigenmodes of the linear stability problem will be indicated by EES in the following. For more details we refer to Ma *et al.* (1998).

Let us now consider some results that we have obtained by using the EES procedure. We repeat that this procedure is based on a linear set of equations and therefore in principle only valid for a very small perturbation amplitudes. To estimate how well the EES solution approximates the real development of disturbances, we have also performed a DNS for the same disturbance conditions. The DNS code used is the same as discussed in the previous section. Only for this case we have used version two, i.e. the disturbance is removed at the end of the computational domain by means of the fringe region. The DNS computations have been carried out for various resolutions with $512 \times 53 \times 32$ collocation points in the axial, radial and tangential direction being the highest.

We first consider a very small disturbance amplitude A_v. The results for the disturbance energy of the fundamental mode and of the $(\pm 2, 0)$ harmonic are shown in figure 8. The energy of the fundamental mode is created in the disturbance region where the blowing and suction boundary condition is imposed on the flow. Thereafter, the disturbance decays almost exponentially. The disturbance energy for the $(\pm 2, 0)$ mode which is created by self interaction of the fundamental mode, is small in the forcing region. It then starts to grow and at a certain value of x it becomes even larger than the energy of the fundamental mode. The background of this growth is connected to the non-normality of the differential operator of the linear perturbation problem. It is thus a linear process and this means that eventually the disturbance energy must decay because all eigenmodes of the linear problem are found to be stable. This process is known as transient growth.

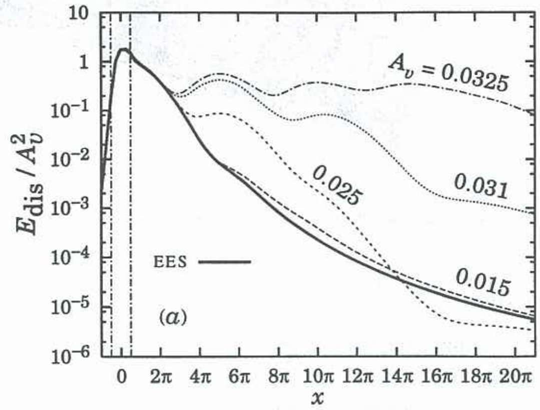

Figure 9: Spatial evolution of the disturbance energy of the fundamental mode for various values of the disturbance amplitude A_v.

The significance of transient growth is that by means of a linear process the energy of some modes can grow despite the fact that the problem is linearly stable. This growth can become quite large and may reach a level where non-linear effects become important and can take over to complete the transition process. Theoretical arguments show that the process of transient growth leads to a scaling law for the critical disturbance amplitude with an exponent $\alpha = -1$. This value is indeed the scaling exponent which we have found in our experiments shown in figure 4.

Figure 8 also shows that the EES and the DNS results are very close which one would expect for a small disturbance amplitude. For larger values of the perturbation amplitude A_v the disturbance energy as a function of axial distance for the case of the fundamental mode is shown in figure 9. It is clear that for the larger values of A_v the DNS results differ substantially from the EES results which are shown in the same figure. So non-linear effects become important. At a value of $A_v = 0.0325$ the disturbance energy seems to saturate and the flow appears to be

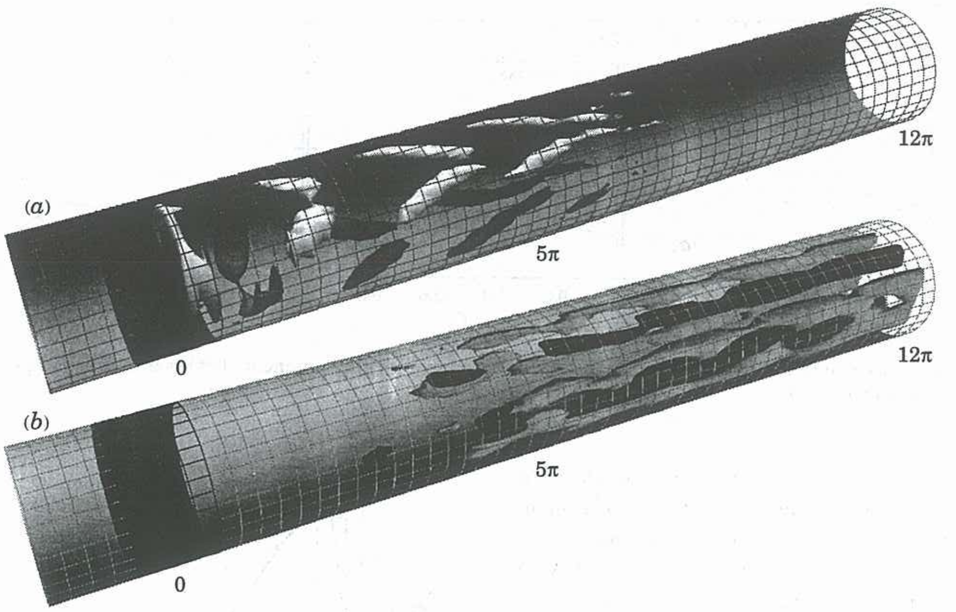

Figure 10: Instantaneous vortices and streamwise streaks for the case of the disturbance amplitude $A_v = 0.031$

close to transition.

A visualization of the flow near transition is shown in Figure 10. Based on this figure we propose the following transition scenario. The periodic blowing and suction generates the fundamental modes $(\pm1, \pm1)$ which have the form of two streamwise counter-rotating vortices which appear alternately at the top and bottom of the pipe as shown in the top of figure 10. By self-interaction these vortices generate the harmonics. The $(\pm2, 0)$ modes which have the shape of low speed streaks as shown in at the bottom of figure 10, are amplified due to transient growth. After the transient growth these low speed streaks may become so strong that they become unstable themselves and the flow becomes turbulent.

CONCLUSION

Based on experimental observations and numerical simulations we have studied the transition of cylindrical pipe flow.

The experiments show multiple transitions. The scaling of the critical amplitude is found to follow a power law with exponents -1 and $-2/3$.

The numerical simulations have been used to study a development of a puff and slug. They show that within the slug the flow is already close to fully developed turbulence. Based on the numerical simulation we propose as possible transition scenario the creation of streamwise vortices which by a process of self-interaction create low-speed streaks. The low-speed streaks grow by the linear process of transient growth until they become unstable themselves leading to turbulence.

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