*13th Australasian Fluid Mechanics Conference*
*Monash University, Melbourne, Australia*
*13-18 December 1998*

# Do particles segregate themselves?

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## ABSTRACT

Metal disks of varying sizes and density are placed at the bottom of a bed of small granular material. We sinusoidally shake the entire system in the vertical direction, and observe that, if the angular acceleration of the shaking is slightly greater than that of gravity, the metal disks rise to the top of the bed. This result has been known for sixty years, but a simple explanation has not been produced, so far.

By using tracer particles, we have shown that this behaviour is not due to any bulk convective motion of the bed, but arises from the disk penetrating through the bed. We use this observation to derive a simple equation that accounts for the disk's motion.

## INTRODUCTION

Of significant interest in the behaviour of granular materials is segregation, where particles of different size or density tend to separate out of the bed when the granular material is agitated by vibrations, pouring or stirring. This can be demonstrated by placing a large particle at the base of a cup of sand. If the cup is shaken vertically, the nut will rise to the surface of the bed. This form of segregation (often termed the "Brazil Nut Effect") has been noted for many years[1], although researchers still do not agree on the fundamental mechanism(s) for its occurrence[2].

At high enough shaking frequencies, the granular bed actively convects all the particles at the same speed to the top of the granular bed. The large particles remain there, because they are too large to travel along the thin downward sheets of the convection rolls[3]. At lower shaking frequencies the ascent speed of a particle is proportional to its size[4,5]. In this case, the upward motion is thought to be due to smaller particles filling the voids generated underneath the large particles during each shaking event[4,6]

In this paper, we experimentally simulate a vibrating granular bed in (partial) two dimensions, using steel disks to represent the segregating particle. Our experiments, and resulting model, show that the disks push themselves through the bed and that the ascent speed of a particle is approximately proportional on the particle's density, and size, while being inversely proportional to the particle's depth in the bed. We show that the large particle "drags" lower bed material with it thereby contradicting the gap or arch models for particle segregation.

To examine this phenomenon, we constructed the experimental test rig shown in Figure 1. It consists of a perspex box with the internal dimensions $20 \times 20 \times 0.6$ cm. Protruding from the ends of this box are perspex blocks, each with a hole drilled at the ends. Two vertical rods extend through these holes from a solid base, constraining the box to move only in the vertical direction. The box is vibrated by an electric motor connected to a gear box that allows the output speed to be altered. The output shaft from the gear box is attached to the base of the box by an eccentric cam, producing an approximately sinusoidal vibration.

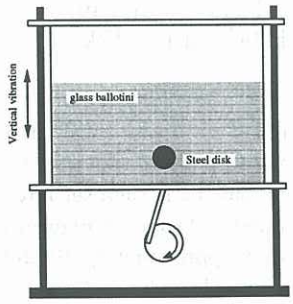



Figure 1: Apparatus for vertical vibration

For our experiment, the bulk particles in the bed were 1 mm diameter glass beads (ballotini). So, the amplitude of vibration to particle diameter ratio is $a/d = 16$. A steel disk was placed at the base of the bed, and the motor switched on. The subsequent behaviour of the disk is shown in Figure 2

Figure 2a shows the initial configuration of the experiment. The disk is at the base of the bed with

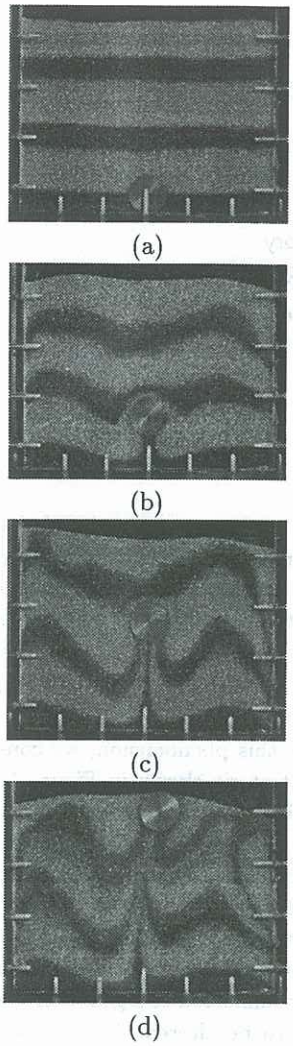



Figure 2: Upward motion of a 30 mm diameter steel disk in a vertically vibrated bed of 1 mm diameter glass beads. Note, in (c), the wake of coloured beads behind the disk

