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SIMULATION OF STRATIFIED FLOW AROUND A SQUARE CYLINDER USING THE *RNGK-ε* TURBULENCE MODEL

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ABSTRACT

The aim of this paper is to investigate the effect of a stable linearly stratified ambient on vortex shedding, circulation and lee wave generation for a two dimensional turbulent flow past a square cylinder. The full unsteady equations of motion along with the ***RNG*** $k-\varepsilon$ turbulence model are solved using a ***SIMPLE*** method with a third order ***QUICK*** discretization for the advective terms on a non-uniform, non-staggered grid with second order Crank-Nicolson time integration. It is shown that the importance of internal waves increases as the stratification increases. A decrease in the non-dimensional frequency of vortex shedding (Strouhal number) and the amplitude occurs with an increased stratification until the complete domination of lee waves.

INTRODUCTION

An exciting recent approach in turbulence modeling is the derivation of a range of turbulence models using Renormalization Group (***RNG***) methods, an effort that was started by Yakhot and Orszag (1986). The ***RNG*** $k-\varepsilon$ model has many advantages compared to standard models and has been shown applicable to flows where Reynolds stress models (***RSM***s) were previously thought to be the only viable approach. Some of the advantages of the ***RNG*** $k-\varepsilon$ model are as follows.

- Low Reynolds number flows may be predicted, so wall functions are no longer required.
- Stratification and rotation (swirl) effects are accounted for by extensions of the ***RNG*** theory to stratified and rotating flow.
- Heat transfer effects are properly handled (no wall function) and Prandtl number effects are taken into account

In this paper results obtained with the ***RNG*** $k-\varepsilon$ model, for flow over a square cylinder with a stratified ambient, will be presented and discussed. The standard non-dimensional control parameters for the flow are the Reynolds number,

$$Re=Ud/\upsilon ,$$

where U is the free stream velocity, d the cylinder width and υ the kinematic viscosity, and the Froude number,

$$F_i = U/Nd ,$$

where N, is the Brunet-Vaisala frequency, is defined as

$$N = \left((-g / \rho_0)(\partial\rho / \partial y) \right)^{1/2} ,$$

with ρ_0 the mean density, g gravity and y the vertical direction.

It is well known that for high enough Reynolds number a vortex shedding wake occurs in non-stratified flow over cylinders. For stratified flow the vortex shedding is influenced by the strength of the stratification, and the critical Reynolds number for the occurrence of shedding is increased for stable stratifications, indicating that vortex shedding is suppressed. Pao *et al* (1968) proposed an empirical stability parameter, as,

$$\kappa = 1/F_i \log(\mathrm{Re}/40) ,$$

that could be used to predict the occurrence of vortex shedding for stratified flows with Re>40. They noted that for a two dimensional circular cylinder vortex shedding would only occur for $\kappa<1$. This was verified by Honji (1984) who observed that the vortex street tend to collapse into internal waves in the near region downstream of the cylinder.

The results presented here were obtained in a finite domain of height 2*D*, which introduce an additional control parameter $K=ND/\pi U$. K parameterises the influence of the vertically bounded domain on the internal wave speed, and appropriately classifies the flow as supercritical ($K<1$), where all internal waves have propagation speed less than U and are consequently swept downstream, and subcritical ($K>1$) where some internal wave modes have propagation speeds greater than U, leading to stationary or upstream travelling waves.

Numerical studies have been carried out for laminar sub-critical flow past a plate perpendicular to the flow by Hanzaki (1989) and Castro (1994), who observed similar behaviour to that described by Pao *et al*. Apparently little attention has been given to supercritical turbulent flow past a bluff body. In the present study results have been obtained for Reynolds numbers in the turbulent range, 1000<Re<22000, for supercritical flow defined by $K<1.0$.

EQUATIONS OF MOTION

The time averaged governing equations of motion for an incompressible, viscous fluid in two dimensions, with the Boussinesq approximation for buoyancy, in dimensionless form, are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(2\upsilon_{eff}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right), \quad (1)$$

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$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left(2\upsilon_{eff}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\upsilon_{eff}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\frac{\theta}{Fr}, \quad (2)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr_t}\nabla\cdot(\upsilon_{eff}\nabla\theta), \quad (3)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (4)$$

in which u and v are the velocity components in the x and y direction respectively, t is the time, θ is the temperature and p is the pressure.The turbulent Prandtl number is defined as:

$$Pr_t=\upsilon_{eff}/\alpha_t,$$

and the Froude number is here defined as

$$Fr=U/2ND,$$

where U, N and D are as above, α_t is the turbulent thermal diffusivity and the effective viscosity is defined as

$$\upsilon_{eff}=\upsilon\left[1+\sqrt{\frac{C_\mu}{\upsilon}\frac{k}{\sqrt{\varepsilon}}}\right]^2,$$

where k is the turbulence kinetic energy and ε is the dissipation of turbulence kinetic energy.

