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PHYSICAL APPRAISAL OF SOME CLASSIC FLOWS IN TERMS OF BOUNDARY GENERATED VORTICITY

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ABSTRACT

Qualitative reconsideration of classic and other flows in terms of boundary generated vorticity may add insight both into the physical nature of particular flows and into the more general behaviour of vorticity at and near boundaries. It can also help to correct misconceptions that may not have been obvious from the velocity fields alone. The flows discussed involve aerofoils, bound vortices, vortices around surface mounted obstacles, mass sources, and vortex rings propagating towards boundaries and through mesh screens.

INTRODUCTION

Vorticity satisfies the Helmholtz equation within fluids, but solutions also require boundary conditions. These are seldom discussed, cannot be deduced from the differential equations, but must be prescribed from an understanding of the physics of vorticity. Simple laws for the generation and decay of vorticity at rigid boundaries in homogeneous flows have been presented (Morton, 1984) but not always accepted. Their acceptance may perhaps be helped by application of the laws either to augment physical understanding of, or to resolve misconceptions that have been associated with classic and other flows. The briefest discussion only will be given here so that a broad range of flows can be introduced. Should this account be found of use then a more detailed analysis may follow elsewhere.

Properties of Vorticity

The following properties are necessary for consistency in solutions to some of the simpler viscous flows (Morton, 1984). They provide a satisfactory basis for understanding a very wide range of heterogeneous as well as homogeneous flows. In *homogeneous* flows, vorticity is generated only at rigid boundaries where tangential pressure gradients act or where those boundaries suffer tangential acceleration, and at curved free boundaries. Generation occurs in an exceedingly thin layer of fluid at the boundary, but vorticity remains always in the fluid and can neither be lost to nor enter from bounding walls. Viscosity plays no part in generation but an essential part in the transport of vorticity away from boundaries. Vortex sheets, corresponding in the inviscid limit to the boundary layers that obtain at lower Reynolds numbers, cannot in general correctly model separation from their boundaries, limiting the usefulness of inviscid fluid dynamics. Decay is due solely to cross-diffusive annihilation of vorticity of opposite senses within the fluid and, in particular, vorticity cannot diffuse to and be lost at rigid boundaries. The kinematic viscosity ν is large among diffusivities, and vorticity may diffuse very rapidly where gradients are high (c.f. Fohl and Turner, 1975, on colliding vortex rings). Dyes and smoke used for visualisation have diffusivities typically three orders of magnitude smaller than ν, and dye traces in liquids or smoke traces in air may persist as 'fossils' long after any initially associated vorticity has dispersed. Thus dye or smoke traces do not in themselves satisfactorily identify the continuing presence of vorticity.

The circulation per unit length of a stationary boundary through the full thickness of the boundary layer is $\underline{n}\times\underline{V}$ where $\underline{n}$ is the unit normal vector to the boundary and $\underline{V}$ the free stream velocity above the boundary layer. The rate of generation of tangential vorticity due to acceleration $d\underline{V}_0/dt$ of a boundary is $-\underline{n}\times d\underline{V}_0/dt$ where $\underline{V}_0$ is the boundary velocity; and the rate of generation at a boundary due to pressure gradient ∇p is $-\rho^{-1}(\underline{n}\times\underline{\nabla})p$. Precisely the same generation mechanisms operate at boundaries in *heterogeneous* flows, but within such flows there is also buoyant generation of vorticity, fully accounted for in the appropriate Helmholtz equation. Uncertainty in handling vorticity arises only at boundaries where, of course, the Helmholtz equation does not apply.

The Blasius boundary layer on a semi-infinite flat plate in a uniform parallel stream U has a line singularity at its leading edge, $x=0$, at which vorticity is generated and advected downstream. In the absence of pressure gradients (except round the curved leading edge of an actual plate which is represented by the leading edge singularity) over a stationary plate, there is no further generation of vorticity at the plate surface ($x>0$) and no loss by cross-diffusion as all vorticity throughout the boundary layer is of the same sign. Thus the boundary layer thickens progressively downstream by diffusion of vorticity alone and both the circulation per unit streamwise length through the full depth of the boundary layer and the flux of vorticity within the boundary layer are invariant.

In an inviscid flow over a stationary boundary a free slip velocity U corresponds to a boundary vortex sheet of circulation U per unit streamwise length. The associated vorticity may be regarded as suffering convection with the flow at speed $U/2$, which is the mean of the speeds on either side of the vortex sheet. It follows that the gross flux of vorticity associated with the vortex sheet may be taken as approximately $U^2/2$.