layers of coloured glass beads acting as tracer particles, so that we can observe movements of the bulk material. The apparatus is then set into motion at a frequency of around 250 rpm. In Figures 2b and c, we show the bed after approximately 10 and 15 seconds of shaking, respectively. Convective bulk motion of the glass beads, in the top corners of the bed, has bent the layers into "smile" shapes. The disk, however, is unaffected by this bulk motion and simply punches through the layers, pulling a trail of coloured beads in its wake. Note that this contradicts the generally held theories that a large particle rises by the compaction of small particles that move underneath it[4,6]. After about 20 seconds, the disk arrives at the top of the bed (Figure 2d).

Other researchers have found bulk convection to be the driving force for segregation[3]. The influence of convection on the bed is dependent on the value of the non-dimensional acceleration $\Gamma$, where $\Gamma = a\omega^2/g$, $a$ the amplitude of the oscillation (1.6 cm), $\omega$ the angular frequency, and $g$ the acceleration due to gravity. These experiments typically operate in the range $1.0 \lesssim \Gamma \lesssim 1.2$, where bulk convective motion appears to have little or no influence on the upward motion of the 30 mm diameter steel disk shown in Figure 2. However, one cannot rule out the influence of bulk convection on the motion of smaller disks. In this experiment we used disks with diameters down to 10 mm. The small disks did suffer lateral motion due to convection of the bed and so their upward motion may also have been affected.

To explain this strong upward movement of the disk through the bed, we have proposed a "penetration" model, based on the behaviour of a particle colliding with a semi-diffuse medium. The distance such a particle will travel into this form of medium is such that the mass of displaced material is equal to the mass of the particle. To illustrate how this works, consider a sphere that is thrown into a low density medium. From Figure 3, the sphere will stop once the mass encountered by the sphere is equal to its own mass.

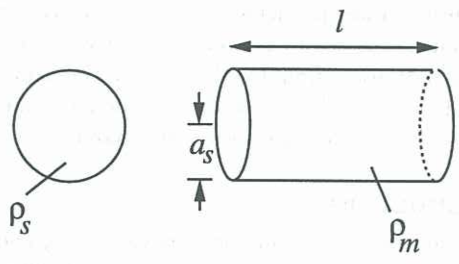



Figure 3: A sphere (density, $\rho_s$, radius $a_s$) impacting on a diffuse medium of density $\rho_m$, penetrates a distance $l$ into the medium

We can, therefore, equate the mass of the sphere to the mass of the displaced cylinder, and so determine the penetration distance:

$$\Delta l = \frac{4}{3}\frac{a_s \rho_s}{\rho_m}, \qquad (1)$$

where $\Delta l$ is the penetration distance, $a_s$ is the radius of the sphere, $\rho_s$ is the density of the sphere, and $\rho_m$ is the density of the medium. This "stopping distance", $\Delta l$, has been noted by researchers conducting high velocity impact experiments of small particles with low density aerogel[7].

The difference between a diffuse medium and the granular bed is the effective mass density of the bed. In a diffuse medium, one can simply take $\rho_m$ as the mass density of the medium. However, in a granular medium a large intruder particle will only see the particles in contact with its surface and these particles will be connected via linear stress chains to many other particles in the bed[8]. So the large intruder particle is subject to an "effective" mass density which must take into account the extra mass from these chains of particles.

In our analysis of the experiment, we assumed that for the disk to move upwards it must move a 'wedge' of material, as shown in Figure 4. Thus, $\rho_m$ is now the mass of the wedge divided by the volume of the 'shell' of particles on the disk surface. The disk cannot move downwards because the bottom of the bed cannot be displaced and so there is an infinite effective mass density for downward motion.

