13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

THE DECAY OF PASSIVE SCALAR FLUCTUATIONS IN SELF-PRESERVING ISOTROPIC TURBULENCE

J.D. LI

School of the Built Environment, Victoria University of Technology,
PO Box 14428, MCMC, Melbourne, 8001, AUSTRALIA

ABSTRACT

Implication of self-preservation on the decay of passive scalar variance in isotropic turbulent flow is considered. It is shown that self preserving solution for decaying passive scalar is possible if the length scale of the variance bearing eddies and the Taylor microscale for scalar grow at the same rate. Also, it is shown that the asymptotic decay law for passive scalar depends on the initial conditions of both velocity and scalar fields. Because of this, the ratio between the time scale for the dissipation of turbulent energy and that of scalar variance will not be constant.

INTRODUCTION

In understanding the development of passive scalar in turbulent flow field, it has been assumed (Tennekes & Lumely, 1972) that a large-scale distortion of the velocity field could be expected to force the scale size of the scalar variance bearing eddies to keep pace with that of the evolving energetic velocity eddies. Because of this, the ratio between the time scale for scalar dissipation and that for energy dissipation, $C_\gamma = (\overline{\gamma'^2}/\epsilon_\gamma)/(K/\epsilon)$, is assumed to be constant. Here $K = u_i u_i/2$ is the turbulent kinetic energy, $\epsilon = \nu\overline{u_{i,j}u_{,i,j}}$ the energy dissipation, γ the scalar fluctuation with $\overline{\gamma'^2}$ being its variance, $\epsilon_\gamma = 2D\overline{\gamma_{,i}\gamma_{,i}}$ the dissipation of scalar variance, u_i the velocity fluctuations, ν the kinetic viscosity, D the molecular diffusitivity, $u_{i,j} = \partial u_i/\partial x_j$ with x_j being the coordinate, and $\gamma_{,i} = \partial\gamma/\partial x_i$.

The experimental results of Warhaft & Lumley (1978) and the DNS results of Mell, Kosaly & Riley (1991) show that the variance of the passive scalar fluctuations decays according to a power law, but the decay exponent is different, depending on how the passive scalar is introduced into the isotropic turbulent field and where it is introduced. It is known that C_γ depends on the exponents of the power law decay of turbulent energy and scalar variance in isotropic turbulence (Warhaft & Lumley, 1978).

In this study, a fixed-point analysis similar to that of Speziale & Bernard (1992) is used to study the asymptotic decay behavior for isotropic scalar fluctuations. Under the assumption of complete self-preserving velocity and scalar fields, the results confirm the power law decay for the scalar variance. However it is found that the decay exponent depends on the initial conditions of both velocity and scalar. Because of this, the ratio between the time scales for energy and variance dissipations cannot be a constant.

THEORETICAL BACKGROUND

Using the conservation equation for passive scalar (Hinze, 1975) and the isotropic condition, it can be shown that

$$\overset{\bullet}{\overline{\gamma'^2}} = \frac{d\overline{\gamma'^2}}{dt} = -\epsilon_r \quad (1)$$

and

$$\overset{\bullet}{\epsilon}_\gamma = (\frac{10}{9}Pe^{1/2}S_\gamma - \frac{5}{9}G_\gamma)\frac{\epsilon_\gamma^2}{\overline{\gamma'^2}} \quad (2)$$

where

$$S_\gamma = -\lambda_\gamma^3[\frac{\partial^3 k}{\partial r^3}]_{r=0}$$

$$G_\gamma = \lambda_\gamma^4[\frac{\partial^4 R_{\gamma,\gamma}}{\partial r^4}]_{r=0}$$

$$R_{\gamma,\gamma}(r,t) = \frac{\overline{\gamma_A\gamma_B}}{\overline{\gamma'^2}}$$

$$k_\gamma(r,t) = \frac{\overline{\gamma_B\gamma_A(u_r)_A}}{\overline{\gamma'^2}u'}$$

$$Pe = \frac{K\overline{\gamma'^2}}{D\epsilon_\gamma}, \quad \lambda_\gamma = [\frac{6D\overline{\gamma'^2}}{\epsilon_\gamma}]^{1/2}$$

and γ_A and γ_B are values of γ at points A and B in space. In deriving (1) and (2), similar procedures as that of Speziale & Bernard (1992) have been used.

