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DECAY RATES OF COHERENT STRUCTURES IN A PLANE TURBULENT WAKE

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ABSTRACT

The rate of decay of coherent structures is characterized by the rate of change of vortex circulation $\mathrm{d}\Gamma/\mathrm{d}t$ and calculated - at constant phase - from the incoherent turbulence statistics for a *nominally* plane turbulent flow. The proposed method is based on the prognostic vorticity transport equation and, consequently, $\mathrm{d}\Gamma/\mathrm{d}t$ is related to the effective turbulent vorticity fluxes. Some experimental data taken in the turbulent near wake of two 2D side-by-side square cylinders are employed to show a good correspondence between the decay rates predicted at constant phase by means of the incoherent turbulence statistics and the decay rates inferred directly from the phase-averaged flow fields at consecutive phases.

INTRODUCTION

The dynamical properties of large-scale organized vortical structures in turbulent flows (often called coherent structures) are closely connected with turbulent vorticity transport. A typical quantity in this regard is the rate of change of vortex circulation $\mathrm{d}\Gamma/\mathrm{d}t$. This paper aims to determine the decay rate of coherent structures - characterized by $\mathrm{d}\Gamma/\mathrm{d}t$ - in a *nominally* plane turbulent incompressible flow with application to a bluff-body wake.

The rate of decay of coherent structures depends on the intensity of turbulent vorticity transport. Neglecting the effect of viscosity the decay rate can be related to the effective turbulent vorticity fluxes introduced in Kolář, Lyn and Rodi (1997, hereafter referred to as KLR). Consequently, the prediction of the decay rates of coherent structures by means of the turbulence statistics at constant phase is proposed.

The recently published experimental data concerning the plane turbulent wake of two side-by-side square cylinders (KLR) are treated in order to permit a comparison of the decay rates carried out by two methods, namely:

1) predicted by the proposed method (i.e. using the turbulence statistics at constant phase),
2) derived directly from the phase-averaged flow fields at consecutive phases.

The first method elucidates the crucial role of the effective turbulent vorticity fluxes in the decay rate of coherent structures characterized by $\mathrm{d}\Gamma/\mathrm{d}t$, whereas the second one shows the estimate $\Delta\Gamma/\Delta t$ in a straightforward manner and, in the present context, provides a certain verification of the first method. The vorticity transport has been already visualized in terms of the effective turbulent vorticity fluxes in KLR, however, the rate of change of vortex circulation $\mathrm{d}\Gamma/\mathrm{d}t$ has not yet been examined (in the turbulent case).

In addition, some interesting and significant consequences of the present procedure are discussed.

TURBULENT VORTICITY FLUXES

Turbulent transport of coherent vorticity is, in 3D flows, characterized by the turbulent vorticity flux-density tensor (KLR)

$$J_{ij} \equiv \langle u'_j \omega'_i \rangle - \langle u'_i \omega'_j \rangle \qquad (1)$$

and can be expressed purely in terms of the gradients of the Reynolds-stress tensor components (at constant phase). In a *nominally* plane turbulent flow there is only one non-zero component of the coherent-vorticity vector $\langle \omega \rangle$ - below denoted in italics simply as $\langle \omega \rangle$ - and the flux-density tensor *de facto* reduces to a vector quantity.

In the following procedure it is sufficient to take into account only the effective part of the turbulent vorticity flux, i.e. the part having a generally non-zero effect upon $\hat{\mathrm{D}}\langle \omega \rangle/\hat{\mathrm{D}}t$ ($\hat{\mathrm{D}}/\hat{\mathrm{D}}t$ is the substantial, or material, derivative in the phase-averaged sense, $\hat{\mathrm{D}}/\hat{\mathrm{D}}t \equiv \partial/\partial t + \langle \mathbf{u} \rangle \cdot \nabla$). The effective turbulent vorticity fluxes (strictly, the flux-density vector components) are, for a *nominally* plane turbulent flow at constant phase, defined as

$$J^x = \left(\frac{\langle v'^2 \rangle - \langle u'^2 \rangle}{2} \right)_y + \langle u'\, v' \rangle_x , \qquad (2a)$$

$$J^y = \left(\frac{\langle v'^2 \rangle - \langle u'^2 \rangle}{2} \right)_x - \langle u'\, v' \rangle_y , \qquad (2b)$$

(KLR, subscripts stand for spatial partial derivatives, superscripts denote the vector components).