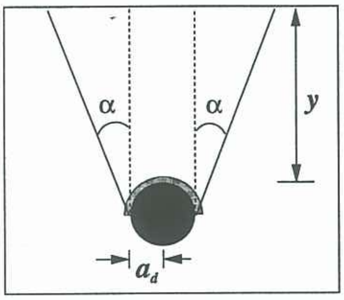



Figure 4: The mass of the wedge of material above the disk gives a large effective mass density in the shell of material directly above and adjacent to the disk.

The width of the shell at the top of the disk is the thickness of the layer of particles in contact with the disk at the disk's surface. We take this thickness to be $2a_p\sqrt{2/3}$, where $a_p$ is the average radius of the bulk particles - in this case the glass beads. This thickness is the distance between the surface of the disk and the second layer of particles away from the disk, where we assume that the first layer of particles is in contact with the disk and the second layer is in contact with the first layer. We also assume that the beads are, to some approximation, hexagonally close packed at the surface of the disk. From this analysis, one can derive an equation for the depth of the disk as a function of time:

$$\left(\frac{\tan\alpha}{3}\right)y^3 + a_d(1+\tan\alpha)y^2$$
$$+a_d^2(\tan\alpha + 2 - \frac{\pi}{2})y + 2\pi\sqrt{\frac{2}{3}}\frac{a_p a_d^2 \rho_d f_S}{\rho_b}t$$
$$-\left(\frac{\tan\alpha}{3}\right)y_i^3 - a_d(1+\tan\alpha)y_i^2$$
$$-a_d^2(\tan\alpha + 2 - \frac{\pi}{2})y_i = 0\ , \qquad (2)$$

where $y$ is the distance between the top of the disk and the surface of the bed at time $t$, $\alpha$ is the angle of the triangular wedge from the vertical, $a_d$ is the radius of the disk, $\rho_d$ is the density of the disk, $\rho_b$ is the bulk density of the bed, $f_S$ is the frequency of shaking and $y_i$ is the initial depth of the disk in the bed.

Figures 5(a), (b) and (c) compare the experimental results for the motion of the disk and the theoretically expected results, from Eqn 2, where we have used a value for $\alpha$ of $7^o$ for Figures 5(a) and (c) and set $\alpha = 6^o$ for Figure 5(b). The $\alpha$ values being determined from the curves of best fit.

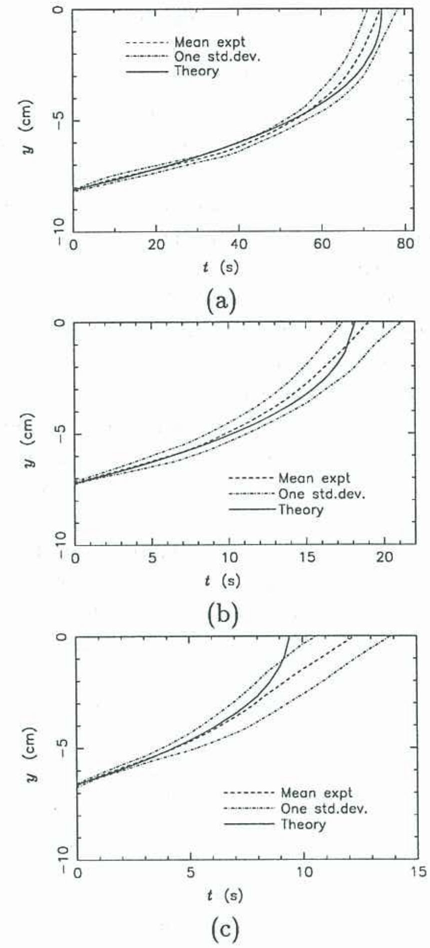



Figure 5: Comparison between experimental and theoretical results for the (a) 10 mm disk, (b) 20 mm disk and (c) 30 mm disk.

Early in the movement of the disk, the theoretical approach seems to fit experimental results well. For later times, however, there is some deviation between the theoretical and experimental lines. This deviation is possibly due to a surface effect. As the disk approaches the top of the bed, the surface gains a "bump" that slows the rise of the disk. This surface effect occurs when the disk is approximately one disk diameter from the top of the bed. For larger disks this bump is bigger, thereby causing a greater deviation from the theoretical prediction, which assumes the granular bed has a flat surface.

