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ELLIPTIC INSTABILITY IN OPEN FLOWS: VORTEX PAIRS AND WAKES

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ABSTRACT

In this paper, we analyze the three-dimensional (3D) instabilities observed in two flows, both of which comprise dominant coherent vortices that are deformed elliptically by strain rate effects. These flows are the counter-rotating vortex pair and the circular cylinder wake in the transition regime. In the case of the vortex pair, we compare the experimental instability growth rate, spatial structure of the internally deforming vortices, and wavelengths of instability, with predictions from the theory of elliptic 3D instability, all of which have good agreement. In the case of the transition in the cylinder wake, we provide evidence that the origin of the "mode A" 3D instability is also an elliptic instability of the highly-strained primary wake vortices. These results provide the first clear evidence of elliptic 3D instability in open flows comprising finite vortices.

INTRODUCTION

The dynamics of a pair of parallel counter-rotating vortices has been the object of a large number of studies in the last three decades. Motivated by Crow's (1970) work, Widnall *et al.* (1974) proposed a mechanism for instability in flows with strained concentrated vortices, of which the counter-rotating vortex pair is one example. No detailed observations or measurements concerning this short-wave instability in controlled laboratory experiments can be found in the literature.

In our experiments, vortex pairs were generated in a water tank at the sharpened parallel edges of two flat plates, hinged on one side to a common base and moved in a prescribed symmetric way by a computer-controlled step motor. The vortex pair characteristics, i.e. the circulation Γ, the core radius a, and the vortex spacing b, are found from flow field measurements using Digital Particle Image Velocimetry (DPIV).

Our second flow considered here is the transition to turbulence in the wake of a circular cylinder, which has been studied for many years (see Williamson 1996*a*)). It was shown experimentally in Williamson (1988) that the wake transition flow regime involves a sequence of two 3D modes of vortex shedding with different spanwise length scales, each of which begins at Reynolds numbers (Re) around 190 and 260, respectively.

ELLIPTIC INSTABILITY IN VORTEX PAIRS

The visualization of the vortex pair in Fig. 1 shows the general features of this short-wavelength instability, from which it is clear that it involves a modification of the internal structure of the vortex cores. Although the vortex centers, marked by bright dye filaments, are perturbed into a wavy shape, one observes the existence, in each vortex, of a dye layer which remains unchanged. Inside and outside of this invariant stream tube, fluid is displaced in opposite radial directions. Simultaneous visualizations from two perpendicular directions reveal that the sinusoidal displacements of the vortex centers lie in planes inclined by 45° with respect to the plane of the pair, i.e. they are oriented in the direction of the mutually induced strain in the vortex cores. We find a remarkably close agreement in the spatial internal structure of the experimental instability with the form of the structure predicted from the elliptic instability theory (see Figs. 1*b* and 1*c*). In particular, the ratio of the axial wavelength λ to the diameter d_i of the invariant stream tube can be measured quite precisely from the visualization in Fig. 1(a) to be $\lambda/d_i = 2.0$, which matches very closely the theoretical value $(\lambda/d_i)_{th} = 1.9898$ for uniform unbounded elliptical flow (Waleffe 1990).

The above observations strongly suggest that this short-wavelength instability is linked to the so-called "elliptic instability", which occurs in flows with elliptical streamlines (a flow resulting from the interaction of a rotational flow and a plane strain). In the present case, it is found in the cores of the vortices. From theoretical studies treating this phenomenon (Landman & Saffmann 1987, Waleffe 1990), one can deduce the following expression for the growth rate σ of the short-wavelength instability for the vortex-pair flow (Leweke & Williamson 1998*a*):

(a)

(b)

(c)