Plane Couette and Poiseuille Flows

These flows exemplify the rules relating to the generation and decay of vorticity. Couette flow between parallel plates may be regarded as evolving in time. All vorticity is generated at the initiation of flow by tangential acceleration of one (or both) plates. Thereafter diffusion leads to the steady asymptotic state comprising a uniform distribution of vorticity (uniform velocity gradient) which is permanent and in which fluid neither gains nor loses vorticity. Poiseuille flow evolves under a pressure gradient between two stationary semi-infinite parallel plates. Vorticity of opposite signs is generated in an entry length at the two plate surfaces, producing two boundary layers which thicken downstream until they meet, after which vorticity is lost by cross-diffusion over the centre plane. Annihilation continues along the mid-plane downstream, but the steadiness of flow is ensured by continuing generation in the pressure gradient along each of the two plates (at which outward normals and vorticity generated are opposite in sense).

APPLICATIONS

The flows to be considered range from 'well-known' and well understood, such as flow past an aerofoil at small incidence, through well-known and perhaps less well understood, such as horseshoe vortices round the base of an obstacle mounted on a plate aligned with the stream in a wind tunnel or water channel, to less well-known and poorly understood, such as the propagation of a vortex ring through a gauze or wire mesh.

The Kutta-Joukowski Condition

This condition specifies in terms of free-stream speed the circulation κ around a two-dimensional aerofoil at small incidence for which the stagnation point on the upper surface is replaced by smooth merging with small offset angle of the upper and lower streams over the sharp trailing edge, as observed in practice. Flow on the streamline *attaching* to the aerofoil is irrotational but there is continuous generation of vorticity of one sign from the pressure maximum of the stagnation point of attachment to the pressure minimum and of the other sign from the pressure minimum to the next pressure maximum corresponding to the *separation* stagnation point. Thus the circulation per unit streamwise length of surface at first increases and then decreases. At the separation stagnation point on the upper surface the tangential components of both velocity and vorticity at the boundary change sign through zero and the flux of vorticity along the aerofoil passes through zero. No further generation of vorticity is possible on the separation streamline along which the flow is irrotational. Thus the inviscid flow cannot in general serve as a model for a thin separated wake because the flux of vorticity from either side of separation to that stagnation point is zero.The sharp trailing edge, however, proves to be a weaker singularity with neither velocity nor vorticity zero; even in the large Reynolds number limit there are non-zero equal and opposite fluxes of vorticity from upper and lower surface vortex sheets past the trailing edge of the aerofoil, which is the only surface location in this limit permitting non-zero fluxes of vorticity from either side and hence providing an in-principle acceptable limit flow for wake formation behind aerofoils at finite Reynolds numbers. It is the only inviscid flow which comes near to simulating separation.

'Bound' or Lifting Line Vortices

These have played a role in analysing the lift on thin wings of finite length. They have provided important insights which may, however, be complemented by consideration of the patterns of vorticity generation and advection over the surface of finite lifting wings. The shear layer generated over a horizontal wing surface and shed as a thin wake may be represented by a succession of discrete closed vortex filaments which form continuously over the surface and are advected into the wake. At zero lift these take the form of a series of vertical ovals, and as the inclination (and lift) are increased the ovals are increasingly skewed forming a streamwise stack, advanced above and retarded below the wing. Averaged vertically, these have mean forward vorticity everywhere to the left of the plane of symmetry and backward vorticity to its right. This representation emphasises: (i) that vorticity is continuously generated at and advected downstream from the entire surface of the wing; (ii) that vortices rendered visible by water droplets in the atmosphere or dye in the laboratory are often the local result of distortions of a more extensive (if less interesting) vorticity field; and (iii) that the vortex pairs observed trailing in the wakes of aircraft have "roots" representing continuous generation over every part of the wing surface. There is no way in which vorticity can be "bound" to a non-rotating surface. The flow field of a single line vortex has infinite moment of momentum and infinite kinetic energy per unit length; single line vortices cannot be created by finite action in finite time and vortices must form in pairs with zero net circulation at distance. As a wing is accelerated into steady motion a starting vortex is shed into its wake, equal and opposite in circulation to that generated around the wing; thereafter in steady flight the fluxes of circulation into the wake from upper and lower wing surfaces, respectively, are equal and opposite and a balance is maintained between surface generation and downstream advection preserving the circulation excess κ in transit in the boundary layer.