In the ***RNG*** $k-\varepsilon$ model the energy and the dissipation equations are written as:

$$\frac{\partial k}{\partial t}+u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\upsilon_{eff}S^2-\varepsilon +\frac{1}{\sigma_k}\nabla\cdot(\upsilon_{eff}\nabla k)-\frac{1}{Fr}\frac{\upsilon_{eff}}{\sigma_t}\frac{\partial\theta}{\partial y}, \quad (5)$$

$$\frac{\partial \varepsilon}{\partial t}+u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=C_{\varepsilon1}\frac{\varepsilon}{k}\upsilon_{eff}S^2-C_{\varepsilon2}\frac{\varepsilon^2}{k}+\frac{1}{\sigma_\varepsilon}\nabla\cdot(\upsilon_{eff}\nabla\varepsilon) -\left(\frac{C_{\varepsilon1}(1-C_{\varepsilon3})}{\sigma_t}\right)\frac{\varepsilon}{k}\frac{\upsilon_{eff}}{Fr}\frac{\partial\theta}{\partial y}-R, \quad (6)$$

in which

$$S^2=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),$$

is the magnitude of the rate of strain and the rate of strain term R is

$$R=\frac{C_\mu\eta^3(1-\eta/\eta_0)}{1+\beta_i\eta^3}\frac{\varepsilon^2}{k},$$

where $\eta=Sk/\varepsilon$.

The *RNG* theory gives values of the constants as

$C_{\varepsilon1}=1.42$, $C_{\varepsilon2}=1.68$, $C_{\varepsilon3}=0.8$, $\sigma_k=\sigma_\varepsilon=0.719$, $\sigma_t=0.5$, $C_\mu=0.0845$, $\eta_0=4.35$ and $\beta_i=0.015$.

Equations (1) to (6), together with appropriate boundary conditions, fully describe the flow.

NUMERICAL METHOD AND BOUNDARY CONDITIONS

The governing equations are discretized using finite volumes with second order, central differences for all but the advective terms, for which a third order *QUICK* scheme is used. Time integration is accomplished via the second order Crank-Nicolson scheme, with the pressure obtained by the solution of a Poisson equation for the pressure correction. A detailed description of the scheme is given in Armfield (1994).

The computational domain (Figure 1) consists of a channel of height $2D$, with no-stress, impervious boundaries and zero normal gradients of k and ε on the top and the bottom. The temperature θ is set to 0.5 at the top and –0.5 at the bottom. A square cylinder of side d is located at the mid-height and at a distance of $10\,d$ from the inlet and $20\,d$ from the outlet boundary. The inlet boundary conditions are u=1.0 and v=0 and zero k and ε, that is a uniform horizontal velocity. The tangential gradient of temperature θ is set to 1.0 at the inlet. For the outlet the normal gradient of all variables is set to zero. Since the generation of lee waves when K is close to 1 is inevitable, special care must be taken at the outlet boundary to avoid wave reflection. Therefore a buffer zone, propose by Streett & Macaraeg (1990), which damps the wave reflection smoothly, is employed at the outlet boundary.

The accuracy of the numerical results depends strongly on the resolution of the boundary layer near the square cylinder walls, so a non-uniform grid, which clusters nodes in the region of the cylinder walls, is used. The finest grid has a magnitude of $0.00165\,d$ in the y direction and $0.0033\,d$ in the x direction adjacent to the square cylinder. The grid is stretched smoothly away from the cylinder using a linear relation. To test the grid dependence solutions were obtained for several grid sizes, (95×59, 129×95, 188×127), and the Strouhal number (*St=fd/U*, in which f is the frequency of shedding) and drag coefficient C_d computed. It was found that the changes in Strouhal number and drag coefficient C_d are about 4% moving from the 95×59 to 129×95 grid and about 1% moving from the 129×95 to 188×127. Therefore, the 129×95 grid was considered sufficiently accurate and used to obtain the results presented here. Results are presented for time step $\Delta t=1\times10^{-4}$, results were also obtained for $\Delta t=5\times10^{-4}$ with no observable variation.

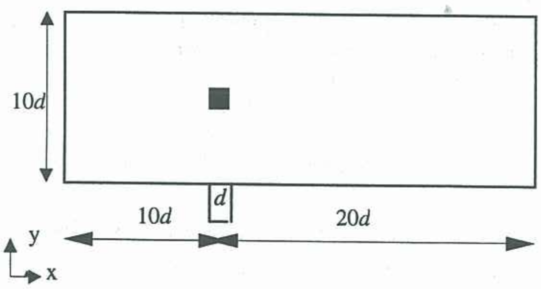

Figure 1: Computational domain.

