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THE SCALING OF MEAN VELOCITY PROFILES IN TURBULENT TAYLOR-COUETTE FLOWS

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ABSTRACT

Measurements have been made using a two component LDV system for turbulent flows between two concentric cylinders with the inner cylinder rotating and the outer cylinder stationary at the radius ratios of 2.88 and 7.1, and at large Taylor numbers. The results are compared with those measured in similar flow geometries at smaller radius ratio using hot wires. The mean velocity profiles have been presented using various scalings, and it is found that under the scaling of wall shear velocity and local distance from the cylinder surface (inner scaling), the data collapse better. A power law is found to fit the data well in the entire interior region apart from the two viscous zones near the cylinder surfaces. The data can also be fitted using a logarithmic law similar to that used in unidirectional wall shear flows, but with different constants and within smaller regions.

INTRODUCTION

Taylor (1935) experimentally studied the turbulent flows between two concentric cylinders with the inner cylinder rotating. Using the experimental results from Pitot tubes, he found that with the outer cylinder stationary, the turbulent flow within the gap essentially conserved its mean angular momentum except within the thin layers close to the surfaces of the two cylinders. Since this pioneering work, Brindley, Lessen and Mobbs (1979) and Koschmieder (1979) have studied, mainly through flow visualization, the large scale Taylor vortices in the turbulent Taylor-Couette flows. Nakamura et al. (1981) measured velocities near a cylinder rotating in essentially unbounded fluid, and Smith & Townsend (1982) studied the turbulent Taylor-Couette flows experimentally using hot wires. Smith & Townsend (1982) found that the mean angular momentum within the gap is nearly conserved and proposed that the mean flow near the cylinder surfaces should scale with the inner scales as those in unidirectional wall shear flows such as pipes, channels, and boundary layers over plane surfaces. Barcilon & Brindley (1984) proposed a theory for this flow based on the idea of marginal stability, and they were able to predict the length scale variations of the Gortler vortex with Reynolds numbers.

In this study, mean velocities for the turbulent Taylor-Couette flows between two concentric cylinders are measured using a two component LDV system at different Reynolds numbers and at two different gap widths between the two cylinders. The results will be presented using various scalings, and comparisons with those of Smith & Townsend (1982) will be made. It is found that the inner scaling suggested by Smith & Townsend (1982) can be used, and a logarithmic law with different constants from those used in unidirectional wall shear flows fit the data well in the usual logarithmic region. However, the data can be well fitted with an algebraic law in nearly all the interior region except within the two thin layers near the cylinder surfaces.

EXPERIMENTAL SET-UP

Figure 1 shows the set-up of the experimental facility used in the present study. The coordinate system is chosen as radial r, tangential θ and axial z with the corresponding velocities u, v and w, respectively. A circular glass tube with a diameter of 180 mm and a length of 430 mm was used as the outer cylinder. The glass tube was located inside a square acrylic tank in order to reduce the curvature effect of the circular tank on the laser light of the LDV system, and it was glued to the square tank to prevent flow between the inside and outside of the glass tube. The tank was filled with distilled water, with the water level being slightly lower than the height of the outer cylinder. The water surface was free. Two different inner cylinders were used, one was made of copper tube with a diameter of 62.5 mm and the other was a solid perspex bar with a diameter of 25.4 mm diameter. The length of the rotating cylinder, L, that submerged into the water was 420 mm. The inner cylinders were

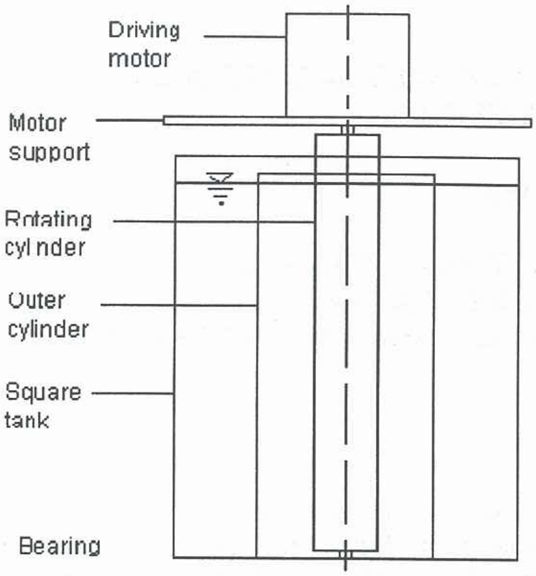

Figure 1: Schematic diagram of the experimental set-up.