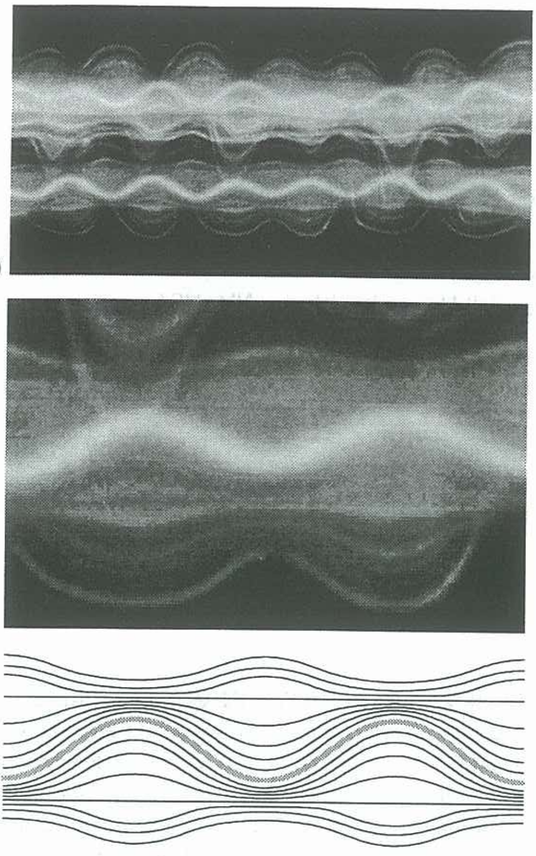

Figure 1: Short-wavelength vortex pair instability. (*a*) Experimental visualization. (*b*) Close-up. (*c*) Theoretical prediction for elliptic instability.

$$\frac{\sigma}{\Gamma/2\pi b^2} = \frac{9}{8} - \frac{32\pi^3}{Re\,(\lambda/b)^2}. \qquad (1)$$

This shows that, for a given wavelength λ, the instability only occurs above a critical Reynolds number

$$Re_c = \frac{2^8\pi^3}{9\,(\lambda/b)^2} \qquad (2)$$

Experimental measurements of the growth rate of the short-wavelength instability are difficult because of the simultaneous development of the long-wavelength Crow instability. However, some values were obtained in cases where the amplitude of the latter remained low for long times. A typical result is $\sigma/(\Gamma/2\pi b^2) = 0.94$ for $\lambda/b = 0.8$ at $Re = 2750$, which lies between the theoretical values 9/8 and 0.56 for inviscid and viscous uniform elliptic flow, respectively (see Eq. 1). The agreement between experiment and theory is satisfactory, considering the idealized nature of the theoretical flow.

An interesting and unexpected observation relates to the phase relationship between the short-wave perturbations on the two vortices. In the front view of Fig. 1, the vortex centers are, at each axial position, displaced in the same transverse direction. This means that the initial reflectional symmetry of the flow, with respect to the plane separating the vortices, is lost. It should be emphasized that no forcing of the phase was applied in the study of the short-wave instability; the initial vortex pair was completely uniform and the flow evolved freely. The symmetry breaking is further illustrated by the cross-sectional view in Fig. 2: the vortex center is displaced to the lower right in the left vortex, and to the upper right in the right one. DPIV measurements in the same plane (Fig. 2(*b*)) confirm that this is not simply an effect caused by the dye visualization method. The maxima of vorticity are displaced in the same way as in Fig. 2(*a*), in close agreement with theoretical predictions for the elliptic instability of a strained vortex (Waleffe 1990). The observed phase relationship between the two vortices can be explained by a kinemanic matching condition for the perturbations on each vortex (see Leweke& Williamson 1998*a*), and we therefore define the flow as resulting from a "co-operative elliptic" instability.

(a)

(b)

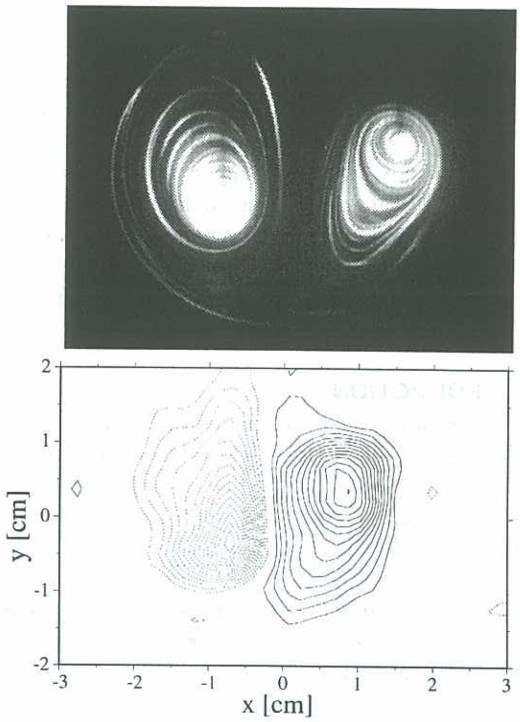

Figure 2: Short-wave perturbation in a cross-cut plane. (*a*) Visualization. (*b*) DPIV vorticity contours.

ELLIPTIC INSTABILITY IN WAKE TRANSITION

We present evidence here that the elliptic instability is also at the physical origin of three-dimensional structure which appears in the wake of a cylinder in the transition regime; namely, the so-called "mode A" wake instability, which appears at $Re = 190$. Further information, including details on the experimental techniques, may be found in Williamson (1996*b*) and in Leweke & Williamson (1998*b*).