For an isotropic scalar field to be self-preserving,

$$R_{\gamma,\gamma}(r,t) = \widetilde{R}_{\gamma,\gamma}(r/L_\gamma), k_\gamma(r,t) = \widetilde{k}_\gamma(r/L_\gamma) \quad (3)$$

and $L_\gamma = L_\gamma(t)$. Using

$$\epsilon_\gamma = -6D\overline{\gamma'^2}[\frac{\partial^2 R_{\gamma,\gamma}}{\partial r^2}]_{r=0} \quad (4)$$

it can be easily shown that

$$\lambda_\gamma^2 [\frac{\partial^2 R_{\gamma,\gamma}}{\partial r^2}]_{r=0} = -1, \qquad \frac{\lambda_\gamma^2}{L_\gamma^2}\widetilde{R}_{\gamma,\gamma}(0) = -1 \quad (5)$$

Hence for a complete self-preserving isotropic scalar field, we must have

$$\lambda_\gamma \propto L_\gamma \quad (6)$$

since $\widetilde{R}_{\gamma,\gamma}(0)$ is constant. Thus it can be concluded that the Taylor microscale for scalar is the only similarity length scale that can yield complete self-preserving solution to the full diffusive equation for an isotropic turbulent scalar field. This conclusion is similar to that given for isotropic turbulence by Speziale & Bernard (1992), and we can set $L_\gamma = \lambda_\gamma$. Thus

$$S_\gamma = -\widetilde{k}_\gamma'''(0) = S_{\gamma 0}.$$

$$G_\gamma = \widetilde{R}_{\gamma,\gamma}^{IV}(0) = G_{\gamma 0}.$$

where the prime denotes derivative with respect to r/L_γ, and subscript 0 denotes the initial value. Substituting these into (1) and (2) gives

$$\dot{\overline{\gamma'^2}} = -\epsilon_\gamma \quad (7)$$

and

$$\dot{\epsilon}_\gamma = (\frac{10}{9}Pe^{1/2}S_{\gamma 0} - \frac{5}{9}G_{\gamma 0})\frac{\epsilon_\gamma^2}{\gamma'^2} \quad (8)$$

FIXED-POINT ANALYSIS

By differentiating the Peclet number with time, we have

$$\dot{Pe} = \frac{1}{D}\frac{\dot{K}\,\gamma'^2\epsilon_\gamma + K\,\dot{\gamma'^2}\,\epsilon_\gamma - K\,\gamma'^2\,\dot{\epsilon}_\gamma}{\epsilon_\gamma^2}. \quad (9)$$

Introducing the dimensionless time τ as $d\tau = (\epsilon_\gamma/\gamma'^2)dt$, it can be shown that

$$\frac{dPe}{d\tau} = Pe[-C_\gamma - 1 - \frac{10}{9}Pe^{1/2}S_{\gamma 0} + \frac{5}{9}G_{\gamma 0}] \quad (10)$$

The fixed point condition is achieved when $dPe/d\tau = 0$ as $\tau \to \infty$. Under this condition, (10) can be written as

$$Pe_\infty[-C_{\gamma\infty} - 1 - \frac{10}{9}Pe_\infty^{1/2}S_{\gamma 0} + \frac{5}{9}G_{\gamma 0}] = 0 \quad (11)$$

Equation (11) has two solutions, one is $Pe_\infty = 0$ and the other $Pe_\infty \neq 0$. These two fixed points are both stable nodes that attract all initial conditions. The solution $Pe_\infty = 0$ will be achieved when $\frac{5}{9}G_{\gamma 0} \leq 1 + C_{\gamma\infty}$. As such, (7) and (8) can be simplified to

$$\dot{\overline{\gamma'^2}} = -\epsilon_r \quad (12)$$

and

$$\dot{\epsilon}_r = -\frac{5}{9}G_{\gamma 0}\frac{\epsilon_\gamma^2}{\gamma'^2} \quad (13)$$

The asymptotic solutions for $\overline{\gamma'^2}$ and ϵ_γ are of the forms:

$$\overline{\gamma'^2} \sim t^{-\alpha}, \quad \epsilon_\gamma \sim t^{-\alpha-1} \quad (14)$$

where $\alpha = 1/(\frac{5}{9}G_{\gamma 0} - 1) \geq 1/C_{\gamma\infty}$. For example, during the final period of decay, $\widetilde{R}(\eta) = \exp(-\frac{1}{2}\eta^2)$ (Hinze, 1975). This gives $G_{\gamma 0} = 3$ and consequently $\alpha = \frac{3}{2}$. This is the well known result for the final period of decay of isotropic scalar field (Hinze, 1975).