A Cylinder Set Impulsively Rotating about its Axis

A cylinder with radius a set impulsively rotating about its axis with angular velocity ω provides one of the few genuine examples of a "bound" vortex. After the initial instant, the *cylinder* has tangential surface velocity $a\omega$ and effective circulation $2\pi a^2\omega$, (which is the circulation, taken positive, of the ring of fluid in contact with the cylinder). The vorticity generated in the fluid by acceleration of the cylinder surface occupies a contiguous thin sheet and is wholly opposed in sense to ω (i.e. is negative) and has gross circulation $-2\pi a^2\omega$ exactly balancing the initial circulation of the rotating cylinder. Thus at the initial instant the gross circulation in any circuit looping the cylinder is precisely zero and the entire body of fluid is at rest. As the flow evolves, negative vorticity diffuses radially outwards from the surface of the cylinder exposing increasing positive circulation in inner circuits, while circulation in sufficiently distant circuits is positive but exponentially small and that at infinity remains zero for all finite time, nicely side-stepping the impossibility of generating a single line vortex.

Horseshoe Vortices

Horseshoe vortices may be observed when the flow past a vertical cylinder mounted in the boundary layer on the floor of a water channel is visualised by releasing dye upstream in the plane of symmetry. In general they form a pattern of vortices in the boundary layer ahead and to either side of the cylinder, although they are absent from the highly diffusive flow at low Reynolds numbers. As the Reynolds number is increased, however, a "horseshoe-like" vortex appears wrapped around the cylinder with arms trailing downstream. With further increase a second vortex with opposite sense of rotation appears beside the first and then a third with the same sense as the first and so on, with as many as five vortices appearing before boundary layer turbulence largely obscures the pattern. The traditional explanation for this well-known phenomenon is that filaments of vorticity representing the boundary layer are advected towards the cylinder and trapped upstream of it while their arms are carried downstream to form a horseshoe, which is stretched and thereby maintained as a vortex. The traditional explanation is, however, a nonsense: for the circulation per unit length of boundary layer is U and its rate of advection in the boundary layer approximately $U/2$; hence the flux of vorticity in the boundary layer towards the cylinder is $U^2/2$, and as these vorticity filaments can neither be severed nor pass over the top of a (high) cylinder the trapped circulation is of order $U^2t/2$, where U represents the free stream speed and t is the time since motion started. According to this argument, the trapped circulation is unbounded and steady flow impossible! Moreover, the traditional argument cannot account for the vortices observed with sense opposite to that of the boundary layer. The traditional explanation has, of course, ignored the vorticity generated in the adverse pressure gradient upstream of the cylinder at a gross rate equal and opposite to that at which vorticity is imported in the boundary layer. Thus the net flux of vorticity to the cylinder is actually zero. However, the (positive) flux with the stream is distributed throughout the boundary layer while the negative vorticity is all generated at the wall, with the result that local concentrations may survive for some distance before cross-diffusive annihilation is complete. These concentrations will suffer stretching and if rendered visible with dye will appear as discrete vortices although they are more of the nature of transients in which the dye (of small diffusivity) persists downstream long after all circulation has decayed.

Symmetric mass sources

Mass sources are commonly regarded as inviscid idealisations, unattainable in the laboratory where flow always separates at the inlet orifice to form a (viscous) jet. A two-dimensional mass source can, however, be produced, if with some difficulty, in a viscous laboratory fluid from a source comprising parallel rubber belts driven at precisely the (uniform) inlet speed of the source fluid. In this case there are no boundary layers on the rubber belts and no flow separation at discharge from the source; streamlines adjacent to a belt do not separate but emerge to follow the containing wall beyond the orifice. For a source orifice in a tank wall, a jet is produced when both belts are stationary; a symmetrical diverging (source) flow when both belts are driven at source speed; and when one belt is driven and one stationary the emerging stream separates from the corner of the stationary belt and is deflected along the wall past the corner of the moving belt.

A Vortex Ring Approaching a Plane Boundary

A vortex ring approaching a plane boundary along a normal is observed to grow somewhat in ring diameter (d) and then to "freeze", that is to come suddenly and completely to rest at distance of order a diameter from the boundary. The propagation speed of the ring is proportional to κ/d, and zero velocity necessarily implies zero circulation (κ). This is in marked contrast to the traditional inviscid solution, in which the ring moves at constant circulation as though under the unencumbered influence of an equal and opposite vortex which is its image in the plane, according to which the ring increases in diameter unboundedly as it moves ever nearer the boundary. The striking difference between observation and inviscid solution results from neglect of the generation of vorticity in the pressure gradient generated inertially at the boundary as the ring approaches. A pressure high is produced where the symmetry axis of the ring intersects the boundary, with pressure gradients directed radially towards this point. The vorticity generated is azimuthal with sense opposite to the ring. This vorticity is swept up by the approaching ring, carried around it and entrained into its rear, then carried forward and finally wound into the core of the vortex producing annihilation of circulation by cross-diffusion in the relatively high gradients of the core. The gross circulation produced at the wall must be equal and opposite to that of the ring before impact begins for total cancellation.