driven by a DC powered servo-motor. Because of the centrifugal force, the water surface near the rotating cylinder was lower than that when the inner cylinder was stationary. However, it was found that the depression was in general less than 5 mm because of the low rotating speeds.

The flow was seeded with 5 μm particles with a specific gravity 1.5. The velocities were measured using a two component LDV system located in the Advanced Fluids Dynamics Laboratory at CSIRO The light source for the LDV system was provided by a Spectra Physics 7 W Argon-Ion laser. Light beams of blue color ($\lambda = 488\ nm$) and green color ($\lambda = 514.5\ nm$) were delivered to a fibre probe using fiber optics. The fibre probe was supported by a robot which can move in three directions with high accuracy. The scattered light from the particles was collected by the back scattering method. The focal length of the focusing lens was 350 mm. In order to avoid directional ambiguity, a frequency shift of 500 kHz was applied to both channels. The signals were filtered with band pass frequencies from 100 kHz to 1 MHz, and sampled for 100 seconds at data rates of around 400 Hz for the copper inner cylinder case and 200 Hz for the small cylinder case. Bias corrections were applied to the data when calculating velocity statistics, and the results were processed using FIND, a Windows based software from TSI.

In order to check the homogeneity of the flow in the axial direction, measurements for flows using the small inner cylinder at $Re = 6105$ (defined later) and at $\Delta z = 50\ mm$ apart were taken. It was found that both the mean velocity and the rms profiles agree

R_1	T_c	$\Omega(1/s)$	T/T_c	Re	Re_d
12.7	17224.7	18.66	2.5×10^4	2791	16994
		39.9	1.2×10^5	6105	37159
31.25	5343.6	18.66	2×10^5	17652	33185
		39.9	10^6	38675	72709

Table 1: Parameters used in the present experiments.

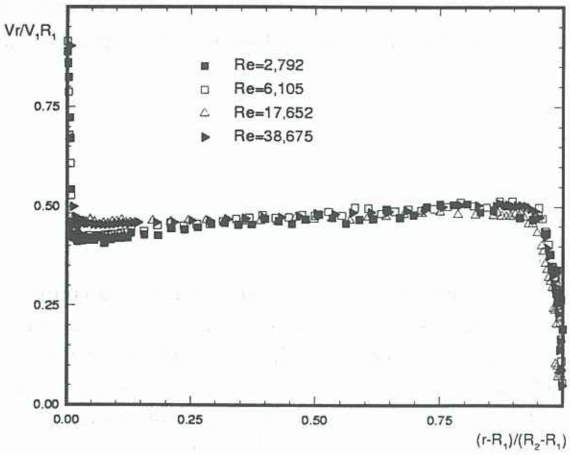

Figure 2: Nomalized mean angular momentum profiles.

with one and another extremely well at the two axial locations.

EXPERIMENTAL RESULTS

Table 1 lists the information in the present study. In the table, R_1 and R_2 are the radii of the inner and outer cylinders, $Re = V_1 R_1/\nu$, $d = R_2 - R_1$, $Re_d = V_1 d/\nu$, $T = 2(R_2 - R_1)Re_d^2/(R_2 + R_1)$ the Taylor number, $V_1 = R_1\Omega$, ν the kinetic viscosity, Ω is the angular speed of the rotating inner cylinder, and T_c is the critical Taylor number. The T_c values in Table 1 were estimated from the results of Roberts (1965). From Table 1, it can be seen that the Taylor numbers for the present study are much higher than the critical Taylor number T_c for instability. Barcilon & Brindley (1984) suggested that steady turbulent flows can be expected for $T/T_c >> 400$. Thus it is expected, and experimental results have confirmed, that the present flows are steady turbulent flows although the Reynolds numbers based on the inner cylinder radius for the small cylinder case are relatively small.