The other solution for (11) with non-zero Pe_∞ is

$$Pe_\infty^{1/2} = \frac{-R_s + \sqrt{R_s^2 + 4\frac{R_t\nu}{D}(\frac{5}{9}G_{\gamma_0} - 1)}}{2} \quad (15)$$

where $R_s = 10\nu S_{\gamma 0}R_{t\infty}/9D$ and $R_t = K^2/\epsilon\nu$. This shows that the asymptotic non-zero Peclet number is a function of $\frac{\nu}{D}$, $R_{t\infty}$, $S_{\gamma 0}$ and $G_{\gamma 0}$. Thus Pe_∞ depends not only on the initial conditions of the scalar field but also those of the velocity field, since

$$R_{t\infty} = (\frac{\frac{7}{15}G_{K0} - 2}{7S_{K0}/3\sqrt{15}})^2 \quad (16)$$

according to Speziale & Bernard (1992) for non-zero asymptotic Reynolds number. Here G_{K0} and S_{K0} are the two point second and triple correlations at $\tau = 0$ for the velocity field. Because of the complex form of (15), (8) cannot be simplified appreciably. It can be written in a functional form as

$$\dot{\overline{\gamma'^2}} = -\epsilon_r \quad (17)$$

and

$$\dot{\epsilon}_r = -\beta\frac{\epsilon_\gamma^2}{\gamma'^2} \quad (18)$$

with $\beta = f(\frac{\nu}{D}, R_{t\infty}, S_{\gamma 0}, G_{\gamma 0})$. Because of this, the final asymptotic solutions for $\overline{\gamma'^2}$ and ϵ_γ at non-zero asymptotic Peclet number will, in general, depend on the initial conditions of both the scalar and velocity fields. This conclusion is quite different from that for the isotropic velocity field (Speziale & Bernard, 1992), where it has been shown that at high Reynolds numbers, the asymptotic decay for the kinetic energy followed a -1 law. The only exception to the above conclusion is when $C_{\gamma\infty} = 1$. In this case, the asymptotic solutions do not depend on the initial conditions, and are in the following forms

$$\overline{\gamma'^2} \sim t^{-1}, \quad \epsilon_\gamma \sim t^{-2} \quad (19)$$

which are the same as that for the asymptotic solution of the velocity field. However, this can only be considered as exception rather than being generally true, since

$$C_{\gamma\infty} = \frac{D}{\nu}\frac{Pe_\infty}{R_{t\infty}} \quad (20)$$

depends on the initial conditions of both the scalar and velocity fields, and on the Schmit number $Sc = \frac{\nu}{D}$.

NUMERICAL ANALYSIS

In order to further investigate the decaying behavior of $\overline{\gamma'^2}$, numerical analysis was carried out. Equations (7) and (8) for scalars and the similar equations for turbulence given by Speziale & Bernard (1992) can be cast into the following nondimensional forms

$$\frac{dK^*}{dt^*} = -\epsilon^* \tag{21}$$

$$\frac{d\epsilon^*}{dt^*} = (\frac{7}{3\sqrt{15}} S_{K_0} R_t^{1/2} - \frac{7}{15} G_0) \frac{\epsilon^{*2}}{K^*} \tag{22}$$

$$\frac{dY^*}{dt^*} = -\epsilon_\gamma^* \tag{23}$$

$$\frac{d\epsilon_\gamma^*}{dt^*} = (\frac{10}{9} Pe^{1/2} S_{\gamma 0} - \frac{5}{9} G_{\gamma 0}) \frac{\epsilon_\gamma^{*2}}{Y^*} \tag{24}$$

where $t^* = t/\tau_0$ with $\tau_0 = K_0/\epsilon_0$, $K^* = K/K_0$, $\epsilon^* = \epsilon/\epsilon_0$, $Y^* = \overline{\gamma'^2}/\overline{\gamma_0'^2}$, and $\epsilon_\gamma^* = \epsilon_\gamma K_0/(\gamma_0^2 \epsilon_0)$. Equations (21)-(24) can be solved numerically with initial values $K^* = \epsilon^* = Y^* = \epsilon_\gamma^* = 1$, and with given S_{K_0}, G_0, S_{γ_0} and G_{γ_0}.

In the numerical results given below, $S_{K0} = 0.5$ has been used. This is the same as that used in Speziale & Bernard (1992). The experimental data of Clay (1973) and direct numerical simulation results of Kerr (1985) both show that S_γ does not depend on Reynolds and Peclet numbers, and is about 0.5. In solving (21)-(24), $S_{\gamma 0}$ =0.5 has been used.