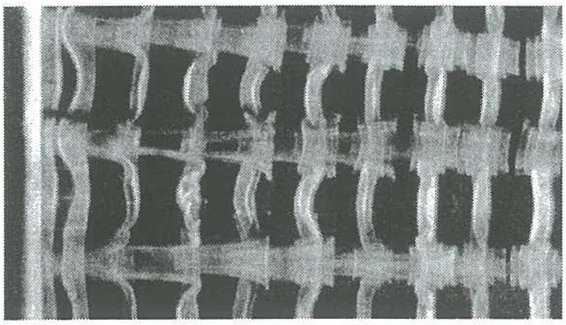

Figure 3: Visualization of the three-dimensional wake transition mode A.

The dye visualization of the near wake in Fig. 3(a) shows the vortex street structure for mode A in plan-view. The primary vortices deform in a wavy fashion along their length. The spanwise wavelength λ_A of these perturbations is a slowly decreasing function of Reynolds number and varies between 4 and 3 cylinder diameters. As the flow progresses, the waviness grows from one cycle to the next, resulting in the local spanwise formation of vortex loops, which become stretched into streamwise vortex pairs in the braid regions between the primary vortices. Mode A is clearly evident in the range $190 < Re < 240$.

Figure 4(a) shows contours of the instantaneous spanwise vorticity, taken from two-dimensional DNS calculations at $Re = 200$ (made available by H. Persillon, see Persillon & Braza 1998). The two main features of the flow are the rolled-up primary vortices and the braid shear layers (Fig. 4b), associated with two different length scales. The cores have a diameter $\delta_{\text{core}} \approx D$, whereas the shear layer (vorticity) thickness is $\delta_{\text{braid}} \approx D/4$.

It was proposed in Williamson (1996b) that the mode A instability is due to an elliptic instability of the vortex cores. In order to demonstrate the link between elliptic flow and the cylinder wake, it is useful to look at the flow field inside a primary vortex in a frame of reference moving with that vortex. This is shown in Fig. 4(c) for the lower vortex of Fig. 4(a). In the core of this vortex, the streamlines are indeed approximately ellipses.

It has been shown by different authors that unbounded linear flows are three-dimensionally unstable for all values of the non-dimensional parameter $\beta = 2\varepsilon/|\omega|$ ($\varepsilon > 0$: strain rate, ω: vorticity), except $\beta = 1$. For inviscid flow, the growth rates σ_i of the most unstable perturbations for the elliptic instability ($0 < \beta < 1$) are given by

$$\frac{\sigma_i}{\varepsilon} = f(\beta) \approx \frac{9}{16}\left(1 - \beta^m\right)^n . \qquad (3)$$

The approximate expression on the r.h.s., with $m = 2.811$ and $n = 0.3914$, was computed here from a least-squares fit to the numerically determined result presented by Landman & Saffman (1987).

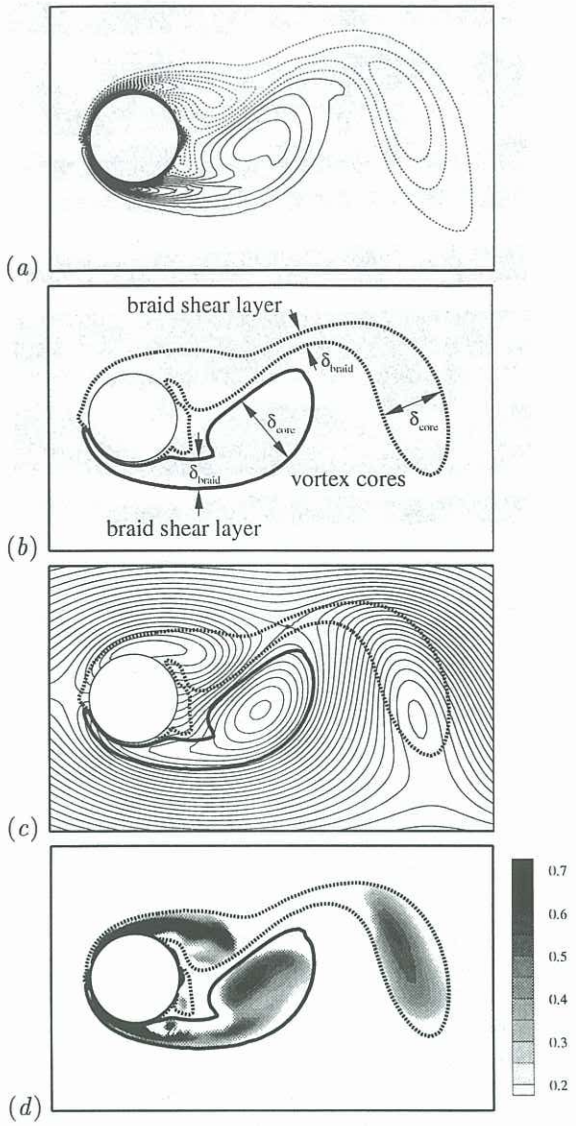

Figure 4: Results derived from 2D DNS by H. Persillon. (a) Vorticity. (b) Principal features. (c) Streamlines in a frame moving with the lower vortex. (d) Local elliptic instability growth rates.