Figure 2 shows the profiles for the mean angular momentum Vr normalized by $V_1 R_1$ at various Reynolds numbers. Here V is the mean tangential velocity. The results are similar to those of Smith & Townsend (1982). Near the inner cylinder, $Vr/V_1 R_1$ quickly decreases to below 0.5, then increases slowly

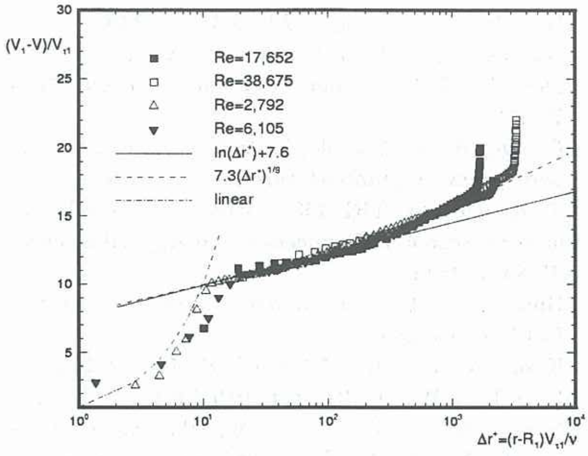

Figure 3: Mean velocity profiles normalized according to inner scaling

towards 0.5 as the outer cylinder is approached, and drops to zero at $r = R_2$. It is clear, from figure 2 and the results of Smith & Townsend (1982), that the gradient for Vr/V_1R_1 in the central region of the gap in general decreases with increasing Reynolds number. It is possible that $Vr/V_1R_1 = 0.5$ at infinite Reynolds number in the interior region away from the walls.

Smith & Townsend (1982) also presented their mean velocity results using the 'inner scaling' as that used often in essentially unidirectional wall shear flows such as channels or boundary layers on plane surfaces. Figure 3 shows the present mean velocity results according to the scaling suggested by Smith & Townsend (1982) at different Reynolds numbers. In presenting results in figure 3, the wall shear velocity at the inner cylinder surface, $V_{\tau 1}$, is needed. Li & Wu (1998) gave the details in determining $V_{\tau 1}$ and found the present results are consistent with those of Smith & Townsend (1982) and Barcilon & Brindley (1984).

In general, the present results collapse not only in the region close to the inner cylinder surface, but also in the interior region. Smith & Townsend (1982) presented their results using the inner scaling up to $\Delta r^+ = 90$, and suggested that their results follow,

$$\frac{V_1 - V}{V_{\tau 1}} = \frac{1}{\kappa}(\ln \Delta r^+ + B) \tag{1}$$

with $\kappa = 0.41$ and $B = 1.8$, for $30 < \Delta r^+ < 90$. These constants are close to those from unidirectional mean flows in channels or boundary layers on plane surfaces. However, it can be seen from figure 3 that the present data between $30 < \Delta r^+ < 200$ can be fitted with a relationship of the form of (1) with $\kappa = 1.0$ and $B = 7.6$.

Smith & Townsend (1982) argued that the streamline curvature will have a strong effect on the flow in the central flow region because parameter

$$\alpha = \frac{V/r}{\partial V/\partial r} \tag{2}$$

is close to -1 in this region, and the viscous effect will dominate the flow near the wall where velocity gradients become large (α is close to zero). Fitting the data to a relation of the form of (1) with $\kappa = 0.41$ and $B = 1.8$ suggests that the streamline curvature has little effect on the flow. Smith & Townsend (1982) analyzed the upper limit for (1) to be applicable with $\kappa = 0.41$ and $B = 1.8$, and they found that for $Re = 40,000$ (their $Re_d = 20,000$), this limit is $\Delta r_c^+ = 47$. Here subscript c stands for the upper limit. Similar analysis to that of Smith & Townsend (1982) would give $\Delta r_c^+ \approx 45$ for $Re = 38,675$, the highest Reynolds number for the results presented in figure 3. In unidirectional wall shear flows, it is generally accepted that the lower limit for (1) to be applicable is between 30 to 60 (Hinze, 1975). Thus it is not surprising that the present data cannot be fitted with (1) using $\kappa = 0.41$ and $B = 1.8$.