Figure 1 shows the decay of energy and scalar variance at $R_{t0} = 5000 = Pe_0$ and $G_{K0} = 10.0$ at various initial G_γ values. The figure show that the decay for both the kinetic energy and scalar variance have an initial flat transient region, this is followed by a rapid decay region. The kinetic energy and scalar variance approach the asymptotic power law decay at large times. In general, the kinetic energy and scalar variance will take a few eddy turnover times to approach asymptotic power law decay. For $0 < t^* < 1.0$, the scalar variance decays quicker than that of the kinetic energy at small $G_{\gamma 0}$, and slower than that of kinetic energy at large $G_{\gamma 0}$.

Figure 2 shows the decay exponents $\alpha_{K*} = (d\log K^*)/(d\log t^*)$ and $\alpha_{Y*} = (d\log Y^*)/(d\log t^*)$ for the results given in figure 1. The figure shows that the decay exponent for the scalar variance in an isotropic turbulence field depends on the initial conditions of the scalar fields. This is consistent with the fixed-point analysis given in the last section. Figure 2 shows that the rate at which the scalar variance achieves its asymptotic power law decay depends on $G_{\gamma 0}$. The higher the $G_{\gamma 0}$, the more rapidly the scalar variance achieves its power law decay.

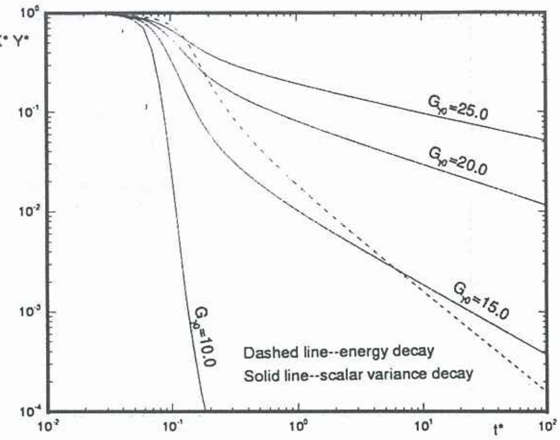

Figure 1: The decay of scalar variance at different initial $G_{\gamma 0}$ with $R_{t0} = Pe_0 = 5000$ and $G_{K0} = 10.0$.

Figure 3 shows the kinetic energy and scalar variance decay at $R_{t0} = Pe_0 = 5000, G_{\gamma 0} = 10.0$ and $G_{K0} = 5.0, 7.5$ and 10.0. The figure shows that the kinetic energy decays with the same asymptotic power law while the scalar variance approaches different asymptotic power laws for different G_{K0}. This clearly shows that the asymptotic behavior for the scalar variance depends not only on the initial conditions of the scalar field but also on that of the velocity field. Figure 3 also shows that the stronger the initial intermittence for the velocity field (higher G_{K0}), the more quickly the scalar variance decays. One interesting point to note is that for all three different G_{K0}, the scalar variance has been reduced to a small fraction of its initial value during the time from $t^* = 0.1$ to $t^* = 0.2$ ($\Delta t^* \approx 0.1$) , that is, one tenth of an eddy turnover time.

Figure 4 shows the scalar variance decay at different initial Peclet numbers with $R_{t0} = 5000, G_{K0} = 10$ and $G_{\gamma 0} = 15$. Also shown in the figure is the energy decay. The figure shows that the asymptotic behavior for the decay of variance is the same for different initial Peclet number. This is consistent with the fixed-point analysis given in the last section, where it has been shown that the asymptotic power law decay for the scalar variance depends on the initial values G_{K0} and $G_{\gamma 0}$ (given $S_{K0} = S_{\gamma 0} = 0.5$), and does not depend on the initial Reynolds and Peclet numbers. The effect of the initial Peclet number on the scalar variance decay is that at low Peclet numbers, the scalar variance decays faster initially.

DISCUSSION AND CONCLUSIONS

In the experiments of Warhaft and Lumley (1978), changing the heating grid configuration will change $G_{\gamma 0}$, while the initial conditions for the velocity field can be considered as the same. Also it is expected

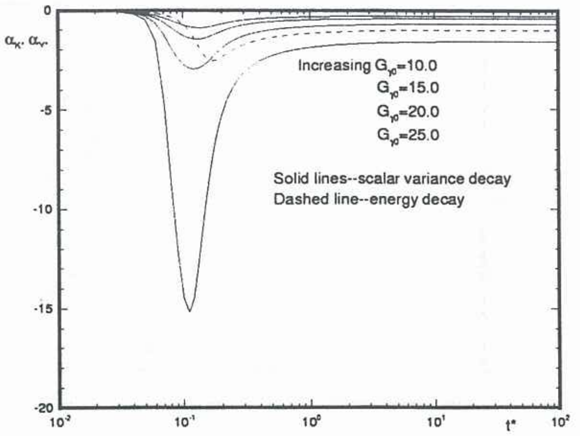

Figure 2: The decay exponents for scalar variance and kinetic energy. Parameters are the same as in figure 1.

