13th Australasian Fluid Mechanics Conference
Monash University, Melbourne, Australia
13-18 December 1998

MODELING A TURBULENT FLOW AROUND A SHARP EDGED OBSTACLE IN THE CLOSED CHANNEL AND IN THE BOUNDARY LAYER

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ABSTRACT

The results of modeling a flow around a two-dimensional square cross-section surface-placed obstacle in the closed plane channel and in the boundary layer are discussed. The mean velocity field was calculated by means of the method of simultaneous iterations with the mean pressure field using the Reynolds-averaged Navier-Stokes equations. Possibilities of the algebraic and differential Reynolds-stress models (RSM) have been investigated.

INTRODUCTION

Turbulent flows in the environment and engineering are characterised by processes of separation and reattachment, include zones of recirculation and stagnation because of sharp edges of different objects situated within a flow or on its rigid boundaries. These factors complicate the problem of adequate description of turbulent flows structure, in particular, the problem of boundary-layer interaction with large-scale roughness elements (for example, a flow around a building in the near-ground atmospheric layer). The local task to solve this problem is elaboration and development of the turbulence model for a flow around a quadratic cross-section surface-placed obstacle. The present paper contains the results of numerical investigation of the two-dimensional turbulent flow around the obstacle and concentrates in achievement of adequate description near the obstacle surfaces where the face, top and rear zones of recirculation are formed.

TURBULENCE MODELS

The Reynolds-averaged Navier-Stokes equations written for two-dimensional case are used to compute the mean velocity and pressure fields. Different RSM are employed to calculate the Reynolds-stress tensor components $<u^2>, <v^2>$ and $<uv>$. The differential model (DM) contains the transport equations for turbulent stresses with the pressure-strain and dissipation terms approximations taken from (Hanjalic and Launder, 1972). For description of processes of turbulence diffusion in these equations, the gradient model is used with the turbulent viscosity coefficient $\nu_t=C_\mu E^2/\varepsilon$ where C_μ=0.09. The transport equations for the turbulence kinetic energy E and for its dissipation rate ε are added to the differential RSM as well as the algebraic one. The non-linear algebraic RSM can be reduced from DM due to assumption of proportionality of diffusion and convection terms of the Reynolds-stress equations to that of the E equation. The explicit algebraic model (AM) does not include the ratio P/ε (where P is the mean shear production of E) and is obtained according to (Pope, 1975) as follows

$$<u^2> = \frac{2}{3}E - 2G\frac{E^2}{\varepsilon}\frac{\partial U}{\partial x} + 2FE\left[\frac{b_3}{2}\frac{E^2}{\varepsilon^2}\left\{\left(\frac{\partial U}{\partial y}\right)^2 - \left(\frac{\partial V}{\partial x}\right)^2\right\} + \frac{b_2}{3}\{S^2\}\right], \quad (1)$$

$$<v^2> = \frac{2}{3}E - 2G\frac{E^2}{\varepsilon}\frac{\partial V}{\partial y} - 2FE\left[\frac{b_3}{2}\frac{E^2}{\varepsilon^2}\left\{\left(\frac{\partial U}{\partial y}\right)^2 - \left(\frac{\partial V}{\partial x}\right)^2\right\} - \frac{b_2}{3}\{S^2\}\right], \quad (2)$$

$$\{S^2\} = \frac{1}{2}\left(\frac{E}{\varepsilon}\right)^2\left[\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2 + 2\left(\frac{\partial U}{\partial x}\right)^2 + 2\left(\frac{\partial V}{\partial y}\right)^2\right],$$

$$b_2 = \frac{5-9C_2}{11},\quad b_3 = \frac{7C_2+1}{11},\quad F = G\left(C_1 - 1 + 2G\{S^2\}\right)^{-1},$$

$$<uv> = -G\frac{E^2}{\varepsilon}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right) + F\frac{E^3}{\varepsilon^2}\left[b_3\left\{\left(\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}\right)\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\right\}\right] \quad (3)$$

where U and V are the horizontal and vertical components of the mean velocity vector; u and v are the corresponding fluctuation velocity components; the symbol <...> means Reynolds averaging; C_1=1.5, C_2=0.4. The function $G=G(E,\varepsilon,\partial U_k/\partial x_m)$ in the expressions (1)-(3) is defined from the cubic equation $G^3+aG^2+bG-c=0$ where

$$b = \frac{(C_1-1)^2 + 2b_3^2 W_{ik}W_{ik} - (b_1 + 2b_2^2/3)\{S^2\}}{4\{S^2\}^2},$$

$$a = \frac{C_1-1}{\{S^2\}},\quad c = \frac{b_1(C_1-1)}{8\{S^2\}^2},\quad W_{ik}W_{ik} = \frac{E^2}{2\varepsilon^2}\left(\frac{\partial U}{\partial y} - \frac{\partial V}{\partial x}\right)^2.$$