RESULTS AND DISCUSSION

To test the code several cases have been run with no stratification, using different Reynolds numbers, to obtain the Strouhal number to compare with previous experimental results. Figure 2 illustrates the comparison of Strouhal number obtained with the ***RNG*** $k-\varepsilon$ model and previous experimental investigations by Davis *et al* (1982) and Okajima (1982).

It can be seen that the results of Davis *et al* and Okajima are not in good agreement for Reynolds numbers in the range $10^2<\mathrm{Re}<4\times10^3$. For Reynolds numbers above 4×10^3 the two sets of experimental data agree well. The numerical results obtained here are in a good agreement with the experimental results of Davis *et al*.

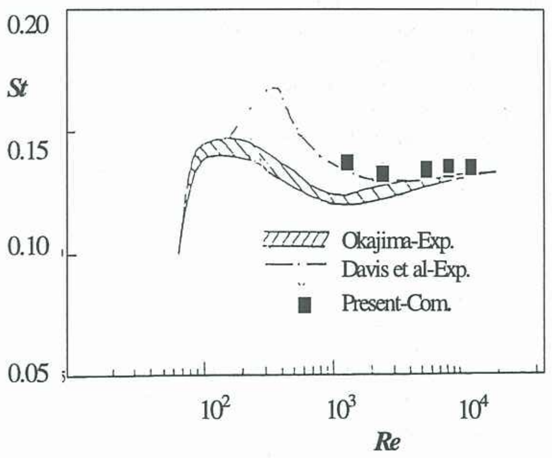

Figure 2: Comparison of Strouhal number with previous result in non-stratified flow.

Another comparison has been carried out at *Re*=22000. This case has been studied experimentally by Lyn (1990) and numerically, using the standard two layer model and the ***RSM*** model, by Franke and Rodi (1993) and using the large eddy simulation (***LES***) model by Murakami *et al.* (1992). The comparison, which is shown in Table 1, has been carried out for the Strouhal number, mean drag coefficient, C_d, and the amplitude of the drag and lift coefficients, $\tilde{C}_d, \tilde{C}_l$.

The standard $k-\varepsilon$ model and two layer ***RSM*** predicted too low and too high a shedding frequency respectively. While the ***RSM*** model with wall function and the present ***RNG*** [illegible] model predict the Strouhal number in good agreement with the measurements of Lyn. The values of the mean drag coefficient show that the results obtained with the ***RSM*** model with wall function and the ***LES*** and ***RNG*** $k-\varepsilon$ model are in better agreement with the measurements of Lyn, While the results obtained with the two layer ***RSM*** are too high and the results obtained with the two layer $k-\varepsilon$ are too low compare to the Lyn measurements.

	Two layer k-ε	*RSM* wall function	Two layer *RSM*	*LES*	*RNG* k-ε	Experiment (Lyn)
St	0.124	0.136	0.159	0.132	0.137	0.135
C_d	1.79	2.15	2.43	2.10	1.97	2.05-2.23
$\tilde{C}_d$	0.0	0.383	0.079	0.12	0.046	_
$\tilde{C}_l$	0.323	2.11	1.84	1.58	0.675	_

Table 1: Comparison of Strouhal number *St*, mean drag coefficient, C_d the amplitude of drag $\tilde{C}_d$, and the amplitude of the lift coefficient $\tilde{C}_l$ for different turbulence models and experiment.

Stratified Flow

The behaviour of vortex shedding behind a square cylinder has been investigated for turbulent flows with $1000\,P\,\mathrm{Re} \le 22000$ and different stratifications with $0<K<1$. The streamlines for *Re*=5000 with *K*=0.32, *K*=0.6, *K*=0.84 and *K*=1.0, all at time t=7 can be seen in Figure 3.

The reduction in the amplitude of the vortex shedding with increasing stratification is observed. At *K*=0.84 the partial suppression of vortex shedding and the formation of lee-waves with long wavelength is observed. When the integration is continued for *K*=0.84, by time t=12 vortex shedding is completely suppressed. For $K\leq0.6$ continued integration has shown the vortex shedding is maintained.

The influence of the stratification on the vortex street may also be seen in Figure 4, which contains plots of the instantaneous circulation in a region downstream of the cylinder ($20\,d<x<25\,d$, $-D<y<D$) for a range of *K* values. It may be seen that the amplitude of the circulation oscillations is reduced as *K* increases and that for *K*=0.6 and lower the amplitude reaches a steady state by approximately time t=1.2.