For the elliptic instability, high values of the locally calculated inviscid growth rate are found mainly in the primary vortex cores, as seen in Fig. 4(d). It seems therefore legitimate to expect the development of an elliptic instability in the primary vortex cores.

Figures 5(a) and 5(b) show a visualization of the very initial stages of the mode A instability. In this picture, the thick dye filament marking the center of a primary vortex, exhibits the same characteristic waviness as would be expected from elliptic instability in Fig. 5(c). Not far from this filament, one can see a layer which does not seem to be disturbed in the spanwise direction, and which possibly corresponds to the invariant stream tube.

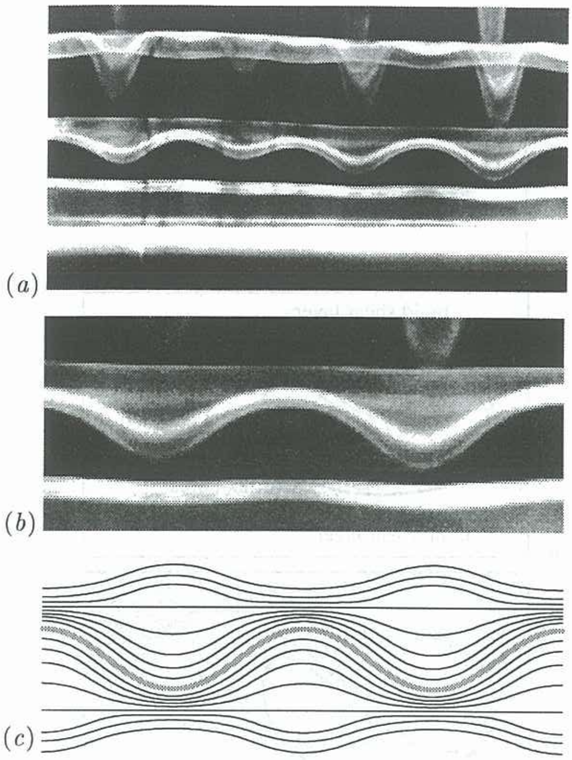

Figure 5: Wake transition mode A. (*a*) Experimental visualization. (*b*) Close-up. (*c*) Theoretical prediction for elliptic instability of an isolated vortex.

A more quantitative comparison can be made concerning the spanwise wavelength of the instability. From the DNS data, the streamline eccentricity is found to be relatively high: $\beta \approx 0.6$. Nevertheless, a relation between cross-sectional and axial length scales can be found for this case from the analysis of Landman & Saffman (1987):

$$\lambda = L \ \tan\theta \left(\frac{2}{1-\beta}\right)^{1/2} \approx \ 3D. \qquad (4)$$

$L = \delta_{\mathrm{core}} \approx D$ is the size of the elliptical flow region, and θ the wave vector inclination with respect to the vortex axis ($\theta \approx 53°$ for $\beta = 0.6$). Although the prediction in Eq. (4) can only be considered as a rough estimate, due to the non-uniformity of β and the time-dempendence of the flow, the agreement with the measured value for mode A is neverteless quite good. These results, and more shown in Leweke & Williamson (1998), strongly suggest a link between 3D wake transition and elliptic instability. Other attempts to explain the origin of three-dimensionality involve a period-doubling route to chaos (Karniadakis & Triantafyllou 1992), a Benjamin-Feir instability of a set of oscillators (Leweke & Provansal 1995), or a centrifugal instability (Brede *et al.* 1996). However, the evidence presented in these studies is either insufficient or disagrees with observations made in experiment and numerical simulation.

CONCLUSIONS

Our studies indicate that certain three-dimensional structure, in flows comprising concentrated vortices, originates from an elliptic instability of the vortex cores. These deductions are based on an agreement of spatial instability structure, growth rate and symmetry, between experiment and theory, in two flows; namely, the counter-rotating vortex pair, and the wake transition "mode A". The link between the elliptic instability theory and concentrated vortices observed experimentally in open flows is shown here for the first time. One might expect such elliptic instabilities to occur in all generic flows containing strained concentrated vortices, such as mixing layers, jets, and other turbulent flows.

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