Neglecting the effect of viscosity the corresponding 2D coherent-vorticity transport equation reads

$$\frac{\hat{\mathrm{D}}\langle \omega \rangle}{\hat{\mathrm{D}}t} + J^x_x + J^y_y = 0 . \qquad (3)$$

THE RATE OF CHANGE OF CIRCULATION IN A NOMINALLY PLANE TURBULENT FLOW

The rate of decay of coherent structures is, in the present context, determined as the rate of change of vortex circulation $\mathrm{d}\Gamma/\mathrm{d}t$. The vortex contour in a turbulent flow can be defined as the boundary of the simply connected area A of an individual vortical structure, where the sign of the phase-averaged vorticity remains constant and is above a certain threshold level to avoid relatively noisy data near the outer edge of the structure (Cantwell, Coles 1983). The circulation Γ is then calculated using Stokes's theorem

$$\Gamma = \int_A \langle \boldsymbol{\omega} \rangle \cdot \mathbf{n}\,\mathrm{d}A = \int_A \langle \omega \rangle \mathrm{d}A . \qquad (4)$$

The determination of $\mathrm{d}\Gamma/\mathrm{d}t$ in the laminar case is based on the idea of a closed material curve moving with the fluid (Whitham 1963, Batchelor 1967, Panton 1984). In the turbulent case, it is assumed that 'a coherent structure is a connected turbulent fluid mass with instantaneously phase-correlated vorticity over its spatial extent' (Hussain 1986). Hence, the determination of $\mathrm{d}\Gamma/\mathrm{d}t$ should be considered in the 'phase-averaged sense'.

For a vector field $\mathbf{f}$ and a material surface A it follows from the kinematics of surface integrals in the frame of the classical field theories (Truesdell, Toupin 1960) that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_A \mathbf{f}\cdot\mathbf{n}\,\mathrm{d}A = \int_A \left(\frac{\mathrm{D}\mathbf{f}}{\mathrm{D}t} - \mathbf{f}\cdot\mathrm{grad}\,\mathbf{u} + \mathbf{f}\,\mathrm{div}\,\mathbf{u} \right)\cdot\mathbf{n}\,\mathrm{d}A . \qquad (5)$$

Returning to the problem of a coherent motion one may replace $\mathbf{f}$ by $\langle \boldsymbol{\omega} \rangle$, $\mathrm{D}/\mathrm{D}t$ by $\hat{\mathrm{D}}/\hat{\mathrm{D}}t$ and $\mathbf{u}$ by $\langle \mathbf{u} \rangle$. Considering a *nominally* plane turbulent incompressible flow the tilting-and-stretching term $\langle \boldsymbol{\omega} \rangle \cdot \mathbf{grad}\langle \mathbf{u} \rangle$ and the compressing term $\langle \boldsymbol{\omega} \rangle \mathbf{div}\langle \mathbf{u} \rangle$ do not enter the RHS of (5). The rate of change of circulation is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int_A \langle \boldsymbol{\omega} \rangle \cdot \mathbf{n}\,\mathrm{d}A = \int_A \frac{\hat{\mathrm{D}}\langle \omega \rangle}{\hat{\mathrm{D}}\mathrm{t}}\mathrm{d}A , \qquad (6)$$

and substitution of (3) into (6) yields the final relation between $\mathrm{d}\Gamma/\mathrm{d}t$ and the effective turbulent vorticity fluxes

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = -\int_A \left(J^x_x + J^y_y \right) \mathrm{d}A \qquad (7)$$

where J^x, J^y are defined by (2a, b).

DECAY RATES OF COHERENT STRUCTURES IN A PLANE TURBULENT WAKE

The experimental data concerning the turbulent wake of two 2D side-by-side square cylinders (KLR, figure 1) are treated in order to permit a comparison of the decay rates carried out by two methods mentioned already in the Introduction.

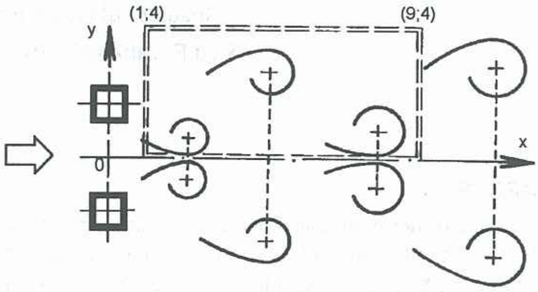

Figure 1 : Sketch of the wake of two side-by-side square cylinders: in-antiphase mode (the measurement region of KLR is framed, with relative dimensions).

KLR provides the phase-averaged characteristics of the near-wake flow at a Reynolds number (based on the average approach velocity U_0 and the cylinder diameter D) ≈ 23 100, with the number of phases assigned as n = 8, using a two-component LDV system and an upstream-trigger method. The work was focussed on the single case of a gap/diameter ratio of 2, for which the resulting individual vortex streets are coupled so as to yield a flow predominantly symmetric about the wake centreline. The phase-averaged characteristics were obtained for the dominant in-antiphase mode (with reference to figure 1, the structures shed towards the wake centreline are termed inner structures, whilst the structures shed towards the freestream are termed outer structures).

The decay rate of an individual vortical structure is predicted at constant phase by means of the incoherent turbulence statistics through the following discretization scheme of the surface integral of (7) (δA is the areal unit of a discretization mesh, i, j are summation indices)

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} \approx -\sum_{i,j} \left(J^x_x + J^y_y \right)_{ij} (\delta A)_{ij} \qquad (8)$$

where J^x, J^y defined by (2a, b) are adequately discretized. The summation concerns exclusively the simply connected area of an individual vortex where the sign of the phase-averaged vorticity remains constant and is above a certain threshold level.