The variation of Strouhal number and circulation amplitude with increasing stratification is shown in Table 2. It can be seen that the Strouhal number has a constant value of *St*=0.122 up to and including *K*=0.6, where there is maintained vortex shedding. For $K\geq0.84$ there is a significant change in the behaviour of flow particularly in the near region of square cylinder. At *K*=1.0 the vortex shedding is suppressed for large time. The Strouhal number shown here is based only on the early part of the integration for which vortex shedding occurs. For $0.84\leq K\leq1.0$ the Strouhal number is 0.09.

There is a significant *K* related influence on the amplitude of the circulation in the region downstream of the square cylinder. The amplitude of circulation decreases continuously with increasing stratification until at *K*=1.0 it has only 1.4% of its magnitude in the unstratified flow (*K*=0.0).

Pao's stability parameter is $\kappa = 0.299$ at *K*=1.0 and *Re* =5000. As noted above for this *K* the complete suppression of shedding occurs at t=12. It is clear that Pao's criterion that shedding occurs for all values of κ less than unity, which was obtained for circular cylinders, is not applicable to square cylinders.

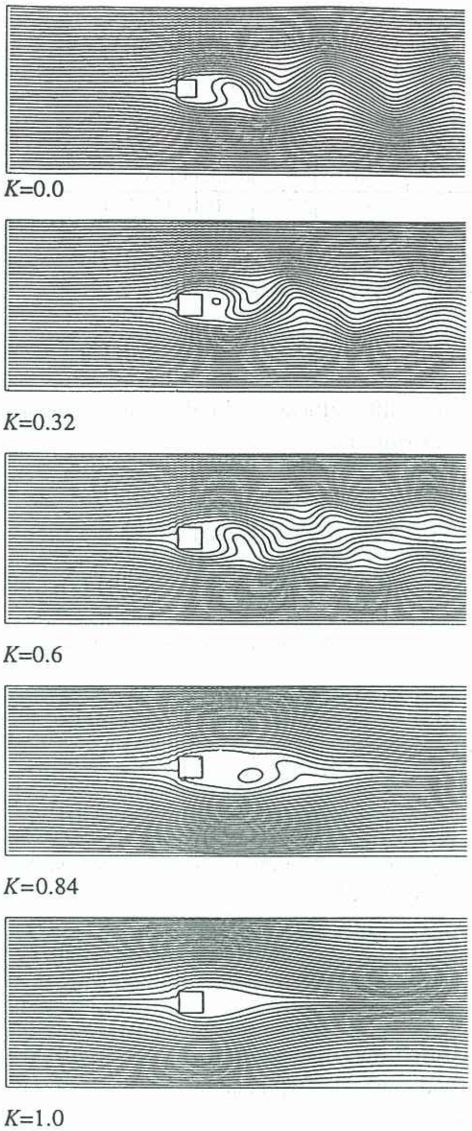

Figure 3: Behaviour of streamlines at different stratification at time t=7 for Re=5000.

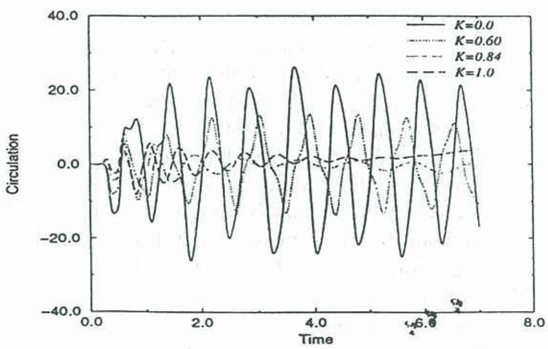

Figure 4: Variation of circulation with time at different stratification for Re=5000.

	St	Circulation amplitude
K=0.0	0.122	17.20
K=0.32	0.122	17.17
K=0.60	0.122	10.30
K=0.84	0.09	2.13
K=0.90	0.09	1.40
K=1.0	0.09	0.257

Table 2. Variation of Strouhal number and amplitude of circulation with K for Re=5000.

CONCLUSION

The *RNG* $k-\varepsilon$ turbulence model, which has been used for the investigation of flow around a square cylinder, shows better prediction of experimental results for unstratified flow than the standard $k-\varepsilon$ turbulence model, and performs as well as the sophisticated wall function ***RSM*** and ***LES*** models. In stratified turbulent flow around a square cylinder results have been presented for Re=5000 showing that stratification inhibits the shedding and that at K=1.0 shedding is entirely suppressed and lee waves dominate the flow behaviour. Additional results for $1000 \leq Re \leq 22000$, which for brevity have not presented, have also demonstrated this behaviour.

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