The rise velocity of the disk is usually defined as the initial depth of the particle divided by the total time of ascent[5]. Using this definition and Eqn (2), we find that the rise speed is given by

$$v_{\text{rise}} = 2\pi\sqrt{\frac{2}{3}\frac{a_p}{\rho_b}}H(f_S - f_C)a_d^2\rho_d f_S / \left(\tan\alpha\, y_i^2 + 2a_d\,|y_i|\,(1+\tan\alpha) + a_d^2(\tan\alpha + 2 - \pi/2)\right) , \quad (3)$$

where $H(x)$ is the Heaviside function and $f_C$ is the critical frequency for segregation. The Heaviside function has a value of 1 for $f_S > f_C$, and a value of 0 for $f_S < f_C$. So below the critical frequency, $f_C$, no segregation occurs. The critical frequency, $f_C$, is the shaking frequency required to dilate the bed. This occurs when $\Gamma \approx 1$. For our experiment, $f_C$ works out to be approximately 3.94 Hz.

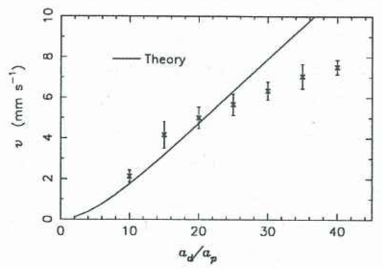
Eqn (3) suggests that the speed of ascent of the disk will increase as the disk approaches the surface of the bed, a result that is most clearly shown in Fig. 5(a). Such a result has also been observed in the experiments undertaken by Duran *et al.*[4]. Eqn (3) also predicts that the speed of the disk's ascent is proportional to the disk density, the shaking frequency, and to the disk size as $y_i$ becomes small. Comparisons of experimental results and theoretical results obtained using Eqn (3) are shown in Figures 6(a), (b), and (c), where we have again used a value for $\alpha$ of $7^o$. Although the agreement between the theory and experiment is not perfect, the experimental results do illustrate the trends predicted from the theory.

To observe the ascent speed as a function of disk size, we measured the ascent speed of steel disks of the same density (7.84 g $\text{cm}^{-3}$), but of different diameters, and as can be seen from Fig. 6(a), the ascent speed is proportional to the disk size. In Fig. 6(a) we have normalised the radii of the disks by dividing them by the radii of the glass beads.

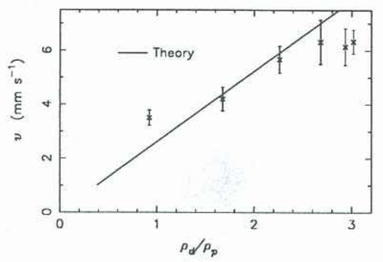
To observe the ascent speed as a function of disk density, we obtained six different density disks of the same size (30 mm diameter), where the densities ranged from 2.19 to 7.84 g $\text{cm}^{-3}$. We normalised these densities by dividing them with the mass density, $\rho_p$, of the glass beads (2.595 g $\text{cm}^{-3}$). As is shown in Fig. 6(b) the ascent speed is proportional to disk density, i.e., the more massive the disk, the faster it will move upwards. This apparently counter-intuitive result is predicted by our theory and is here experimentally verified.

As expected, the frequency results (Fig. 6(c)) show that there is a critical frequency for the movement of the disk and this frequency does occur at $\Gamma \approx 1$. However, the ascent speed of the disk appears to be greater than the simple direct proportionality expected in Eqn (3). This raises the possibility that the bulk density of the bed, $\rho_b$, decreases, with increasing shaking frequency, a property that is not included in our model.

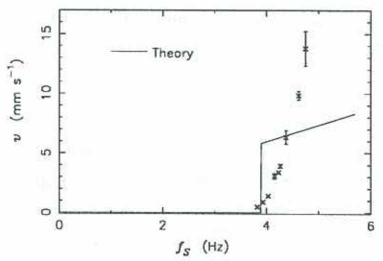
So, why do brazil nuts go to the surface? Our results suggest that, for particulate material, the bigger and denser you are the further and faster you go.




(a)




(b)




(c)

Figure 6: Comparison between experimental and theoretical results for the ascent speed of the disk vs (a) disk diameter, (b) disk density and (c) shaking frequency.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