Figure 2 shows the comparison of the decay rates given by (8) with the values of $\Delta\Gamma/\Delta t$ derived directly from the phase-averaged flow fields at consecutive phases.

Explicitly, figure 2 compares the values of predicted decay rates, non-dimensionalized by the average approach velocity U_0 and the cylinder diameter D, given by

$$-\sum_{i,j}\left(\frac{\left(J_x^x+J_y^y\right)D^2}{U_0^2}\right)_{ij}\left(\frac{\delta A}{D^2}\right)_{ij}$$

with the non-dimensionalized values of $\Delta\Gamma/\Delta t$ calculated from the phase-averaged flow fields at two consecutive phases, denoted by subscripts k, k+1, for the point k+0.5 (the equal-sized time difference between consecutive phases is the average vortex-shedding period T divided by the number of phases assigned n) expressed as

$$\left(\left[\sum_{i,j}\left(\frac{\langle\omega\rangle D}{U_0}\right)_{ij}\left(\frac{\delta A}{D^2}\right)_{ij}\right]_{k+1} - \left[\sum_{i,j}\left(\frac{\langle\omega\rangle D}{U_0}\right)_{ij}\left(\frac{\delta A}{D^2}\right)_{ij}\right]_{k}\right)\cdot\frac{nD}{TU_0}$$

With regard to the experimental method adopted for obtaining the employed database (see KLR) the results indicate a good correspondence between the decay rates predicted at constant phase by means of the incoherent turbulence statistics and the decay rates inferred from the phase-averaged flow fields at consecutive phases.

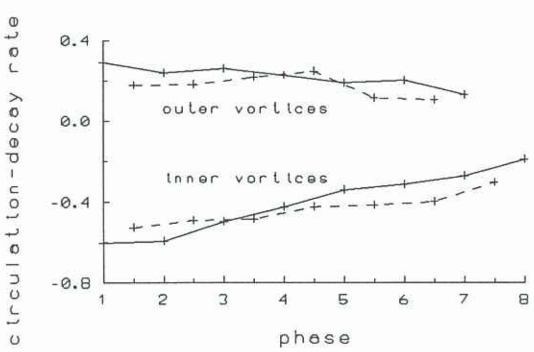

Figure 2 : The (non-dimensionalized) rate of decay of vortex circulation $\mathrm{d}\Gamma/\mathrm{d}t$: predicted at constant phase by means of the incoherent turbulence statistics (solid line), direct estimates calculated as $\Delta\Gamma/\Delta t$ from the phase-averaged flow fields at consecutive phases (dashed line). The phase is, for both inner and outer vortices, formally denoted from 1 to 8. Phase 1 corresponds to the most upstream observed position of an individual vortex fully within the measurement region (figure 1). The sign of circulation is taken into account and determines the sign of the values of circulation-decay rates.

DISCUSSION

The prediction of decay rates seems to be dependent upon the second spatial partial derivatives of the Reynolds-stress tensor components. However, basically, it depends merely upon the first partial derivatives. If Green's theorem is applied to (7) it holds that

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}t} &= \int_A \frac{\hat{\mathrm{D}}\langle\omega\rangle}{\hat{\mathrm{D}}t}\,\mathrm{d}A \\ &= -\int_A \left(J_x^x + J_y^y\right)\mathrm{d}A \\ &= \oint_C \left(J^y\,\mathrm{d}x - J^x\,\mathrm{d}y\right)\end{aligned} \qquad (9)$$

where C denotes a closed vortex contour. According to (9) the decay rate is determined only by the values of J^x, J^y at the vortex contour C. Consequently, the rate of change of circulation (at constant phase) depends merely upon the (plane) structure of incoherent turbulence (anisotropy, inhomogeneity) at the vortex contour. Note that for the case of purely viscous diffusion the rate of change of circulation depends (besides kinematic viscosity) only on the spatial rates of vorticity changes at the vortex contour (Whitham 1963, Batchelor 1967).

The proposed determination of $\mathrm{d}\Gamma/\mathrm{d}t$ can be further extended to include 3D turbulent flows. This extension is beyond the scope of the present paper.

CONCLUSIONS

The rate of decay of coherent structures in a *nominally* plane turbulent (incompressible) flow is calculated as the rate of decay of vortex circulation from the turbulence statistics at constant phase. The proposed method is based on the prognostic vorticity transport equation and, consequently, $\mathrm{d}\Gamma/\mathrm{d}t$ is related to the effective turbulent vorticity fluxes expressed in terms of the gradients of the Reynolds-stress tensor components. The decay rate (at constant phase) depends merely upon the structure of incoherent turbulence (anisotropy, inhomogeneity) at the vortex contour.

The phase-averaged characteristics of the plane turbulent wake of two side-by-side square cylinders show a good correspondence between the decay rates predicted at constant phase by means of the incoherent turbulence statistics and the decay rates derived directly from the phase-averaged flow fields at consecutive phases.

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