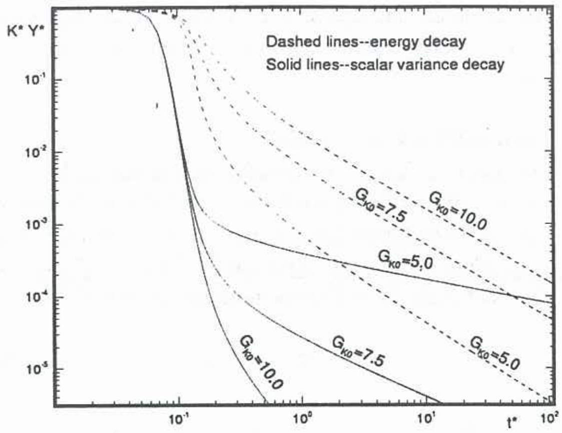

Figure 3: The decay of scalar variance at different G_{K0} with $R_{t0} = Pe_0 = 5000$ and $G_{\gamma 0} = 10.0$.

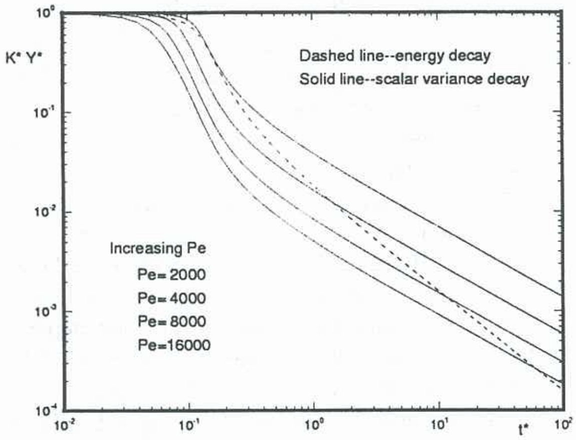

Figure 4: The decay of scalar variance at different initial Peclet numbers with $R_{t0} = 5000$ and $G_{K0} = 10.0$ and $G_{\gamma 0} = 15.0$.

that the decay exponent from different experiments will be different because the solidity of the turbulence generating grid and the shape of the grid (i.e. square or round bar) are different. This will cause the initial conditions for both the velocity and scalar fields to be different. In the DNS data of Mell et al. (1991), although the velocity field decayed faster than the power law, it was found that the decay exponent for the scalar variance increased as the initial velocity-to-scalar length-scale ratio increasing. This finding is consistent with the present predictions under the self-preserving assumption.

Because the asymptotic power law decay exponent of the passive scalar depends on the initial conditions of both the velocity and scalar fields, the time scale ratio $C_\gamma = t_\gamma/t_K = (\overline{\gamma'^2}/\epsilon_\gamma)/(K/\epsilon)$ will also depend on the initial conditions of both the velocity and scalar fields. Warhaft and Lumley (1978), from their experimental results, concluded that there was no evidence in the relaxation of C_γ from its initial value to an equilibrium value in their wind tunnel, which extends to nearly one turbulence decay time.

REFERENCES

Clay, J.P 1973 *Turbulent mixing of temperature in water, air and mercury*. PhD. Thesis, University of California at San Diego.

Hinze, J.O. *Turbulence*. McGraw-Hill (1975).

Kerr, R.M. 1985 Higher-order derivative correlations and the alignment of small-scale structures in isotropic numerical turbulence. *J.Fluid Mech.* **153**, 31-58.

Mell, W.E., G. Kosaly and Riley, J. The length-scale dependence of scalar mixing. Phys. Fluids A **3** (10), 2474-2476 (1991).

Speziale, C.G. and Bernard, P. The energy decay in self-preserving isotropic turbulence revisited. *J.Fluid Mech.* **241**, pp645-667 (1992).

Sreenivasan, K.R., Tavoularis,S., Henry, R. and Corrsin, S. 1980 Temperature fluctuations and scales in grid-generated turbulence. *J.Fluid Mech.* **100**, 597-621.

Warhaft, Z. and Lumley, J.L. An Experimental study of the decay of temperature fluctuations in grid-generated turbulence. *J.Fluid Mech.* **88**, 659-684 (1978).