The linear Boussinesq model (BM) of gradient type

$$\begin{Bmatrix} <u^2> = \frac{2}{3}E - 2C_\mu \frac{E^2}{\varepsilon}\frac{\partial U}{\partial x}, \\ <v^2> = \frac{2}{3}E - 2C_\mu \frac{E^2}{\varepsilon}\frac{\partial V}{\partial y}, \end{Bmatrix}$$
(4)

$$<uv> = -C_\mu \frac{E^2}{\varepsilon}\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)$$
(5)

uses the isotropic coefficient $\nu_t = C_\mu E^2/\varepsilon$ and contains the constant C_μ instead of the complex function G in the linear and quadratic terms of the AM expressions (1)-(3).

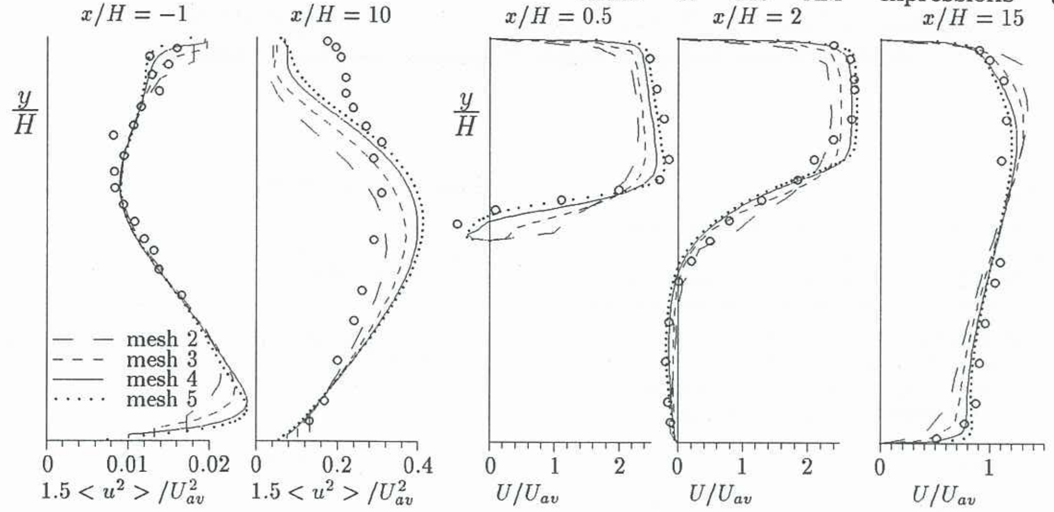


Figure 1 : Vertical profiles of horizontal components of E and mean velocity vector calculated at mesh 2-5 by means of DM.

NUMERICAL REALIZATION OF MODELS

Plane-channel flow has been chosen to examine the turbulence models applicability because of the presence of measurements data of (Durst and Rastogi, 1979) denoted by open circles in Figures 1-3. At calculation of the flow in the channel of height $2H$, the Reynolds number based on height $H = 2.5$ sm of a quadratic cross-section obstacle and velocity U_{av} averaged on the flow cross section was $Re_H = 8500$ (Durst and Rastogi, 1979). At calculation of the turbulent boundary layer with the quadratic cross-section obstacle of height $H = 20$ sm and inflow velocity U_H at this height, the Reynolds number $Re_H = U_H H/\nu \cong 70000$ (Murakami and Mochida, 1988) was sufficiently high too. The wall boundary conditions were taken as «wall law» (Durst and Rastogi, 1979; Murakami and Mochida, 1988). The developed turbulent inflow was set at the upstream boundary at x/H=-11 in the channel and at x/H=-15 in the boundary layer with the vertical profiles of U,V,E taken from experiments. Normal gradients of E,ε,U,V (turbulent stresses in DM) were set to zero at the downstream boundary far from the obstacle. Other details of initial distributions, boundary conditions, equations discretization and computation procedure can be seen in (Kurbatskii and Yakovenko, 1996).