In figure 3, an algebraic power law

$$\frac{V_1 - V}{V_{\tau_1}} = C(\Delta r^+)^{1/n} \tag{3}$$

with $C = 7.3$ and $n = 9$, is also given. It can be seen that (3) fits the present data very well in the wide interior region between the two cylinders except within the viscous regions near the surfaces. A similar relation to (3) has been used in unidirectional flows such as pipes and boundary layers over plane surfaces with $C = 8.3$ and $n = 7$ (Hinze, 1975). Recent theoretical works of Barenblatt (1993), George, Castillo & Knecht (1993) and Oberlack (1998) suggest that algebraic relation (3) will fit the data better over larger regions than that of (1) in parallel turbulent wall shear flows. The present results show that this may also be applicable to the turbulent Taylor-Couette flows.

Nakamura et al (1981), from the measured velocities near a cylinder rotating in nearly unbounded fluid, found that their results fit the relationship,

$$V_1 - \frac{VR_1}{r} = \frac{V_{\tau_1}}{æ}[\ln \frac{V_{\tau_1}(r^2 - R_1^2)R_1}{2\nu r^2} + D] \tag{4}$$

with $æ = 0.64$ and $D = 1.8$. In figure 4, the present results are plotted according to the scaling of Nakamura et al (1981). Also shown in the figure is relation (4) with $æ = 0.64$ and $D = 3.9$. It can be seen from the figure that the present data from the bounded flows at two different gap widths collapse together using the scaling of Nakamura et al (1981) for results near the inner cylinder surfaces, with a different values for D. Away from the surface, the data departure from (4) systematically. For $\frac{V_{\tau_1}(r^2-R_1^2)R_1}{2\nu r^2} < 10$, the

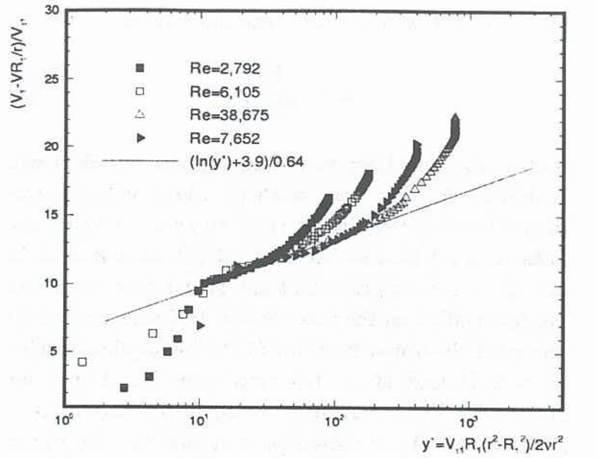

Figure 4: Mean velocity profiles normalized using the scaling of Nakamura et al (1981).

data from the small cylinder do not follow the scaling of Nakamura et al (1981).

DISCUSSION AND CONCLUSIONS

The velocity fields between the turbulent Taylor-Couette flows have been measured using a two component LDV system at two large gap widths and high Taylor numbers and analyzed using various scalings. The results are consistent qualitatively with the results from Smith & Townsend (1982), measured using hot wires for the same flow at smaller gap width. It is found that mean velocities collapse under the inner scaling as suggested by Smith & Townsend (1982), and fit the logarithmic law as used in unidirectional wall shear flows, but, with different constants in the near wall regions. The same law with the same constants as in unidirectional flow cannot be fitted to the data in any region. Also it was found that a power law relationship as suggested by recent theoretical works seems to fit the data over the wide gap except within the two thin layers near the cylinder surfaces ($\Delta r^+ < 10$). The fact that different laws from those used in unidirectional wall shear flows need to be used suggests that the streamline curvature has an effect on the flow almost to the surface of the cylinder.

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