Collision at slant impact of two vortex rings

Fohl and Turner (1975) have reported an experiment on the slant collision of two vortex rings which deserves to be better known than perhaps it is. They used two small impulse vortex generators in a tank of water to generate simultaneously two equal laminar vortex rings, one dyed red and one blue, so that they propagated along symmetrically inclined intersecting paths in the vertical plane through the generators. On collision, the lowest limbs of the two vortices interacted first and there followed an extremely rapid reorganisation without change in vertical momentum, from which emerged a single vortex ring, half red and half blue with sharply defined colour junctions, and which propagated along the forward bisector in the plane of the two approach paths. Higher speeds of impact resulted in two rings, each half red and half blue, which propagated symmetrically out in a plane at right angles to the plane of approach. Although the collision process was rapid, we may judge that the lower limbs of the colliding vortices on close approach respond as a vortex pair propagating downwards, probably becoming unstable, wrapping round each other and creating large but highly localised gradients where cross-diffusion produces reconnection. The pressure field of the vortex core then restores the deformed filament(s) into ring(s). The "junction points" at which diffusive reconnection has taken place are shown clearly by the sharp colour transition, although it should be emphasised that there is no question of cutting vortex filaments. It is difficult to grasp the speed at which cross-diffusive reconnection of vortex filaments with opposite sense can occur, but the Fohl and Turner experiment provides convincing evidence that it does so.

Passage of a vortex ring through a fine wire mesh

At low Reynolds numbers, a vortex ring impacting normally on a fine wire mesh senses the mesh as a rigid wall and exhibits the characteristic behaviour described above. At moderately larger Reynolds numbers the ring passes through the mesh, shedding some material because of the negative impulse received, and if care is taken to prevent instability and breakup *may continue as a smaller but still coherent vortex ring*. This remarkable piece of vortical microsurgery can be explained qualitatively in terms of boundary generation and cross-diffusive reconnection of vorticity filaments. At larger Reynolds numbers still, vortex rings may have partially turbulent cores and on passage through a mesh are likely to experience breakup, sharing residual momentum with ambient fluid and suffering substantial loss of energy through turbulent dissipation.

When a vortex ring or pair approaches an impervious wall its loss of momentum corresponds with an inertial increase in pressure. There are two consequences: (i) a source-like flow is driven back from the wall and to the sides; and (ii) azimuthal vorticity is generated in rings centred on the point of intersection of the propagation axis with the wall (or in the appropriate pairs for an incident vortex pair). When, however, the ring (or pair) approaches a permeable mesh or grid, part only of its impulse is converted to an increase in pressure and the remainder is carried with fluid through the mesh holes. At low Reynolds number the viscous resistance to flow through the "pores" is high and there is negligible passage of momentum through the screen, which therefore acts in essence as an impervious wall. As the Reynolds number is increased an increasing proportion of the incident momentum is carried by "pore flow" and a decreasing proportion is realised in the upstream pressure field. Such inertial pressure excess as remains on the upstream side has two effects relevant to our consideration. Fresh vorticity is generated in the pressure *gradient* over faces of the wires , bars or other impermeable parts of the screen; and fresh vorticity is generated also at the sides of the wires and bars as the pressure *difference* between the upstream and downstream faces of the screen drives fluid through the "pores". To understand the consequences of this fresh generation of vorticity, consider one line core of a vortex pair which is parallel to, approaching, and much closer to a plane screen than the separation of the two cores. The line of pressure high on the wall will lie not directly under the core axis but displaced slightly towards the other line vortex of the pair. The vorticity generated directly under the approaching line core at facing parts of the screen will be opposite in sense to the core. The pressure difference between up- and down-stream faces will drive fluid through a "pore" , with two consequences for vorticity. Part of the approaching vortex core will be carried through the screen in a forward loop and fresh vorticity will be generated in azimuthal rings over the sides of the "pore" which will be advected out in association with the mini-jets that penetrate the screen and emerge downstream of it. There will also be generation at the "pore" sides of vorticity normal to the screen associated with the loops of incident vortex core as they are advected through the screen. Finally, the forward loops have associated tangential flow over the downstream face of the screen, with corresponding surface vorticity.These distributions of vorticity and the vortex filaments with which they may be related will be illustrated and described in greater detail, and it will be shown that as the filaments are drawn out over each other a series of cross-diffusive reconnections may be expected which will leave at the screen a series of narrow closed vorticity filaments that will quickly annihilate together with a reconstituted line filament downstream of the screen.

CONCLUSION

The rules for the generation of vorticity at flow boundaries presented previously are both confirmed and add insight to a series of applications which have been introduced and discussed briefly.

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