The series of calculations has been performed to test the approximation convergence of difference equations solutions to exact ones with non-uniform grid refinement and to clear up the detail structure of turbulent flow around an obstacle. The mesh interval δ near the obstacle surfaces was taken in successive runs to be equal to $H/6$, $H/12$, $H/24$, $H/48$, $H/96$ (meshes 1-5, respectively). Differences between the results of computations using mesh 1-5 decrease continuously with grid refinement (Figure 1) and are localized near the top where the third recirculation region forms in calculations at mesh 4 and 5.

In the case of the turbulent boundary layer, computations give exactly the measured value $x_R = 6.8H$ (Logan and Phataraphruk, 1990) of the rear recirculation zone length if δ=$H/48$, underestimate the value of x_R at more rough grids and show convergence with the growth of H/δ..

It can be seen that mesh 4 with δ=$H/48$ provides the satisfactory approximation convergence. The deviations between the results of computations by means of differential RSM at various meshes 4 and 5 (Figure 1) are much less than differences between the results obtained at mesh 4 or 5 by means of different RSM (Figure 2).

Possibilities of different RSM have been investigated. The model BM (4)-(5) (long dashed lines in Figure 2) with an isotropic coefficient of turbulent viscosity gives large quantitative discrepancies with experiments and even negative non-physical values of horizontal component $<u^2>$ of E (at x/H=–1 in Figure 2a). The non-linear algebraic model AMN (1)-(2) for the normal Reynolds stresses (short dashed lines in Figure 2) is able to reproduce their anisotropy and to correct shortcoming of the linear model. The differential model DMN with the transport equations for $<u^2>$, $<v^2>$ and the expression (5) for $<uv>$ (dotted lines in Figure 2) describes behaviour of $<u^2>$ in front of the obstacle (where processes of convection and diffusion are essential) better than algebraic model. The complete differential model DM (solid lines in Figure 2) with the transport equations for all Reynolds-stress tensor components gives most exact reproducing of mean velocity field in front of the obstacle, above it and in the recovery region behind it. The latter model describes (Figure 3d) also the joint of recirculation zones above the obstacle and behind it at meshes 4 and 5.

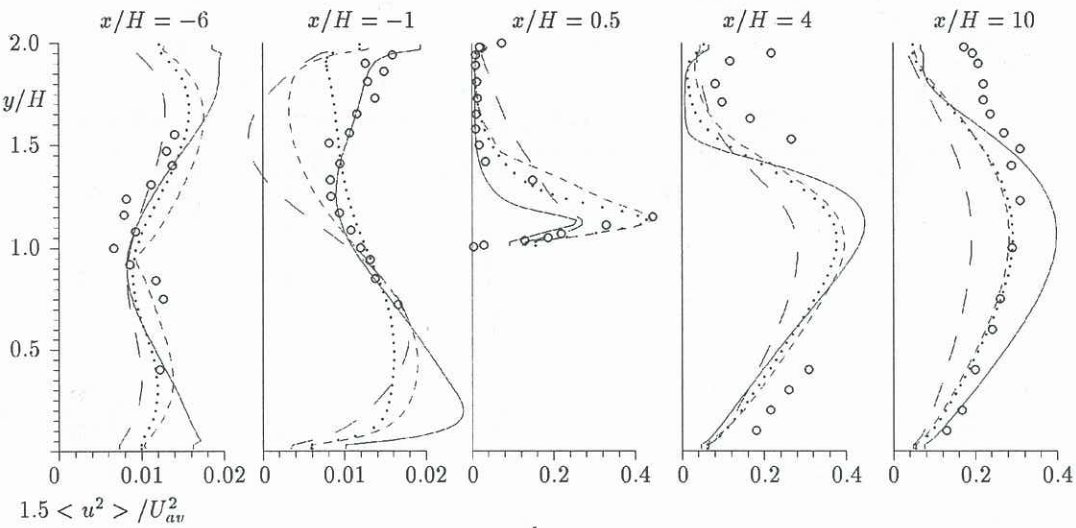
The effect of use of various difference schemes for convective terms in the transport equations for calculation of flows around sharp-edged obstacles at high Reynolds numbers has been examined. In particular, the differences between the results of computation (Figure 3a-d) with the different schemes (e.g. first-order upwind scheme and QUICK one) by the same model at the refined grid are much less of the discrepancies of sought functions and streamlines calculated by the various turbulence models (Figures 2-3) but with the same convective scheme.

CONCLUSION

The results of modeling turbulent flows around surface-placed obstacle of the quadratic cross section in the closed plane channel and in the boundary layer have been

Results of modeling a propagation of passive scalar from a surface-placed line source in the turbulent boundary layer with the obstacle and without it have been obtained. Calculations of the mean concentration field were followed by careful examining an applicability of turbulence models for description of the boundary layer characteristics with use of the measurements (Gibson et al, 1984; Poreh and Cermak, 1964) and DNS (Spalart, 1988).
.obtained. Possibilities of the algebraic and differential RSM have been compared. The differential RSM with the transport equations for all turbulent stresses components gives most satisfactory reproducing of the main flow

a



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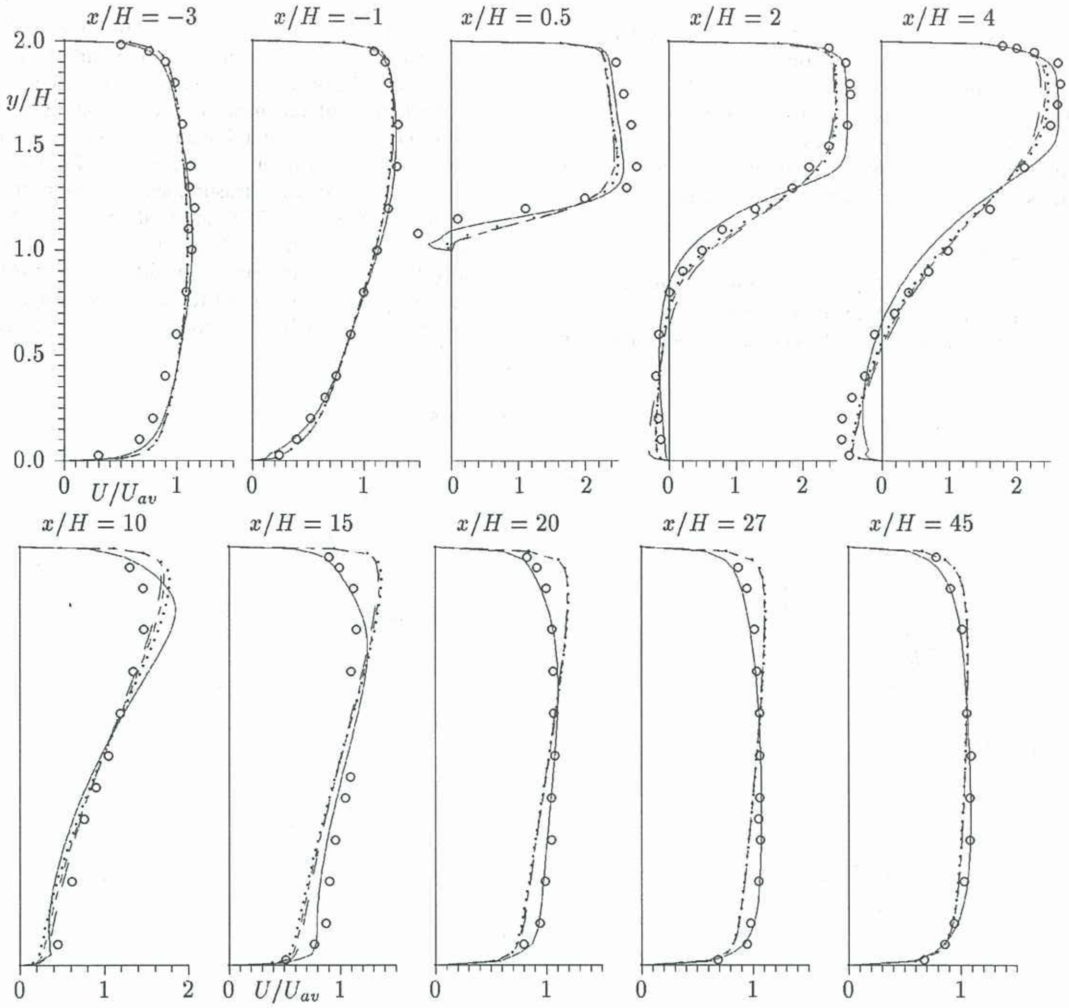

Figure 2 : Measured profiles and results of computation by different models at the refined grid with δ=*H*/48.

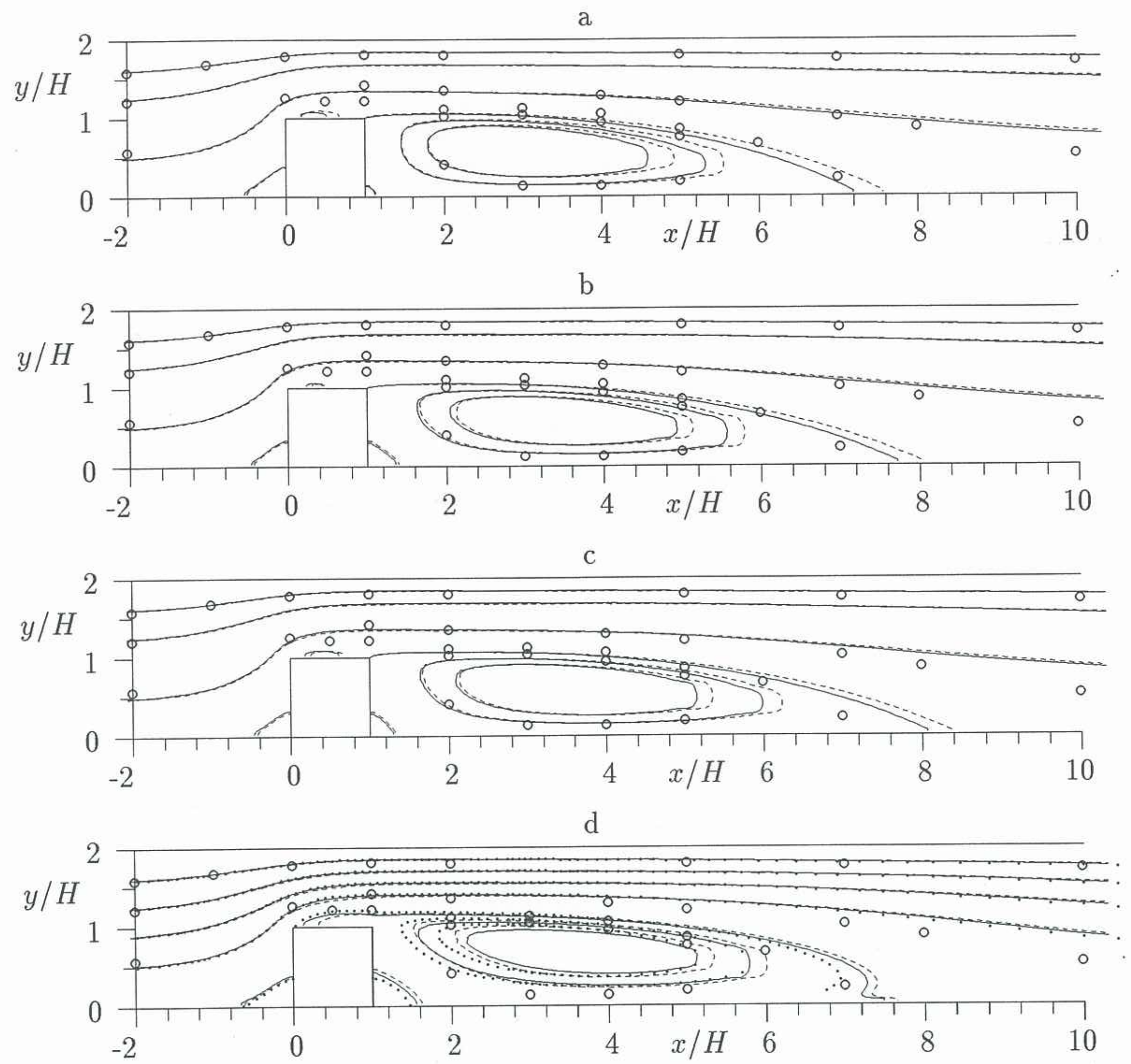

Figure 3 : Streamlines near the obstacle. Computations by means of the RSM models: (a) BM, (b) AMN, DMN, (d) DM ; solid lines correspond to calculations at $\delta = H/48$ with upwind scheme for convective terms in equations for all sought quantities, dashed lines correspond to calculations at $\delta = H/48$ with second-order scheme for convective terms in equations for U and V, dotted lines correspond to calculation by DM at the refined grid with $\delta = H/96$.

characteristics. Results of modeling a flow near the obstacle can be used in calculations at natural conditions (e.g., a flow around a building).

ACKNOLEDGEMENT

This work was supported by the Russian Foundation of Basic Research (Grant 96-05-64007).

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